# EFFECT OF TENSOR NUCLEON-NUCLEON FORCES ON THE NUCLEON-NUCLEUS OPTICAL POTENTIAL 

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#### Abstract

In the approximation of unpolarized nuclear matter, the optical potential for nucleon-nucleus scattering is calculated on the basis of the effective Skyrme interaction with allowance for tensor nucleon-nucleon forces. It is shown that the tensor Skyrme forces make a significant contribution to the imaginary part of the optical potential. The effect of tensor nucleon-nucleon forces on the radial distribution of the imaginary part of the optical potential is investigated by considering the example of elastic neutron scattering by ${ }^{40} \mathrm{Ca}$ nuclei at scattering energies of about a few tens of megaelectronvolts.


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## 1. INTRODUCTION

In the approximation of nuclear matter, the nucleonnucleus optical potential obtained by calculating the mass operator for the single-particle Green's function $[1,2]$ is considered here for the effective densitydependent nucleon-nucleon Skyrme interaction [3-5]. It is well known [5, 6] that the effective nucleonnucleon Skyrme interaction involves components describing tensor interaction between intranuclear nucleons. However, tensor nucleon-nucleon forces are usually disregarded in studying the structure of nuclei and various collective phenomena in them [7], as well as nucleon-nucleus scattering [8-10]. It should also be noted that the disregard of tensor nucleonnucleon forces is characteristic of various calculations based on Skyrme forces or on other effective nucleonnucleon interactions [7]. In many cases, this is justified, since any density-dependent effective nucleonnucleon interaction, including the Skyrme interaction, takes partly into account the contribution of tensor forces [11]. For example, the effect of tensor Skyrme forces on the single-particle spectra of some even-even nuclei was analyzed in [5], where it was shown that the contribution of these forces is not decisive for the features of these spectra. The optical potential for the interaction of nucleons with even-even nuclei was investigated in [12] without taking into account tensor forces. As was shown in [13], however, the tensor component of the effective Skyrme interaction makes a significant contribution to the central and the tensor spin-spin potential for the interaction of nucleons with odd nuclei. Hence, it is of particular interest to investigate the effect of tensor Skyrme
forces on the optical potential for the interaction of nucleons with even nuclei. In the present study, the nucleon-nucleus optical potential is analyzed on the basis of a calculation of the mass operator for the single-particle Green's function [8-10, 12], this calculation being performed with allowance for the tensor component of the effective Skyrme nucleon-nucleon interaction. It is shown, among other things, that, in the Hartree-Fock approximation (within the method used here, this is the zero-order approximation [12]), the tensor nucleon-nucleon interaction does not contribute to the mass operator that is, to the real part of the optical potential. In this mass operator, the expression that determines the imaginary part of the optical potential develops, however, in the second order of perturbation theory in the effective nucleonnucleon interaction [12], additional terms that are quadratic in the parameters of the tensor Skyrme interaction. The effect of the tensor Skyrme forces on the radial dependence of the optical potential is analyzed here by considering the example of neutron scattering by ${ }^{40} \mathrm{Ca}$ nuclei.

## 2. OPTICAL POTENTIAL

Let us represent the effective Skyrme nucleon-nucleon interaction in the form $[4,5]$

$$
\begin{equation*}
v=v_{1}+v_{2}(\rho)+v_{t e}+v_{t o}, \tag{1}
\end{equation*}
$$

where $v_{1}$ and $v_{2}(\rho)$ are the components that are, respectively, independent of and dependent on the density $\rho$, while $v_{t e}$ and $v_{t o}$ are, respectively, the even and the odd component of the tensor interaction. The terms appearing in expression (1) are given by

