# PARAMETERS OF DEFORMATION OF THE EXCITED STATES IN sd-SHELL ODD NUCLEI 

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#### Abstract

The review of the major models used for the description of the deformed nuclei is given. The difficulties conditioned by the uniqueness of the choice of nuclear deformation parameters in rotational bands are traced. The calculations of the reduced probabilities of electromagnetic transitions in the sd-shell odd nuclei performed within the modified Nilsson model show that the use of different values of deformation parameters for different excited states gives better fits to the experimental data.


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## 1. INTRODUCTION

The decay schemes at low excitation energies of several odd nuclei in the sd-shell are usually interpreted with help of the generalized collective rotational model [1]. The levels are grouped in the rotational bands each of which is based on the internal single-particle states calculated in the deformed nuclear potential. Assuming a cylinder symmetry, the analysis starts with the determination of the form of the nuclear shape from the measured quadrupole momenta, magnetic moments or unbinding parameters. When the value and the sign of deformation become known the detailed comparison of the calculated and measured values becomes possible by the use of the eigenvalues and eigenfunctions corresponding to the one-particle shell model. Those values are the level excitation energies, the probabilities of gamma and beta transitions, the reduced nucleon widths which characterize such nuclear reactions as stripping and nuclear pick-up, the magnetic and quadrupole momenta of nuclear levels and their spins and parities. Although the major striking achievements of the generalized collective model were associated with the $150<A<190$ and $A>222$ nuclei the remarkable correlations with the data were found for the sd-shell light odd nuclei as well [1]. The application of the generalized model to the light deformed nuclei is more complicated than to the heavier ones because one must take into account the overlap of rotational bands and the hole states in the low-energy part of the spectrum. In general, the model with symmetric spheroid core is applicable to this shell due to the mix of two or more rotational bands with $\Delta K= \pm 1$. In this case the R.P.C. operator used and it has the form $O_{R P C}=\alpha\left(I_{1} J_{1}+I_{2} J_{2}\right)([7])$. It is usually assumed that the deformation does not depend on the excita-
tion energy. At the same time, the data point on the existence of deformation of light nuclei in the excited states as well as in the ground ones. of The shell model built at the end of 1940-th, having explained great amount of data associated with the ground and weakly excited states of atomic nuclei, faced substantial problems. Particularly, the measured values of quadrupole momenta for several nuclei appeared to be much higher the same values calculated due to the shell model. On the basis of the method developed in NSC KhIPT the nuclear deformation in the excited states was accounted for, which allowed for better description of the reduced probabilities of electromagnetic transitions [2]. We have considered the nuclei with $n=11$ nucleons of definite kind ${ }^{21} N e,{ }^{23} N a$, ${ }^{23} \mathrm{Mg}$ with pronounced rotational structure of levels. We have confirmed the influence of deformation on the values of the probabilities of electromagnetic transitions to the ground state from both the oneparticle and collective initial states. For systematic study the computer codes which calculate the matrix elements of the electromagnetic transitions on the basis of the developed method which uses Nilsson's wave functions are needed.

## 2. NILSSON'S MODEL

In Nilsson'n model, the interaction of nucleons with the nuclear field is described with help of the Hamiltonian of the form [3]

$$
\begin{equation*}
H=H_{0}+C \cdot l \cdot s+D \cdot l^{2} \tag{1}
\end{equation*}
$$

where $H_{0}$ is the oscillatory potential to which the spin-orbit potential $C \cdot l \cdot s$ and $D \cdot l^{2}$ correction for the states with higher angular momenta are added. The weights of these two terms are chosen to replicate the known sequence of one-particle states in the shell model with spherical potential. Nilsson studied

[^0]the role of deformation of the nuclear potential with a cylinder symmetry. Neglecting for a while the $l \cdot s$ and $l^{2}$ terms, he found that the Hamiltonian with deformed potential could be divided into the spherically symmetric term $H_{0}$ and the $H_{\delta}$ term describing the binding of the particle to the symmetry axis. The representation is chosen to be diagonal with respect to $H_{0}$. All operators $l^{2}, l_{z}, s_{z}$ commute with $H_{0}$. The respective quantum numbers are denoted as $l, \bigwedge, \Sigma$. Then the total Hamiltonian has the form
\[

$$
\begin{equation*}
H=H_{0}+H_{\delta}+C \cdot l \cdot s+D \cdot l^{2} \tag{2}
\end{equation*}
$$

