Section D. QED PROCESSES IN STRONG FIELDS

NEW PERTURBATION THEORY IN QED

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The new perturbation theory in QED without the ultraviolet catastrophe is constructed.

PACS: 21.45+v,11.80.-m

1. INTRODUCTION

In the conventional Dyson-Feynman approach in quantum electrodynamics all quantities are calculated within the perturbation theory with using the zero order photon and electron propagators (see, e.g., the famous monographs [1-5]). Fundamental difficulty of the approach is a divergence of integrals representing radiative corrections to the electron mass, electron charge or power of interaction between electrons and photons. A renormalization procedure allows us to eliminate the divergences with the help of any form of subtracting the infinitely large terms in the perturbation theory. Some of the known physicists (including Feynman [6], Landau and Lifshitz, Gell-Mann [5]) estimated this procedure as incorrect.

There is a number of attempts to solve the problem with divergences in the framework of conventional theory. In the last few years some works was published where the authors find the possibility to avoid the divergence problem as in QED as well as in the other field theories with the help of "clothing" procedure firstly proposed in [7] (see, e.g., [8-12] and references therein). In this connection one should note, that the "clothing" is fulfilled within the conventional perturbation theory. On the other hand, it is interesting to know, whether this difficulty is the property only of the perturbation theory or of the quantum electrodynamics itself? To answer this question one should construct any other form of the perturbation theory, quantitatively different from the Dyson-Feynman approach.

We would like to show that the appropriate form of the theory where the infrared as well ultraviolet divergences turn out to be absent indeed can be constructed. The main idea consists in accounting for the proper part of interaction on the first stage of calculations. The general scheme of the theory was described in [13]. The relativistic units ($\hbar$=1, c=1) are used throughout the paper. The normalization volume is set equal to unity. We use the common notations of the four-dimensional relativistic field theory. The Greek indices run over 0,1,2,3; the Latin one over 1,2,3; $g_{\alpha\beta}$ is the metric of the pseudo-Euclidean space-time with the signature (1,-1,-1,-1).

2. BASIC DEFINITIONS

Consider electron as quantum of the spinor field with the wave operator in usual form

$$\hat{\psi} = \sum_{\lambda,\mathbf{p}} \left[u_{\lambda\mathbf{p}} e^{-i\mathbf{px}} \hat{a}_{\lambda\mathbf{p}} + v_{\lambda\mathbf{p}} e^{i\mathbf{px}} \hat{d}^{+}_{\lambda\mathbf{p}} \right], \tag{1}$$

where $\mathbf{p}$ is the momentum of the electron, $\lambda = \pm 1/2$ denotes the projection of the electron spin on the specific axe of quantization (below we choose it as z-axis). Bispinors in Eq. (1) are defined, e.g., in [5],

$$u_{\lambda\,\mathbf{p}} = \frac{1}{\sqrt{2\varepsilon_{\mathbf{p}}}} \begin{pmatrix} \sqrt{\varepsilon_{\mathbf{p}} + m} \;\; w_\lambda \\ \sqrt{\varepsilon_{\mathbf{p}} - m} \left(\mathbf{n_p}\boldsymbol{\sigma}\right) \;\; w_\lambda \end{pmatrix};$$

$$v_{\lambda\mathbf{p}} = u_{-\lambda,-\mathbf{p}} = \frac{1}{\sqrt{2\varepsilon_{\mathbf{p}}}} \begin{pmatrix} \sqrt{\varepsilon_{\mathbf{p}} - m} \;\; \left(\mathbf{n_p}\boldsymbol{\sigma}\right) w'_\lambda \\ \sqrt{\varepsilon_{\mathbf{p}} + m} \;\; w'_\lambda \end{pmatrix}.$$