[^0]\[

$$
\begin{gather*}
v_{1}=t_{0}\left(1+x_{0} P_{\sigma}\right) \delta(\mathbf{r})+\frac{1}{2} t_{1}\left(1+x_{1} P_{\sigma}\right)\left[\mathbf{k}^{\prime 2} \delta(\mathbf{r})+\delta(\mathbf{r}) \mathbf{k}^{2}\right]+t_{2}\left(1+x_{2} P_{\sigma}\right) \mathbf{k}^{\prime} \delta(\mathbf{r}) \mathbf{k}+ \\
+i W_{0}\left[\mathbf{k}^{\prime} \times \delta(\mathbf{r}) \mathbf{k}\right]\left(\boldsymbol{\sigma}_{1}+\boldsymbol{\sigma}_{2}\right)  \tag{2}\\
v_{2}(\rho)=\frac{1}{6} t_{3}\left(1+x_{3} P_{\sigma}\right) \rho^{\gamma}(\mathbf{R}) \delta(\mathbf{r})+\frac{1}{2} t_{4}\left(1+x_{4} P_{\sigma}\right)\left[\mathbf{k}^{\prime 2} \rho(\mathbf{R}) \delta(\mathbf{r})+\delta(\mathbf{r}) \rho(\mathbf{R}) \mathbf{k}^{2}\right]+t_{5}\left(1+x_{5} P_{\sigma}\right) \mathbf{k}^{\prime} \rho(\mathbf{R}) \delta(\mathbf{r}) \mathbf{k}  \tag{3}\\
v_{t e}=\frac{1}{2} T\left\{\left[\left(\boldsymbol{\sigma}_{1} \cdot \mathbf{k}^{\prime}\right)\left(\boldsymbol{\sigma}_{2} \cdot \mathbf{k}^{\prime}\right)-\frac{1}{3}\left(\boldsymbol{\sigma}_{1} \cdot \boldsymbol{\sigma}_{2}\right) \mathbf{k}^{\prime 2}\right] \delta(\mathbf{r})+\delta(\mathbf{r})\left[\left(\boldsymbol{\sigma}_{1} \cdot \mathbf{k}\right)\left(\boldsymbol{\sigma}_{2} \cdot \mathbf{k}\right)-\frac{1}{3}\left(\boldsymbol{\sigma}_{1} \cdot \boldsymbol{\sigma}_{2}\right) \mathbf{k}^{2}\right]\right\}  \tag{4}\\
v_{t o}=U\left\{\left(\boldsymbol{\sigma}_{1} \cdot \mathbf{k}^{\prime}\right) \delta(\mathbf{r})\left(\boldsymbol{\sigma}_{2} \cdot \mathbf{k}\right)-\frac{1}{3}\left(\boldsymbol{\sigma}_{1} \cdot \boldsymbol{\sigma}_{2}\right)\left(\mathbf{k}^{\prime} \cdot \delta(\mathbf{r}) \mathbf{k}\right)\right\} \tag{5}
\end{gather*}
$$
\]

where the notation used is identical to that in $[4,5,7,9,10]$. In the approximation of unpolarized nuclear matter (that is, for even-even nuclei), the real part of the optical potential $U_{\alpha \alpha}$ based on the interaction in (1) with $v_{t e}=v_{t o}=0$ was calculated in [12]. The corresponding expression for $U_{\alpha \alpha}$ presented in the Appendix.

Since the potential $U_{\alpha \alpha}$ determined by the antisymmetrized diagonal matrix elements of the interaction in (1) [7, 12], the tensor nucleon-nucleon interaction specified by Eqs. (4) and (5) does not contribute to the real part of the optical potential in the approximation of unpolarized nuclear matter.

In the second order of perturbation theory in the effective nucleon-nucleon interaction, the imaginary part of the mass operator for the single-particle

