ACCOUNT OF THE LONGITUDINAL TEMPERATURE IN CYCLOTRON SUPERRADIANCE

O.P. Novak[1]*, ***A.P. Fomina***[2], ***R.I. Kholodov***[1]

[1] *Institute of Applied Physics NAS of Ukraine, 40000, Sumy, Ukraine;*

[2] *Bogolyubov Institute for Theoretical Physics NAS of Ukraine, 03680, Kiev, Ukraine*

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The phenomenon of cyclotron Dicke superradiance (SR) in the inverted system of nonrelativistic electrons in low density magnetized plasma is considered. It is shown, that account of the longitudinal temperature increases the critical electron density which is needed for the nonequilibrium phase transition to the SR regime.

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1. INTRODUCTION

The phenomenon of superradiance (SR) was considered first by Dicke [1] on the example of two-level model. SR was studied in a number of works (see, for example, reviews [2, 3, 4]), but a lot of interesting and physically important questions remain not investigated.

In the Ref. [5] (see, also [6, 7]) the theory of collective coherent SR in the system of inverted electrons occupying high Landau levels [8] in low density magnetized plasma with

$$E_{\perp} = n\hbar\omega_H, \quad n \gg 1, \tag{1}$$

$$\omega_H = \frac{eH}{mc}, \tag{2}$$

has been developed.

It was shown in [5, 6], that under certain conditions the polarization phase transition occurs to the Dicke SR state [1] in such system due to the dipole-dipole interaction between rotating electrons. The phenomenon of SR arises in "coherence domains" with the sizes R_0 smaller than the radiation wavelength λ when all N_0 radiating dipoles gradually align in the same direction due to the dipole-dipole interaction in a "near zone" $R_0 \ll \lambda$. As a result, the total dipole moment of a domain becomes proportional to the number of electrons N_0 and the intensity of collective coherent dipole radiation of a domain increases in N_0 times in contrast to the intensity of uncorrelated dipole radiation and becomes proportional to N_0^2. The transition to such correlated polarized state is similar to the phase transition in magnetics and the Weiss method of mean self-consistent field [9] was used to find the criteria of self-polarization in a such system.

The resulting nonlinear equation is similar to the Weiss equation and determines the threshold of polarization phase transition in a domain on the density of inverted electrons. It is defined by the relation [5, 6]

$$n_e > n_{ec} = \frac{0.18H^2kT}{mc^2E_{\perp}}. \tag{3}$$

In Ref. [7] the SR phenomenon theory is used to explain the nature and the main features of the super power decameter radiation (DCM) of the Jupiter-Io system.

The sporadic DCM radiation of Jupiter was discovered in 1955. Despite considerable progress in studying the features of DCM radiation, there are no generally accepted and consistent answers to many important questions yet. The most important problem is the nature of the coherent collective mechanism of radiation providing a gigantic peak power of the DCM-pulses. It reaches $\sim 10^{17} \div 10^{18}$ erg/s that corresponds to the brightness temperature of the source about $\sim 10^{17}$ K. Introducing the SR mechanism simplifies the problem and allows us to explain the observed power of DCM-pulses without involving of free parameters.

However, in References [6, 7] the authors assumed that electrons in a magnetic field rotate at circular orbits with fixed centers and do not move along the field. The purpose of the present paper is to investigate how the longitudinal motion of electrons affects the main features of the cyclotron SR.

2. ACCOUNT OF LONGITUDINAL TEMPERATURE

It is convenient to consider first the situation when the electrons in a coherence domain move along and opposite to the magnetic field with the same constant speed v. Thus, the electron trajectories are de-

*Corresponding author E-mail address: novak-o-p@ukr.net

fined by the expressions

$$\vec{R}_j(t) = \vec{r}_j(t) + \vec{r}_{j\perp}(t), \quad (4)$$
$$\vec{r}_j(t) = \vec{r}_j(0) \pm \vec{v}t, \qquad \vec{v} = (0,0,v), \quad (5)$$
$$\vec{r}_\perp(t) = r_L[\cos(\omega t + \alpha), \sin(\omega t + \alpha), 0]. \quad (6)$$

Here, $\vec{r_j}$ defines the position of the orbit center and r_L is the Larmor radius. Thus, it is assumed that $N_0/2$ electrons move along the field with the velocity $\vec{v}$ while the others $N_0/2$ electrons move opposite to the field with the same speed.