Here $\boldsymbol{n_p}$ is the unit vector along the $\mathbf{p}$-direction, $\boldsymbol{\sigma}$ is the vector of Pauli matrices, $\varepsilon_{\mathbf{p}} = p^0 = \left(p^2 + m^2\right)^{1/2}$ is the energy of the electron with the momentum $\mathbf{p}$. The unit spinors w_λ, w'_λ describe the states with the spin projections equal to λ. In Eq. (1) $\hat{a}_{\lambda\mathbf{p}}$ is the electron annihilation operator in the state with the defined polarization, momentum and the energy $\varepsilon_{\mathbf{p}}$, $\hat{d}^{+}_{\lambda\mathbf{p}}$ is the positron creation operator for the same state. All the operators $\hat{a}_{\lambda\mathbf{p}}, \hat{a}^{+}_{\lambda\mathbf{p}}, \hat{d}_{\lambda\mathbf{p}}, \hat{d}^{+}_{\lambda\mathbf{p}}$ obey the standard Fermi commutation relations. In the current density operator,

$$\hat{j}^{\mu} = e : \hat{\bar{\psi}} \gamma^{\mu} \hat{\psi}, \tag{2}$$

where the semicolons mean the normal ordering, e is the electron charge, $\hat{\bar{\psi}} = \hat{\psi}^{+}\gamma^{0}$ is the usual Dirac conjugate operator. In the following we need the Fourier representation of the current density operator. Note, as a rule, this operator does not conserve the spin of the particle. Now we need to construct the "approximate" current operators which have to commute if related to different times. For the one-electron problem the appropriate expression may be written in the form

$$\hat{j}^{\mu}_{\mathbf{q}(0)} = f^{\mu}(\mathbf{q},t)\hat{\rho}_{\mathbf{q}}(t). \tag{3}$$

The 4-vector $f^{\mu}(\mathbf{q},t)$ is defined below and

$$\hat{\rho}_{\mathbf{q}} = \sum_{\lambda,\mathbf{k}} \hat{a}^{+}_{\lambda,\mathbf{k}} \hat{a}_{\lambda,\mathbf{k}+\mathbf{q}}$$

is the Fourier component of the time-independent electron density operator. Note, the operators defined by Eq. (3) include only diagonal terms in polarization indices and if even refer to different times, obey the commutative relation

$$\left[\hat{j}^{\mu}_{\mathbf{q}(0)}(t),\hat{j}^{\nu}_{\mathbf{q}(0)}(t')\right]_{-}=0. \qquad (4)$$

We consider $\hat{j}^{\mu}_{\mathbf{q}(0)}(t)$ as the zero order approximation to $\hat{j}^{\mu}_{\mathbf{q}}(t)$. To define vector $f^{\mu}(\mathbf{q},t)$ we impose a physical condition: the mean values of current operators defined by Eqs. (3) and (2) are the same, namely,

$$\left(t\,|\,\hat{j}^{\mu}_{\mathbf{q}}(t)\,|\,t\right)=f^{\mu}(\mathbf{q},t)\left(t\,|\,\hat{\rho}_{\mathbf{q}}\,|\,t\right). \qquad (5)$$

This choice of $f^{\mu}(\mathbf{q},t)$ ensures the rapid convergence of series in modified perturbation theory. Obviously, Eq.(5) may be solved only approximately because the exact vector of state is unknown.

3. SOLUTION TO THE GROUND STATE PROBLEM

Denote the deviation of the current from its "zero" value as

$$\Delta\hat{j}^{\mu}_{\mathbf{q}}(t)=\hat{j}^{\mu}_{\mathbf{q}}(t)-\hat{j}^{\mu}_{\mathbf{q}(0)}(t)$$

and represent the electromagnetic interaction as a sum of two parts, $\hat{H}_{\text{int}}(t)=\hat{H}^{(0)}_{\text{int}}(t)+\hat{H}^{(1)}_{\text{int}}(t)$, where

$$\hat{H}^{(0)}_{\text{int}}(t)=e\int\hat{j}^{\mu}_{(0)}(\mathbf{r},t)\hat{A}_{\mu}(\mathbf{r},t)dV; \qquad (6)$$

$$\hat{H}^{(1)}_{\text{int}}(t)=e\int\Delta\hat{j}_{\mu}(\mathbf{r},t)\hat{A}_{\mu}(\mathbf{r},t)dV. \qquad (7)$$

The 4-vector-potential operator of electromagnetic field is defined by the ordinary way as

$$\hat{A}^{\mu}(\mathbf{r},t)=\sum_{\alpha,\mathbf{q}}g_{\mathbf{q}}\left\{\hat{b}_{\alpha\mathbf{q}}e^{\mu}_{\alpha\mathbf{q}}e^{i\mathbf{qr}}+\hat{b}^{+}_{\alpha\mathbf{q}}e^{\mu*}_{\alpha\mathbf{q}}e^{-i\mathbf{qr}}\right\}. \qquad (8)$$