Green's function is given by $[8-10,12]$

$$
\begin{align*}
& M_{\alpha \alpha}(\varepsilon)=-\pi \sum_{\lambda \mu \nu}\langle\alpha \nu| v|\lambda \mu\rangle\langle\lambda \mu| v(1-P)|\alpha \nu\rangle \times \\
& \quad \times\left(1-n_{\lambda}\right)\left(1-n_{\mu}\right) n_{\nu} \delta\left(\varepsilon+\varepsilon_{\nu}-\varepsilon_{\lambda}-\varepsilon_{\mu}\right) \tag{6}
\end{align*}
$$

where $\alpha, \nu(\lambda, \mu)$ are quantum numbers that characterize states of two interacting nucleons, $\varepsilon_{\mu}$ stands for single-particle energies, $n_{\mu}$ are occupation numbers, and $P$ is the operator executing the permutations of spatial, spin, and isospin variables.
Substituting the Skyrme interaction specified by Eqs.(1)-(5) into (6) and replacing single-particle wave functions by plane waves, we represent the imaginary part of the optical potential in the form

$$
\begin{equation*}
W_{\alpha \alpha}(r)=-\frac{1}{64 \pi^{5}} \sum_{i=1}^{7} W_{i} \tag{7}
\end{equation*}
$$

$$
\begin{align*}
& W_{1}=\left(2 g_{00}+\frac{1}{18} g_{33} \rho^{2 \gamma}+\frac{2}{3} g_{03} \rho^{\gamma}\right)\left[I_{1}\left(\tau_{\alpha}, n\right)+I_{1}\left(\tau_{\alpha}, p\right)\right]-\left(2 h_{00}+\frac{1}{18} h_{33} \rho^{2 \gamma}+\frac{2}{3} h_{03} \rho^{\gamma}\right) I_{1}\left(\tau_{\alpha}, \tau_{\alpha}\right),  \tag{8}\\
& W_{2}=\left(2 g_{01}+2 g_{04} \rho+\frac{1}{3} g_{13} \rho^{\gamma}+\frac{1}{3} g_{34} \rho^{\gamma+1}\right)\left[I_{2}\left(\tau_{\alpha}, n\right)+I_{2}\left(\tau_{\alpha}, p\right)\right]- \\
& -\left(2 h_{01}+2 h_{04} \rho+\frac{1}{3} h_{13} \rho^{\gamma}+\frac{1}{3} h_{34} \rho^{\gamma+1}\right) I_{2}\left(\tau_{\alpha}, \tau_{\alpha}\right),  \tag{9}\\
& W_{3}=\left(\frac{1}{2} g_{11}+\frac{1}{2} g_{44} \rho^{2}+g_{14} \rho+\frac{1}{3} T^{2}\right)\left[I_{3}\left(\tau_{\alpha}, n\right)+I_{3}\left(\tau_{\alpha}, p\right)\right]- \\
& -\left(\frac{1}{2} h_{11}+\frac{1}{2} h_{44} \rho^{2}+h_{14} \rho+\frac{1}{3} T^{2}\right) I_{3}\left(\tau_{\alpha}, \tau_{\alpha}\right),  \tag{10}\\
& W_{4}=2\left(2 g_{02}+2 g_{05} \rho+\frac{1}{3} g_{23} \rho^{\gamma}+\frac{1}{3} g_{35} \rho^{\gamma+1}\right)\left[I_{4}\left(\tau_{\alpha}, n\right)+I_{4}\left(\tau_{\alpha}, p\right)\right],  \tag{11}\\
& W_{5}=2\left(g_{12}+g_{15} \rho+g_{24} \rho+g_{45} \rho^{2}+\frac{2}{3} U T\right)\left[I_{5}\left(\tau_{\alpha}, n\right)+I_{5}\left(\tau_{\alpha}, p\right)\right],  \tag{12}\\
& W_{6}=2\left(g_{22}+g_{55} \rho^{2}+2 g_{25} \rho+\frac{2}{3} U^{2}\right)\left[I_{6}\left(\tau_{\alpha}, n\right)+I_{6}\left(\tau_{\alpha}, p\right)\right]+ \\
& +2\left(h_{22}+h_{55} \rho^{2}+2 h_{25} \rho+\frac{2}{3} U^{2}\right) I_{6}\left(\tau_{\alpha}, \tau_{\alpha}\right),  \tag{13}\\
& W_{7}=\left(4 W_{0}^{2}+U^{2}-T^{2}\right)\left[I_{7}\left(\tau_{\alpha}, n\right)+I_{7}\left(\tau_{\alpha}, p\right)\right]+\left(4 W_{0}^{2}+U^{2}+T^{2}\right) I_{7}\left(\tau_{\alpha}, \tau_{\alpha}\right), \tag{14}
\end{align*}
$$