The potential energy of a trial dipole is

$$U = -\vec{d}_0 \vec{E}, \quad (7)$$

where $\vec{d_0} = e\vec{r}_{0\perp}(t)$ and the total electric field $\vec{E}$ in a near zone decomposes in the sum

$$\vec{E} = \sum_j^{N-1} \frac{3\vec{n}_j(\vec{n}_j\vec{d}_j) - \vec{d}_j}{|\vec{r}_0 - \vec{r}_j|^3}, \quad (8)$$
$$\vec{n}_j = \frac{\vec{r}_0 - \vec{r}_j}{|\vec{r}_0 - \vec{r}_j|}. \quad (9)$$

Averaging (7) over the rotation period one should note that radius vectors $\vec{r}_j$ vary with time slowly in comparison with $\vec{r}_L$. Substituting Eqs. (8), (9) into (7) and averaging over the period we obtain

$$\langle U(\vec{r}_0)\rangle = -\frac{d_0^2}{2}\left[\sum_{j=1}^{N/2} \frac{1 - 3\vec{n}_{jz}^2}{|\vec{r}_0 - \vec{r}_j|^3}\cos(\alpha_0 - \alpha_j) + \sum_{j=N/2}^{N} \frac{1 - 3\vec{n}_{jz}^2}{|\vec{r}_0 - \vec{r}_j|^3}\cos(\alpha_0 - \alpha_j)\right], \quad (10)$$

where $d_0 = er_L$ and the first sum is taken over the electrons which have the same longitudinal velosity as the trial one while the second sum is taken over the other group of electrons.

The position of the trial dipole should not be preferred, therefore it necessary to average (10) over $\vec{r}_0$. After the replacing of variables according to the relations $\{\vec{r}_0, \vec{r}_j\} \to \{\vec{r} = \vec{r}_0 - \vec{r}_j, \vec{R} = \frac{1}{2}(\vec{r}_0 + \vec{r}_j)\}$, the averaged potential energy can be expressed in the form

$$\langle U\rangle = -\frac{d_0^2 n_e}{4}\langle\cos\Delta\alpha\rangle \times \left[\int_{V_{c1}} \frac{r^2 - 3z^2}{r^5} d\vec{r}_j + \int_{V_{c2}} \frac{r^2 - 3z^2}{r^5} d\vec{r}_j\right], \quad (11)$$

where the sums are replaced with the integrals. Note that $\cos(\alpha_0 - \alpha_j)$ is replaced by its averaged over the ensemble value $\langle\cos\Delta\alpha\rangle$ in accordance with the Weiss method [9].

Aligning of dipoles is energetically favourable because it increases the negative contribution to the potential energy $\langle U\rangle$. Therefore, the correlations will occur only in the "coherence domain" defined by the condition

$$r^2 - 3z^2 > 0. \quad (12)$$

In the near-by domains the directions of the average polarization vectors should be close to opposite to minimize the total potential energy of the system. In the similar way magnetics and ferroelectrics are broken into domains too.

As follows from Eq. (12), the "coherence domains" in relative coordinates look like flattened cylinders with conical covers, as shown in Fig. 1,a.

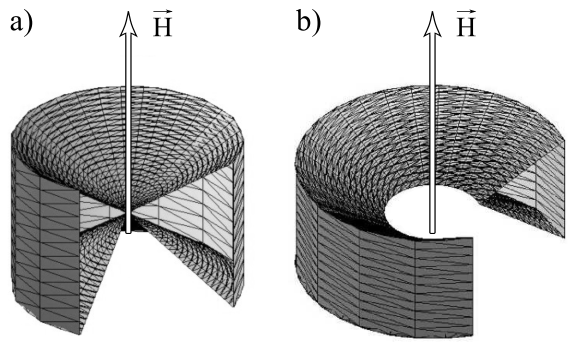

Fig.1. *Overal view of the coherence domains in the cases of zero longitudinal temperature (a) and presence of the motion along the magnetic field (b)*

Both coherence domains V_{c1} and V_{c2} in Eq. (11) have the same shape defined by Eq. (12). However, V_{c2} moves along z axis since the second summand describes the contribution from dipoles that move relative to each other. Consequently, aligning between such dipoles is possible only in the area that belongs to the both volumes during the phasing time τ It results in changing of the coherence domain shape, as shown in Fig. 1,b, Fig. 2.