Here $\hat{b}^{+}_{\alpha\mathbf{q}}$ and $\hat{b}_{\alpha\mathbf{q}}$ are the creation and annihilation operators of photons in states with polarization α $(\alpha=0,1,2,3)$, momentum $\mathbf{q}$ and energy $\omega=q$. The coupling function $g_q=\sqrt{2\pi/\omega}$ is defined, e.g., in [5]. By definition the different unit vectors of polarization $e^{\mu}_{\alpha\mathbf{q}}$ are orthogonal to each other and obey the normalization conditions (see, e.g., [2])

$$g_{\mu\nu}e^{\mu*}_{\alpha\mathbf{q}}e^{\nu}_{\beta\mathbf{q}}=g_{\alpha\beta}\,;\quad \sum_{\alpha}e^{\mu*}_{\alpha\mathbf{q}}e^{\nu}_{\alpha\mathbf{q}}g_{\alpha\alpha}=g^{\mu\nu}.$$

The conditions of the relativistic invariance (see, e.g., [1,2,3]) are satisfied when the photon operators obey the commutation relations

$$\left[\hat{b}_{\alpha\mathbf{q}},\hat{b}^{+}_{\beta\mathbf{q}'}\right]_{-}=-g_{\alpha\beta}\cdot\Delta(\mathbf{q}-\mathbf{q}')\,;$$

$$\left[\hat{b}_{\alpha\mathbf{q}},\hat{b}_{\beta\mathbf{q}'}\right]_{-}=\left[\hat{b}^{+}_{\alpha\mathbf{q}},\hat{b}^{+}_{\beta\mathbf{q}'}\right]_{-}=0.$$

Here $\Delta(\mathbf{q}-\mathbf{q}')$ equal to unity only if $\mathbf{q}=\mathbf{q}'$ and equal to zero in the opposite case. It is well known that the non-usual commutation relations for the scalar photons lead to the indefinite metric in the photon state space. There are also some peculiarities in the definition of physical quantities calculated below.

By virtue of (4), the equation

$$i\frac{d}{dt}\,|\,t)=\hat{H}^{(0)}_{\text{int}}(t)\,|\,t) \qquad (9)$$

has the exact solution expressed as the direct product of extended coherent states [13], namely

$$|\,t)_0=\prod_{\alpha,\mathbf{q}}\exp\left\{-i\hat{\chi}_{\alpha\mathbf{q}}(t)-\hat{b}_{\alpha\mathbf{q}}\hat{Q}^{+}_{\alpha\mathbf{q}}(t)+\hat{b}^{+}_{\alpha\mathbf{q}}\hat{Q}_{\alpha\mathbf{q}}(t)\right\}|\,0). \qquad (10)$$

Here

$$\hat{Q}_{\alpha\mathbf{q}}(t)=\hat{\rho}_{\mathbf{q}}Q_{\alpha\mathbf{q}}(t)\,;\quad \hat{\chi}_{\alpha\mathbf{q}}(t)=\hat{\rho}^{+}_{\mathbf{q}}\hat{\rho}_{\mathbf{q}}\chi_{\alpha\mathbf{q}}(t) \qquad (11)$$

and

$$Q_{\alpha\mathbf{q}}(t)=-ig_q\int_0^t dt'e^{\mu*}_{\alpha\mathbf{q}}f_{\mu}(\mathbf{q},t')e^{i\omega t'}, \qquad (12)$$

$$\chi_{\alpha\mathbf{q}}(t)=\int_0^t \text{Im}\left\{\dot{Q}^{*}_{\alpha\mathbf{q}}(t')Q_{\alpha\mathbf{q}}(t')\right\}dt'. \qquad (13)$$

Within our approach the initial vector of state |0) is the direct product of the electromagnetic field vacuum state |vac) and the vector $|\,\varphi_0)$, describing the initial state of a particle, $|\,0)=|\,\varphi_0)\otimes|\,vac)=|\,\varphi_0,vac)$. Assume the vector $|\,\varphi_0)$ is a free wave packet which widens in course of time. We must take into account that at t>0 in our interacting system each part of it is not independent and cannot be described exactly. E.g., there is no wave function for the electron but the simplest way to get explicit information about it consists in the calculation of its density matrix.