where $\rho=\rho(r), g_{i j}=t_{i} t_{j}\left(1+x_{i} x_{j}+\frac{x_{i}+x_{j}}{2}\right)$, $h_{i j}=t_{i} t_{j}\left(x_{i}+x_{j}+\frac{1+x_{i} x_{j}}{2}\right), i, j=0,1, \ldots, 5 ;$ and

$$
\begin{array}{r}
I_{i}\left(\tau_{\alpha}, \tau_{\mu}\right)= \\
=\int \mathrm{d} \mathbf{K}_{\mu} \mathrm{d} \mathbf{K}_{\lambda} \mathrm{d} \mathbf{K}_{\nu} f_{i}\left(\mathbf{K}_{\alpha}, \mathbf{K}_{\mu}, \mathbf{K}_{\lambda}, \mathbf{K}_{\nu}\right) \times \\
\times \delta\left(E+\varepsilon_{\mu}-\varepsilon_{\lambda}-\varepsilon_{\nu}\right) \delta\left(\mathbf{K}_{\alpha}+\mathbf{K}_{\mu}-\mathbf{K}_{\lambda}-\mathbf{K}_{\nu}\right) \times \\
\times n_{\mu}\left(1-n_{\lambda}\right)\left(1-n_{\nu}\right), i=1, \ldots, 7 . \tag{15}
\end{array}
$$

In expressions (15) $\tau_{\mu}$ is the isospin index of the $\mu$ state ( $\tau_{\mu}=n$ for neutrons and $\tau_{\mu}=p$ for protons) with $\tau_{\lambda}=\tau_{\alpha}, \tau_{\nu}=\tau_{\mu}$, while the functions $f_{i}\left(\mathbf{K}_{\alpha}, \mathbf{K}_{\mu}, \mathbf{K}_{\lambda}, \mathbf{K}_{\nu}\right)(i=1, \ldots, 7)$ are given by [8]

$$
\begin{array}{r}
f_{1}=1, f_{2}=\mathbf{K}_{\alpha \mu}^{2}+\mathbf{K}_{\lambda \nu}^{2}, f_{3}=\left(\mathbf{K}_{\alpha \mu}^{2}+\mathbf{K}_{\lambda \nu}^{2}\right)^{2} \\
f_{4}=\mathbf{K}_{\alpha \mu} \mathbf{K}_{\lambda \nu}, f_{5}=\left(\mathbf{K}_{\alpha \mu}^{2}+\mathbf{K}_{\lambda \nu}^{2}\right)\left(\mathbf{K}_{\alpha \mu} \mathbf{K}_{\lambda \nu}\right) \\
f_{6}=\left(\mathbf{K}_{\alpha \mu} \mathbf{K}_{\lambda \nu}\right)^{2}, f_{7}=\left(\mathbf{K}_{\alpha \mu} \times \mathbf{K}_{\lambda \nu}\right)^{2} \tag{16}
\end{array}
$$

where $\mathbf{K}_{\alpha \mu}=\left(\mathbf{K}_{\alpha}-\mathbf{K}_{\mu}\right) / 2, \mathbf{K}_{\lambda \nu}=\left(\mathbf{K}_{\lambda}-\mathbf{K}_{\nu}\right) / 2$.
The integrals in (15) can be calculated analytically [8]. For the case of symmetric nuclear matter (that is, for nuclear matter consisting of an equal number of neutrons and protons) considered below, the corresponding expressions are presented in the Appendix. Thus, we see that, in expressions (10) and (12)-(14), which determine the imaginary part of the nucleonnucleus optical potential, there arise terms describing the contribution of the tensor nucleon-nucleon interaction [recall that it is specified by Eqs.(4) and (5)], which are quadratic in the parameters $T$ and $U$ of this interaction.