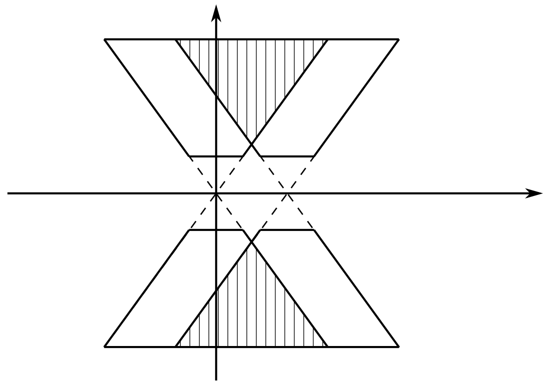

Fig.2. *Changing of the coherence domain due to the electron motion along the magnetic field*

It is convenient to carry out integration in Eq. (11) in cylindrical coordinates.

In the integral over the variable r, the lower limit should be chosen about Larmor radius r_L since at the smaller distance the electron-electron interaction can not be described by dipole formulas. The upper limit should be chosen $\sim R_0$, the characteristic size of the "coherence domain" determined by the condition $r_L \ll R_0 \ll \lambda$. After integration we obtain

$$\langle U \rangle = -\frac{1}{2} d_0^2 n_e \langle \cos \Delta\alpha \rangle J(\eta) \,, \qquad (13)$$

where

$$J(\eta) = \frac{4\pi}{3\sqrt{3}} \left[\ln \frac{\sqrt{2}(x_2 - \chi_1)(x_2 - \chi_2)}{\eta(x_1 - \chi_1)(x_1 - \chi_2)} + \frac{\sqrt{6}x_1 - 3\sqrt{3}}{1 + x_1^2} + \frac{\sqrt{6}x_2 - 3\sqrt{3}}{1 + x_2^2} \right], \quad (14)$$

$$\begin{cases} x_1 = \frac{\sqrt{3}-1}{\sqrt{2}}, & \eta\sqrt{2} < 1; \\ x_1 = \sqrt{1 + (10\eta - \frac{1}{\sqrt{2}})^2} - (10\eta - \frac{1}{\sqrt{2}}), & \eta\sqrt{2} > 1; \end{cases}$$

$$x_2 = \sqrt{1 + (\eta - \frac{1}{\sqrt{2}})^2} - (\eta - \frac{1}{\sqrt{2}}) \qquad (15)$$

and η is the dimensionless longitudinal speed,

$$\eta = \frac{v\tau}{R_0}. \qquad (16)$$

Let us proceed to the case of Maxwell distribution $f(v)$ for the dipole longitudinal speed,

$$f(v) = \sqrt{\frac{m}{2\pi k T_\parallel}} \exp\left(-\frac{mv^2}{2kT_\parallel} \right). \qquad (17)$$

In this case the total electric field of the electron system in Eq. (7) can be written in the form

$$\vec{E} = \sum_i \sum_j \vec{E}_{i,j} \,, \qquad (18)$$

where the sum over particle longitudinal speed is introduced. Following the same procedure as before and introducing the additional averaging over the speed of the trial dipole, we obtain the average interaction energy in the form

$$\langle U \rangle = -\frac{1}{2} e^2 r_L^2 n_e \langle \cos \Delta\alpha \rangle \int_{-\infty}^{\infty} dv' f(v') \int_{-\infty}^{\infty} dv J(|v - v'|) \,, \qquad (19)$$

where the quantity $J(|v - v'|)$ is the previously found integral over the modified coherence volume,

$$J(|v - v'|) = \int_V d\vec{r} \, \frac{r^2 - 3z^2}{r^5}. \qquad (20)$$

After changing of variables according to the relations $\{\eta, \eta'\} \to \{\xi = \eta - \eta', \xi' = \eta'\}$, the averaged potential energy can be expressed in the form

$$\langle U \rangle = -\frac{d_0^2 n_e \langle \cos \Delta\alpha \rangle}{4\eta_0\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-\frac{\xi^2}{4\eta_0^2}} J(|\xi|) d\xi \,, \qquad (21)$$

where the function $J(x)$ is defined by the Eq. (14) and the quantity η_0 is dimensionless longitudinal temperature,