4. THE NEW PERTURBATION THEORY

If we neglect the corrections producing by $\hat{H}^{(1)}_{\text{int}}$, then Eq. (10) completely solves the problem of calculation of the physical quantities of interest. The quantity $|\,Q_{\alpha\mathbf{q}}(t)\,|^2$ has the direct physical meaning of the mean number of photons with polarization α and the momentum $\mathbf{q}$ created by the particle to the given time t. Therefore,

$$\Delta\mathbf{k}(t)=\sum_{\alpha,\mathbf{q}}\mathbf{q}\,|\,Q_{\alpha\mathbf{q}}(t)\,|^2 \qquad (14)$$

is the mean momentum loss and

$$\Delta E(t)=\sum_{\alpha,\mathbf{q}}\omega\,|\,Q_{\alpha\mathbf{q}}(t)\,|^2 \qquad (15)$$

is the mean energy loss of the particle. Sometimes (e.g., for particle at rest) the energy and momentum losses equal to zero.

In the considered approximation we can calculate also many other physical quantities (e.g., the Green function) for the particle with taking into account the back influence of electromagnetic quanta on particle's state. To account for the corrections due to interaction operator $\hat{H}^{(1)}_{\text{int}}$ a modified perturbation theory should be constructed. With this aim we introduce a "zero" order evolution operator

$$\hat{U}_0(t)=\exp\left\{\sum\hat{Q}_{\alpha\mathbf{q}}(t)\hat{b}^{+}_{\alpha\mathbf{q}}-\hat{Q}^{+}_{\alpha\mathbf{q}}(t)\hat{b}_{\alpha\mathbf{q}}-i\hat{\chi}_{\alpha\mathbf{q}}(t)\right\}. \qquad (16)$$

The vector of state defined by Eq. (10) can be rewritten as $|t)_0 = \hat{U}_0(t)|0)$. Introduce the new representation of operators as follows

$$\widetilde{A}(t) = \hat{U}_0^+(t)\hat{A}(t)\hat{U}_0(t)\,. \quad (17)$$

The vector of state $|t> = \hat{U}_0^+(t)|t)$ in this representation obeys the equation

$$i\frac{d}{dt}|t> = \hat{H}_{\text{int}}^{(1)}(t)|t>\,. \quad (18)$$

The solution to this equation may be found in a rigorous way via *T*-exponent.

5. APPLICATION TO ONE-PARTICLE PROBLEM

An approximate solution of Eq. (5) can be found with the help of a self-consistent procedure, when we use as a state |t) an approximate expression which functionally depends on $f^{\mu}(\mathbf{q},t)$. In the first approximation we suppose $|t) \approx |t)_0$. Generally speaking, the quantity $f^{\mu}(\mathbf{q},t)$ must be chosen in a strong connection to the particular problem. We consider now only the evolution of one-particle initial state on the background of photon vacuum. We begin with evaluation of the left-hand side of Eq.(5).

Assume φ_0 is a Gauss wave packet

$$|\varphi_0) = \left(8\pi\delta^2\right)^{3/4}\sum_{\mathbf{k}} e^{-\delta^2(\mathbf{k}-\mathbf{k}_0)^2}\hat{a}^+_{\lambda',\mathbf{k}}|vac)\,, \quad (19)$$

where λ' is a some initial polarization of the electron. Let us represent Eq.(5) in a more convenient form. Using the well-known Baker-Hausdorff formula and applying the formalism of extended coherent states [13], we can reduce Eq. (5) to the form

$$f^{\mu}(\mathbf{q},t) = e\sum_{\mathbf{p}}\bar{u}_{\lambda',\mathbf{p}-\mathbf{q}/2}\gamma^{\mu}u_{\lambda',\mathbf{p}+\mathbf{q}/2}$$
$$\times\exp[it(\varepsilon_{\mathbf{p}-\mathbf{q}/2}-\varepsilon_{\mathbf{p}+\mathbf{q}/2})]\int\frac{d^3s}{(2\pi)^3}e^{-s^2/8\delta^2} \quad (20)$$
$$\times\exp\left\{i\mathbf{s}(\mathbf{k}_0-\mathbf{p}) - \sum_{\alpha',\mathbf{q}'}|Q_{\alpha',\mathbf{q}'}|^2\,(1-e^{-\mathbf{1q's}})\right\}.$$