## 3. RESULTS

For the example of neutron scattering by ${ }^{40} \mathrm{Ca}$ nuclei, in which case the approximation of unpolarized symmetric nuclear matter is valid, we further consider the effect of tensor nucleon-nucleon forces on the radial distribution of the imaginary part of the optical potential.
As was mentioned above, virtually no parametrizations that are currently used for the effective Skyrme interaction include the parameters $T$ and $U$, which characterize the strength of tensor forces. They were taken into account only in $[5,6]$; it should be noted that three-particle velocity-dependent Skyrme forces were used in [6], whereas a two-particle densitydependent effective interaction is employed here.
Two approaches to parametrizing tensor Skyrme forces were proposed in [5]. The first approach reduces to calculating the parameters $T$ and $U$ on the basis of a specific realistic potential for free nucleonnucleon scattering, while the second one consists in fitting these parameters on the basis of an analysis of the single-particle spectra of the ${ }^{48} \mathrm{Ca},{ }^{56} \mathrm{Ni}$, and ${ }^{208} \mathrm{~Pb}$ nuclei.
In [13], both these parametrizations of the tensor forces were used in studying the real part of the optical potential for the interaction of nucleons with odd
nuclei. As a result, it was shown that the second approach is preferable in describing experimental data on elastic nucleon-nucleus scattering. In this study, we therefore use the $T$ and $U$ values that were obtained from the analysis of the single-particle nuclear spectra.
According to [5], the admissible values of the parameters $T$ and $U$ (under the condition that the remaining parameters of the effective interaction (1) are preset) are constrained as

$$
\begin{equation*}
\alpha_{0}+\beta_{0} \geq 0,-80 \leq \alpha_{0} \leq 0,0 \leq \beta_{0} \leq 80 \tag{17}
\end{equation*}
$$

where

$$
\begin{gather*}
\alpha_{0}=\frac{5}{12} U+\frac{1}{8}\left(t_{1}-t_{2}\right)-\frac{1}{8}\left(t_{1} x_{1}+t_{2} x_{2}\right),  \tag{18}\\
\beta_{0}=\frac{5}{24}(T+U)-\frac{1}{8}\left(t_{1} x_{1}+t_{2} x_{2}\right) . \tag{19}
\end{gather*}
$$

In the particular case of $\alpha_{0}=\beta_{0}=0$, we obtain $T=-U=163.5 \mathrm{MeV} \mathrm{fm}{ }^{5}$ for the $S k M^{*}$ parametrization [14] and $T=-U=191.574 \mathrm{MeV} \mathrm{fm}{ }^{5}$ for the $S k a$ parametrization [15].

Table 1. Volume integrals and root-mean-square radii of the imaginary part of the optical potential for elastic neutron scattering by ${ }^{40}$ Ca nuclei at $E=0$ and $E=50 \mathrm{MeV}$ according to the calculation performed with $(W(r))$ and without $\left(W_{0}(r)\right)$ allowance for the tensor Skyrme forces; (also presented are the analogous results for the phenomenological optical potential $\tilde{W}(r)$ [18])

|  |  | $J_{W}, \mathrm{MeV} \mathrm{fm}^{3}$ |  | $r_{W}, \mathrm{fm}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $S^{*} k M^{*}$ | $W(r)$ | 75.40 | 462.40 | 4.03 | 4.39 |
|  | $W_{0}(r)$ | 71.31 | 414.89 | 4.08 | 4.50 |
| $S k a$ | $W(r)$ | 43.91 | 268.48 | 4.06 | 4.34 |
|  | $W_{0}(r)$ | 40.75 | 230.27 | 4.12 | 4.48 |
| $S 3 m$ | $W(r)$ | 57.99 | 374.59 | 3.90 | 4.18 |
|  | $W_{0}(r)$ | 55.08 | 312.34 | 3.94 | 4.27 |
| $S 4$ | $W(r)$ | 25.93 | 170.76 | 4.05 | 4.20 |
|  | $W_{0}(r)$ | 23.68 | 141.54 | 4.11 | 4.31 |
| $S 5$ | $W(r)$ | 22.29 | 155.97 | 4.09 | 4.20 |
|  | $W_{0}(r)$ | 20.18 | 127.36 | 4.15 | 4.29 |
|  | $\tilde{W}(r)$ | 52.20 | 81.64 | 4.84 | 4.35 |