$$\eta_0^2 = \frac{kT_\parallel \tau^2}{mR_0^2}. \qquad (22)$$

Thus, it is seen that the aligning of dipoles with radiation of the released energy is energetically favourable because of the negative contribution of dipole-dipole interaction to the total potential energy of the system. The thermal fluctuations resist to this tendency. They are realized in low density magnetized plasma mainly in the form of plasma fluctuations and Alfven waves. To determine the threshold values of the parameters, when the transition to the polarized state become possible, one can use the Weiss method of mean self-consistent field [9]. According to the results of Ref. [6], the criteria of such transition can be written as

$$\frac{\langle U \rangle}{2kT \langle \cos \Delta\alpha \rangle} > 1. \qquad (23)$$

It is convenient to introduce an auxiliary normalized function $g(\eta_0)$,

$$g(\eta_0) = \frac{\langle U \rangle}{\langle U_{T_\parallel = 0} \rangle} = \frac{3\sqrt{3}}{8\sqrt{\pi^3}} \frac{\int_{-\infty}^{\infty} e^{-\frac{\xi^2}{4\eta_0^2}} J(|\xi|) d\xi}{\eta_0 \ln(\frac{mc^2}{5E_\perp})} \,, \qquad (24)$$

where $\langle U_{T_\parallel=0} \rangle$ is the potential energy of the dipole-dipole interaction at zero longitudinal temperature, obtained in Ref. [6],

$$\langle U_{T_\parallel=0} \rangle = -\frac{2\pi}{3\sqrt{3}} \ln\left(\frac{mc^2}{5E_\perp} \right) n_e d_0^2 \langle \cos \Delta\alpha \rangle. \qquad (25)$$

Evidently, the condition $g(0) = 1$ is true. Function $g(\eta_0)$ has been calculated numerically and it is shown in Fig. 3.

Finally, to determine the criteria of the transition to the SR regime, it is necessary to substitute the found potential energy $\langle U \rangle = \langle U_{T_\parallel=0} \rangle g(\eta_0)$ to the relation (23). With account of Eq. (25) the criteria takes on the form

$$\frac{2\pi}{3\sqrt{3}} \ln\left(\frac{mc^2}{5E_\perp} \right) \frac{mc^2}{H^2} \frac{n_e E_\perp}{kT_\perp} \cdot g(\eta_0) > 1. \qquad (26)$$

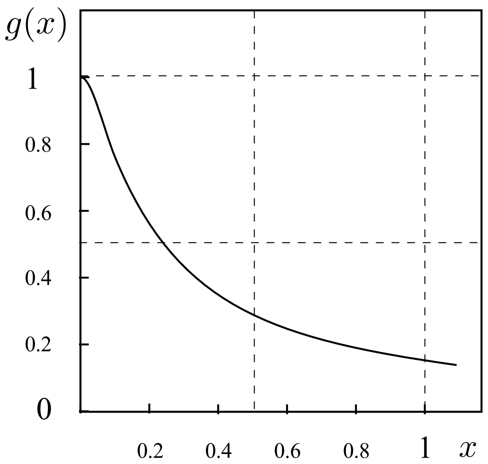

Fig.3. *The dimensionless function $g(x)$ defining the normalized potential energy of the dipole-dipole interaction*

3. PHASING DYNAMICS

As follows from the above consideration, longitudinal motion in low density magnetized plasma results in electron outflow from the coherence domain. Consequently, the total number of phased dipoles at each moment of time t decreases with $T_\|$ growing.

Let $t' < t$ be the moment of time when the number of phased dipoles at zero longitudinal temperature N equals to this number $N'(t)$ at $T_\| \neq 0$,

$$N'(t) = N(t'). \tag{27}$$

To find the number of phased dipoles in the moment $t + dt$ one should take into account the outflow dN:

$$N'(t + dt) = N(t' + dt) - dN(t). \tag{28}$$

Assuming for estimation that the coherence domain is a cylinder with radius R_0, one can find the outflow of electrons having speed withing the interval $(v, v + dv)$ in the form

$$dN_v = 2\pi R^2 dt \cdot v dn_v\,, \tag{29}$$

where the fraction dn_v is given by the Maxwell distribution,

$$dn_v = \frac{N'(t)}{V}\sqrt{\frac{m}{2\pi k T_\|}}\exp\left[-\frac{mv^2}{2kT_\|}\right]dv. \tag{30}$$