Because $Q_{\alpha\mathbf{q}}$ depends on f^{μ}, Eq. (20) is actually an integral equation for $f^{\mu}(\mathbf{q},t)$. Let us assume the initial width of the particle's wave packet is large compared to the Compton wavelength, i.e., $\delta >> 1/mc$. Under this condition the main contribution to the sum in the right-hand side of Eq. (20) comes from the small neighborhood of the point $\mathbf{p} = \mathbf{k}_0$. Assuming that in this region the quantity $|\mathbf{p}-\mathbf{k}_0|$ is small enough and comparable with $|\Delta\mathbf{k}|$, we can expand the integrand in exponent in the right-hand side of Eq. (20) with account for the terms of only the first and second order with respect to $(\mathbf{p}-\mathbf{k}_0)$. In the non-relativistic case the stationary point of exponent in Eq. (20) is found to be $\mathbf{p} = \mathbf{k}_0 - \Delta\mathbf{k}(t)$.

Accepting this result, in the first approximation we can calculate the sum involved in (20) in the stationary phase approximation and get

$$f^{\mu}(\mathbf{q},t) \approx e\cdot\bar{u}_{\lambda',\mathbf{p}_0-\mathbf{q}/2}\gamma^{\mu}u_{\lambda',\mathbf{p}_0+\mathbf{q}/2}\cdot e^{-it(\varepsilon_+-\varepsilon_-)}\,, \quad (21)$$

where

$\mathbf{p}_0 = \mathbf{k}_0 - \Delta\mathbf{k}(\mathbf{k}_0,t),\quad \varepsilon_{\pm} = (m^2 + (\mathbf{p}_0 \pm \mathbf{q}/2)^2)^{1/2}$,

$\Delta\mathbf{k}(\mathbf{k}_0,t)$ is the mean momentum loss of the particle.

6. MEAN ELECTROMAGNETIC FIELD

Consider now the problem of corrections to the Coulomb field. The mean vector-potential of the field is given as $A^{\mu} = <t|\widetilde{A}^{\mu}|t>$. Let us calculate this vector in the zero approximation setting $|t> \approx |0)$. By using this assumption we obtain

$$A^{\mu} = 2e\,\mathrm{Re}\left\{i\sum_{\alpha,\mathbf{q}}g_q^2 g_{\alpha\alpha}e^{\mu}_{\alpha\mathbf{q}}e^{i\mathbf{qr}-i\omega_q t}e^{\nu}_{\alpha\mathbf{q}}\right.$$
$$\left.\times\int_{-\infty}^{t}e^{i\omega_q t'}f_{\nu}(\mathbf{q},t')\rho_{\mathbf{q}}(t')dt'\right\}. \quad (22)$$

where the function $\rho_{\mathbf{q}}(t)$ is the Fourier component of particle probability distribution in the rest frame.

The vector-potential A^{μ} is represented by the convolution of the potential for point particle and the probability distribution for the particular state. It is interesting to analyze the universal case, corresponding to the point charge, for which $\rho_{\mathbf{q}} = 1$. For the uniformly moving particle we have $\mathbf{p}_0 = \mathbf{k}_0$ and arrive at

$$A^0 = 2e\,\mathrm{Re}\left\{\sum_{\mathbf{q}}\frac{g_q^2}{\sqrt{4\varepsilon_+\varepsilon_-}}w^+_{\lambda'}\Big(\sqrt{(\varepsilon_+ + m)(\varepsilon_- + m)}\right.$$
$$+\frac{\sqrt{(\varepsilon_+ - m)(\varepsilon_- - m)}}{|\mathbf{k}_0-\mathbf{q}/2||\mathbf{k}_0+\mathbf{q}/2|}(k_0^2 - q^2/4 + i\boldsymbol{\sigma}[\mathbf{k}_0\times\mathbf{q}]\Big)w_{\lambda} \quad (23)$$
$$\left.\times\frac{e^{i\mathbf{qr}-i(\varepsilon_+-\varepsilon_-)t}}{\omega_{\mathbf{q}}+\varepsilon_- - \varepsilon_+}\right\};$$