In order to characterize the radial distribution of $W(r)$, the values of the volume integrals $J_{W}$,

$$
\begin{equation*}
J_{W}=-\frac{1}{A} \int \mathrm{~d}^{3} r W(r) \tag{20}
\end{equation*}
$$

where $A$ is the number of intranuclear nucleons, and the values of the root-mean-square radii $r_{W}$,

$$
\begin{equation*}
r_{W}=\frac{\int \mathrm{d}^{3} r r^{2} W(r)}{\int \mathrm{d}^{3} r W(r)} \tag{21}
\end{equation*}
$$

are listed in Table 1. We note that the inclusion of the tensor Skyrme forces leads to an increase in the volume integrals and to a decrease in the root-meansquare radii, the relative contribution of the tensor
forces becoming greater at higher scattering energy. Table 1 also displays the values of $J_{W}$ and $r_{W}$ for the $S 3 m, S 4$, and $S 5$ forces [17] and for the phenomenological optical potential (last row in the table) that was obtained from an analysis of experimental data on elastic nucleon-nucleus scattering [18]. The best agreement is achieved if we use the $S 4$ and $S 5$ forces (up to 50 MeV ) and the $S k a$ and $S 3 m$ forces (up to 20 MeV ). At the same time, the best description of the real part of the optical potential (A.1) is obtained with the $S 3 m, S k a$, and $S k M^{*}$ Skyrme forces (Table 2).

Table 2. Volume integrals and root-mean-square radii of the real part of the optical potential for elastic neutron scattering by ${ }^{40}$ Ca nuclei at $E=0$ and $E=50 \mathrm{MeV}$ according to the calculation that employs various parametrizations of the Skyrme forces (given additionally in the last row are the analogous results for the phenomenological optical potential from [18])

|  | $J_{U}, \mathrm{MeV} \mathrm{fm}^{3}$ |  | $r_{U}, \mathrm{fm}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| SkM | 498.75 | 426.93 | 3.91 | 3.98 |
| $S k a$ | 464.29 | 315.05 | 3.89 | 4.01 |
| $S 3 m$ | 463.15 | 374.59 | 3.82 | 3.88 |
| $S 4$ | 431.01 | 206.92 | 3.86 | 4.02 |
| $S 5$ | 420.47 | 145.30 | 3.87 | 4.05 |
|  | 500.67 | 373.75 | 4.06 | 4.06 |

In Table 2, the quantities $J_{U}$ and $r_{U}$ are defined in the same way as the quantities $J_{W}(20)$ and $r_{W}$ (21) and the values in the last row correspond to the phenomenological optical potential from [18].

## 4. CONCLUSIONS

Thus, we have demonstrated that the tensor nucleonnucleon forces determine, to a considerable extent, the imaginary part of the optical potential for nucleon-nucleus scattering, the main contribution coming from the component of the tensor forces that corresponds to the interaction of two nucleons in odd states of their relative motion.
The present analysis has revealed that, at least for $E<20 \mathrm{MeV}$, the optical potential for nucleonnucleus scattering can be calculated by using the $S 3 m$ and $S k a$ forces. In order to achieve better agreement between the microscopic optical potential and phenomenological ones, it seems necessary to calculate simultaneously the basic properties of nuclei and the relevant optical potentials. Preliminary calculations show that even a small (smaller than $5 \%$ ) variation in the parameters $t_{0}$ and $t_{3}$ of the Skyrme forces makes it possible to improve considerably the accuracy in describing elastic nucleon-nucleus scattering, basic properties of bound nuclear states concurrently undergoing only minor changes (within 1\%). Another way to improve the results obtained here consists in studying more comprehensively and consistently taking into account those components of the Skyrme
forces that are proportional to the parameters $t_{4}$ and $t_{5}$. These terms make a dominant contribution to the Hartree-Fock potential (and, hence, to the optical potential) only near the nuclear surface and, in principle, enable one to describe better the smearing of the surface nuclear layer.