Performing simple integration, we obtain the following expression for the total outflow:

$$dN = N'(t)\sqrt{\frac{2}{\pi}}\frac{\eta_0}{\tau}dt. \tag{31}$$

Finally, expanding Eq. (28) into series in dt and taking into account Eq. (27), one can find the following equations determining the sought function $N'(t)$,

$$\frac{dN'(t)}{dt} = \left.\frac{dN(t)}{dt}\right|_{t=t'} - N'(t)\frac{\eta_0}{\tau}\sqrt{\frac{2}{\pi}}, \tag{32}$$

$$N(t') = N'(t). \tag{33}$$

To solve Eqs. (32), the function $N(t)$ describing the phasing at zero longitudinal temperature is needed, which is unknown. Nevertheless, some conclusions of how the longitudinal motion affects the phasing can be inferred from treatment of a model function $N(t)$. Apparently, it should satisfy the following conditions:

1. $N(t) \sim \exp(\frac{t}{\tau})$ when $t \to 0$.
2. $N(t) \to N_0$ when $t \to \infty$.
3. $N(0) \sim 1$.

For example, let us choose the model function in the form

$$N(t) = \frac{N_0}{1 + N_0 \exp(-\dfrac{t}{\tau})}. \tag{34}$$

Then the solution of Eqs. (32), (33) can be expressed as

$$N'(t) = \frac{\gamma N_0}{1 + [\gamma N_0 - 1]\exp(-\gamma \frac{t}{\tau})}\,, \tag{35}$$

where

$$\gamma = 1 - \eta_0\sqrt{\frac{2}{\pi}} \tag{36}$$

and the initial condition is $N'(0) = 1$. The plots of Eq. (35) at different temperatures is shown in Fig. 4.

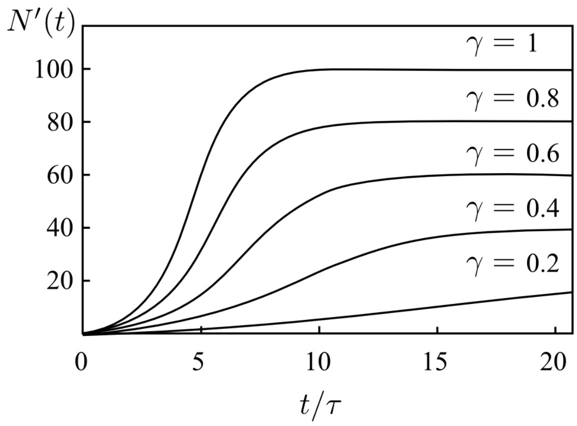

Fig.4. *Phasing dynamics at different longitudinal temperatures, $N_0 = 100$*

Thus, the total amount of phased electrons decreases by a factor γ while the phasing time increases by the factor γ^{-1}. In other words, account of the longitudinal temperature results in the following. The coherence domain size decreasing is proportional to the temperature, while phasing time is in inverse ratio to the temperature.

Moreover, phasing is impossible when $\gamma < 0$. Consequently, the following condition should be met:

$$\eta_0 < \sqrt{\frac{\pi}{2}}. \tag{37}$$

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УЧЕТ ПРОДОЛЬНОЙ ТЕМПЕРАТУРЫ В ЦИКЛОТРОННОМ СВЕРХИЗЛУЧЕНИИ

А.П. Новак, А.П. Фомина, Р.И. Холодов

Рассмотрено явление циклотронного сверхизлучения Дике (СИ) в инвертированной системе нерелятивистских электронов в разреженной замагниченной плазме. Показано, что учет продольной температуры увеличивает критическую концентрацию электронов, необходимую для неравновесного фазового перехода в СИ-режим.

ВРАХУВАННЯ ПОВЗДОВЖНЬОЇ ТЕМПЕРАТУРИ В ЦИКЛОТРОННОМУ НАДВИПРОМІНЮВАННІ

О.П. Новак, А.П. Фоміна, Р.І. Холодов

Розглянуто явище циклотронного надвипромінювання Діке (НВ) в інвертованій системі нерелятивістських електронів у розрідженій замагніченій плазмі. Показано, що врахування поздовжньої температури збільшує критичну концентрацію електронів, необхідну для нерівноважного фазового переходу в НВ-режим.