$$\mathbf{A} = 2e\,\mathrm{Re}\left\{\sum_{\mathbf{q}}\frac{g_q^2}{\sqrt{4\varepsilon_+\varepsilon_-}}w^+_{\lambda'}\big(\boldsymbol{\sigma}(\mathbf{p}_+\boldsymbol{\sigma}) + (\mathbf{p}_-\boldsymbol{\sigma})\boldsymbol{\sigma}\big)w_{\lambda'}\right.$$
$$\left.\times\frac{\exp[i\mathbf{qr}-i(\varepsilon_+-\varepsilon_-)t]}{\omega_{\mathbf{q}}+\varepsilon_- - \varepsilon_+}\right\}, \quad (24)$$

where $\mathbf{p}_{\pm} = (\mathbf{k}_0 \pm \mathbf{q}/2)\sqrt{(\varepsilon_{\pm}+m)(\varepsilon_{\mp}+m)}$.

First we evaluate the scalar potential for the particle being at rest. We substitute $\mathbf{k}_0 = 0$ and get

$$A_0 = e\frac{2}{\pi r}\int_0^{2mr}K_0(\xi)\,d\xi\,, \quad (25)$$

where $K_0(\xi)$ is the MacDonald function of zero order. It is important to note that at $r \to 0$ the potential (25) diverges only logarithmically as $(Zm/\pi)\ln(e/mr)$, $r << 1/m$. At $r >> 1/2m$ it coincides with the Coulomb potential. The interaction with the electromagnetic field changes the particle form-factor at distances which do not exceed several Compton wavelengths. The same behavior is manifested by the potential which takes into account the radiation corrections in the Feynman perturbation theory.

If we attempt to represent formulas (23) as the usual retarded potential corresponding to any charge distribution, then we see, that it is possible only if this distribution sufficiently depends on the mass of the particle. The dependence of the field on the mass can be connected to the effect of the reaction force acting at photon radiation. This circumstance means not the charge distribution but the sample action of electromagnetic quanta wrapped around the charge and creating the field which we observe as the source of the Coulomb force.

Consider the self-energy for particle at rest. The electromagnetic contribution to it can be obtained in the usual manner as the energy of electric field

$$E = \frac{1}{8\pi}\int (-\nabla A_o)^2 dV = -\frac{1}{8\pi}\int A_0 \Delta A_0 dV .$$

The integral can be evaluated exactly. First we introduce $\Phi = rA_0$ and using the spherical symmetry and $\Delta A_0 = (1/r)d^2\Phi/dr^2$, bring the energy to the form

$$E = \frac{8}{\pi^2}(em)^2 \int_0^\infty K_0^2(2mr)\, dr = me^2 .$$

As we see, the electromagnetic contribution to the self-energy of elementary particle is comparatively small. In the case of electron as well as proton the electromagnetic correction is equal to the self-energy multiplied by the fine structure constant. We can confirm the known result that the main part of the self-energy for elementary particles has the non-electromagnetic nature.

We should make once more important notation. We can calculate the general energy of all the photons wrapping around the particle and found that the total its energy exactly coincides with the electromagnetic contribution to the self-energy. This result allows us to consider this fact as a new method to calculate the energy of electromagnetic field, without using the explicit expressions for the electric and magnetic fields.

Consider the case of non-relativistic particle. We can set $\mathbf{k}_0 = m\dot{\mathbf{r}}_0$, where $\mathbf{r}_0(t)$ is the current mean coordinate of the particle,

$$\varepsilon_+ - \varepsilon_- \approx \frac{m\mathbf{q}\dot{\mathbf{r}}_0(t)}{\varepsilon_{\mathbf{q}/2}}; \quad \varepsilon_+\varepsilon_- \approx m^2(1 + q^2/4m^2) .$$