## APPENDIX

In the approximation of unpolarized nuclear matter, the real part of the optical potential is given by [12]

$$
\begin{gather*}
U_{\alpha \alpha}(r, E)=\frac{m_{\alpha}^{*}}{m_{\alpha}}\left\{g_{0} \rho-h_{0} \rho_{\alpha}+\frac{1}{4} \kappa\left[g_{1}+g_{2}+\left(g_{4}+g_{5}\right) \rho\right]-\right. \\
-\frac{1}{4} \kappa_{\alpha}\left[h_{1}-h_{2}+\left(h_{4}-h_{5}\right) \rho\right]+\frac{1}{6} \rho^{\gamma}\left(g_{3} \rho-h_{3} \rho_{\alpha}\right)+ \\
+\frac{1}{4}\left(g_{4}+g_{5}\right) \rho \kappa-\frac{1}{4}\left(h_{4}-h_{5}\right) \sum_{q} \rho_{q} \kappa_{q}+ \\
\left.+\frac{1}{12} \gamma \rho^{\gamma-1}\left(g_{3} \rho^{2}-h_{3} \sum_{q} \rho_{q}^{2}\right)\right\}+ \\
+\left(1-\frac{m_{\alpha}^{*}}{m_{\alpha}}\right) \frac{M}{M+m_{\alpha}} E \tag{A1}
\end{gather*}
$$

where $m_{\alpha}$ is the mass of the incident nucleon; $M$ is the target-nucleus mass; $E$ is the scattering energy; $\alpha=\{n, p\}$ is the isospin index; $g_{i}=t_{i}(1+$ $\left.x_{i} / 2\right)$ and $h_{i}=t_{i}\left(1 / 2+x_{i}\right)$ with $i=0,1, \ldots, 5$, $\kappa=\frac{3}{5}\left(3 \pi^{2} \rho / 2\right)^{2 / 3} \rho, \kappa_{\alpha}=\frac{3}{5}\left(3 \pi^{2} \rho_{\alpha}\right)^{2 / 3} \rho_{\alpha}$; and $m_{\alpha}^{*}$ is an effective mass that satisfies the relation

$$
\begin{align*}
\frac{m_{\alpha}}{m_{\alpha}^{*}}= & 1+\frac{1}{4} \frac{2 m_{\alpha}}{\hbar^{2}}\left\{\left[g_{1}+g_{2}+\left(g_{4}+g_{5}\right) \rho\right] \rho-\right. \\
& \left.-\left[h_{1}-h_{2}+\left(h_{4}-h_{5}\right) \rho\right] \rho_{\alpha}\right\} \tag{A2}
\end{align*}
$$

In the approximation of symmetric nuclear matter, we have $\rho_{n}=\rho_{p}=\rho / 2, m_{n}^{*}=m_{p}^{*}=m^{*}$, while the integrals in (15) become [8]