In this case we arrive at

$$A^0 = \frac{2}{\pi^2} em \int_{-\infty}^{t} dt' \int_0^\infty dk \int_0^\infty d\tau \frac{\cos(k\tau)}{|\mathbf{r} - m\mathbf{r}_0(t')/\varepsilon_{k/2}|}$$
$$\times\left\{K_0\left(2m\,|\,t - t' - \tau - |\mathbf{r} - m\mathbf{r}_0(t')/\varepsilon_{k/2}|\,|\right) - K_0\left(2m\,|\,t - t' - \tau + |\mathbf{r} - m\mathbf{r}_0(t')/\varepsilon_{k/2}|\,|\right)\right\} . \quad (26)$$

Within the region $r \geq 1/2m$ the main contribution to the potential (26) occurs from the small $k < 2m$. In this region one can neglect all the nonlinear in $\lambda = k/2m$ terms in the integrand and get

$$A^\mu = \frac{2}{\pi} Zmc^2 \int_0^\infty d\tau \frac{\dot{r}^\mu_{(0)}(t-\tau)}{|\mathbf{r} - \mathbf{r}_0(t-\tau)|}$$
$$\times\left[K_0(2mc\,|\,c\tau - |\mathbf{r} - \mathbf{r}_0(t-\tau)|\,|) - K_0(2mc\,|\,c\tau + |\mathbf{r} - \mathbf{r}_0(t-\tau)|\,|)\right]. \quad (27)$$

Note, the 4-potential (27) obeys the Lorentz condition. Comparing this result with Eq. (26) we see that the previous expression contains the additional retardation time τ. The physical explanation of this effect can be found on the basis of action of the reaction force appearing at the virtual fast ($k > 2m$) photon radiation. At this radiation the particle gets a great (virtual) deflection from the center of the probability distribution. This deflections display themselves in Eq. (26) in form of the retardation time. At distances $r >> 1/2m$ the MacDonald functions change very fast compared to the characteristic distance of the potential inhomogeneity. If one replaces in Eq. (27) the MacDonald functions by the corresponding delta-functions, the Lienard-Wichert potentials will result.

The additional contribution appears from the mean value of the vector potential, which general expression is given by the formula

$$\mathbf{A}_{mag}(\mathbf{r}) = \left[\nabla\Phi \times \boldsymbol{\mu}_e\right], \quad (28)$$

where

$$\Phi(r) = \frac{2}{\pi r}\int_0^{2mr} K_0(x)dx , \quad (29)$$

and $\boldsymbol{\mu}_e = e\mathbf{s}/m$ is the electron magnetic momentum. Apparently, the expression (28) represents the vector potential of magnetic field caused by the electron magnetic momentum. At $r \to \infty$ we have $\Phi(r) \to 1/r$, what is right for the point magnetic momentum. But at $r < 1/m$ the function $\Phi(r)$ has depends only logarithmically on r, simulating the behavior of the main part of the potential.

In Sec. 2 it was noted, that the interaction with the photon field could change the electron spin direction. In our approximation, when only diagonal part of Hamiltonian included in the ground state definition, we do not take into account the spin-flip effects during the radiation. In the more strict consideration one needs to take into account the above speculations and gets the regular form for the magnetic momentum field.

7. CONCLUSION

We see that problems with divergences in the QED are more the problems of the perturbation theory than the QED itself. The divergences in the Feynman-Dyson perturbation theory are eliminated due to the physical effects: i) displacement of the particle as the reaction on its own radiation, ii) spin-flip processes due to radiation. In the Feynman approach the commonly used zero order propagators do not take into account the electromagnetic interaction at all. Apparently, if we use more correct propagators within the new (modified) perturbation theory (which is equivalent to the prior "clothing" of particles), the infrared as well ultraviolet divergences at the calculation of different physical quantities will be eliminated.

I am thankful to Dr. V.Yu. Korda for drawing my attention to refs. [7-12] and related works.

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НОВАЯ ТЕОРИЯ ВОЗМУЩЕНИЙ В КЭД

Г.М. Филиппов

Построен новый вариант теории возмущений в КЭД, в котором отсутствует ультрафиолетовая катастрофа.

НОВА ТЕОРІЯ ЗБУРЕНЬ У КЕД

Г.М. Філіппов

Побудовано новий варіант теорії збурень у КЕД, у якому відсутня ультрафіолетова катастрофа.