$$
\begin{gather*}
I_{1}=\frac{2 m^{*}}{\hbar^{2}} \frac{2 \pi^{2}}{15 K_{\alpha}}\left[\left(5 K_{\alpha}^{2}-7 K_{F}^{2}\right) K_{F}^{3}+\right. \\
\left.+2\left(2 K_{F}^{2}-K_{\alpha}^{2}\right)^{5 / 2} \theta\left(2 K_{F}^{2}-K_{\alpha}^{2}\right)\right],  \tag{A3}\\
I_{2}=\frac{2 m^{*}}{\hbar^{2}} \frac{\pi^{2}}{105 K_{\alpha}}\left[\left(35 K_{\alpha}^{4}-14 K_{\alpha}^{2} K_{F}^{2}-45 K_{F}^{4}\right) K_{F}^{3}+\right. \\
\left.+4\left(K_{\alpha}^{2}+5 K_{F}^{2}\right)\left(2 K_{F}^{2}-K_{\alpha}^{2}\right)^{5 / 2} \theta\left(2 K_{F}^{2}-K_{\alpha}^{2}\right)\right],(\mathrm{A} 4)  \tag{A4}\\
I_{3}=\frac{m^{*}}{\hbar^{2}} \frac{\pi^{2}}{945 K_{\alpha}}\left[\left(315 K_{\alpha}^{6}+441 K_{\alpha}^{4} K_{F}^{2}-\right.\right. \\
\left.-747 K_{\alpha}^{2} K_{F}^{4}-473 K_{F}^{6}\right) K_{F}^{3}- \\
-8\left(5 K_{\alpha}^{4}-20 K_{\alpha}^{2} K_{F}^{2}-43 K_{F}^{4}\right)\left(2 K_{F}^{2}-K_{\alpha}^{2}\right)^{5 / 2} \times \\
\left.\times \theta\left(2 K_{F}^{2}-K_{\alpha}^{2}\right)\right],  \tag{A5}\\
I_{4}=I_{5}=0,  \tag{A6}\\
\left.+711 K_{\alpha}^{2} K_{F}^{4}-803 K_{F}^{6}\right) K_{F}^{3}-16\left(2 K_{\alpha}^{2}-13 K_{F}^{2}\right) \times
\end{gather*}
$$

$$
\begin{gather*}
\left.\times\left(2 K_{F}^{2}-K_{\alpha}^{2}\right)^{7 / 2} \theta\left(2 K_{F}^{2}-K_{\alpha}^{2}\right)\right],  \tag{A7}\\
I_{7}=I_{3}-I_{6}, \tag{A8}
\end{gather*}
$$

where $\theta(x)$ is a theta function, $K_{F}=\left(3 \pi^{2} \rho / 2\right)^{1 / 3}$ is the Fermi momentum, and

$$
\begin{equation*}
K_{\alpha}^{2}=\frac{2 m_{\alpha}}{\hbar^{2}}\left(\frac{M}{M+m_{\alpha}} E-U_{\alpha \alpha}\right) . \tag{A9}
\end{equation*}
$$

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## ВЛИЯНИЕ ТЕНЗОРНЫХ НУКЛОН-НУКЛОННЫХ СИЛ НА НУКЛОН-ЯДЕРНЫЙ ОПТИЧЕСКИЙ ПОТЕНЦИАЛ

## С.М. Кравченко

В приближении неполяризованной ядерной материи рассчитан оптический потенциал нуклон-ядерного рассеяния на основе эффективного взаимодействия Скирма с учетом тензорных нуклон-нуклонных сил. Показано, что тензорные силы Скирма дают существенный вклад в мнимую часть оптического потенциала. На примере упругого рассеяния нейтронов ядрами ${ }^{40} \mathrm{Ca}$ исследовано влияние тензорных нуклон-нуклонных сил на радиальное распределение мнимой части оптического потенциала при энергиях рассеяния в несколько десятков мегаэлектронвольт.

## ВПЛИВ ТЕНЗОРНИХ НУКЛОН-НУКЛОННИХ СИЛ НА НУКЛОН-ЯДЕРНИЙ ОПТИЧНИЙ ПОТЕНЦІАЛ

## С.М. Кравченко

У наближенні неполяризованої ядерної матерії розрахований оптичний потенціал нуклон-ядерного розсіяння на основі ефективної взаємодії Скірма з урахуванням тензорних нуклон-нуклонних сил. Доведено, що тензорні сили Скірма дають істотний внесок в уявну частину оптичного потенціалу. На прикладі пружного розсіяння нейтронів ядрами ${ }^{40} \mathrm{Ca}$ досліджено вплив тензорних нуклон-нуклонних сил на радіальний розподіл уявної частини оптичного потенціалу при енергіях розсіяння в декілька десятків мегаелектронвольт.


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