\]

in which the last three terms are treated as a perturbation. By the proper choice of the parameters (shown here for convenience) Nilsson arrived to

$$
\begin{equation*}
H=H_{0}+k \eta \omega_{0} R \tag{3}
\end{equation*}
$$

where $k=-C / 2 \eta \omega_{0}$ and $R$ were given as

$$
\begin{equation*}
R=\eta R-2 l s-\mu l^{2} \tag{4}
\end{equation*}
$$

Here $\mu=2 D / C$ and $\eta=(\delta / k)\left(\omega_{0}(\delta) / \omega_{0}(0)\right)$. The parameter $\delta$ is connected to the deformation of the nuclear potential $U$ and is defined from

$$
\begin{equation*}
H_{\delta}=\delta \eta \omega_{0} U=k \eta^{2} \omega_{0} U \tag{5}
\end{equation*}
$$

where

$$
\begin{equation*}
U=-\frac{4}{3} \sqrt{\frac{\pi}{5}} r^{2} Y_{20} \tag{6}
\end{equation*}
$$

From the diagonalization of the operator R Nilsson found the eigenvalues $\mathrm{r}^{N \Omega}(\eta)$ and the corresponding eigenvalues of the total Hamiltonian

$$
\begin{equation*}
E^{N \Omega}=(N+3 / 2) \eta \omega_{0}+k \eta \omega_{0} r^{N \Omega} \tag{7}
\end{equation*}
$$

The general quantum number N is the number of of oscillator quanta and $\Omega$ is the quantum number corresponding to the operator $\mathrm{j}_{z}=\mathrm{l}_{z}+\mathrm{s}_{z}$ which commutates with H . Thus, $\Omega$ is the component of the total angular momentum along the nuclear axis. The state vectors use are $|N l \Lambda \Sigma\rangle$ represented as where $\Lambda$ and $\Sigma$ are the eigenvalues of the operators $\mathrm{l}_{z}$ and $\mathrm{s}_{z}$ and, thus, $\Omega=\Lambda+\Sigma$.

For the d-shell which is the outer shell for the considered nuclei Nilsson diagonalized R with $\mu=0$ for $\mathrm{N}=0,1,2$ when for the higher values of N the values of $\mu$ (0.35-0.55) were used. It seems valuable to study the influence of $\mu \mathrm{l}^{2}$ term on the eigenvalues and eigenfunctions for $\mathrm{N}=2$. For $\mathrm{N}=2, \Omega=5 / 2$ (orbit 5) there exists only one eigenvector $|222+\rangle$ with the eigenvalue $\mathrm{r}=2 / 3 \eta-2-6 \mu$. For $\mathrm{N}=2$ and $\Omega=3 / 2$ there exist two eigenvectors $|221+\rangle$ and $|221-\rangle$. Because the term $\mu \mathrm{l}^{2}$ is diagonal and each of these eigenvectors has $l=2$ the influence is in the simple addition of the constant diagonal matrix - the unity matrix $-6 \mu$ times to matrix diagonalized by Nilsson. Therefore, the new eigenvalues appear as those ones tabulated by Nilsson and divided by $-6 \mu$ and the eigenfunctions are the same both for the orbit 8 and the orbit 7 .

For $\mathrm{N}=2, \Omega=1 / 2$ however, the eigenvectors are $|220+\rangle,|200+\rangle$ and $|221-\rangle$. The argument used for $\Omega=3 / 2$ is not valid any more and the matrix should be diagonalized for each values of $\eta$ and $\mu$. The results of these calculations are presented in [4], giving the eigenvalues and the eigenfunctions for the orbits 11,9 and 6 . In this work the influence of the term related to the centrifugal correction $l^{2}$ for the sequence of levels of the sd-shell nuclei is studied. The graphs of the eigenvalues and the unbinding parameters for the orbit $\Omega=1 / 2$ are given. The other approach to the modification of Nilsson's model is associated with the use of the other representations of the eigenvectors of the harmonic oscillator [3,5].

## 3. APPLICATION TO THE SCHEMES OF EXCITED STATES

Now we are going to discuss several examples of the decay schemes for which the calculations giving better agreement with the data are presented.

$$
\text { 3.1. }{ }^{25} \mathrm{Mg} \text { AND }{ }^{27} \mathrm{Al} \text { [6]. }
$$

The configuration of the ground state for the nucleons are taken in the form
$\pi\left(O^{16} ; d_{5 / 2}\left(1 / 2^{2}, 3 / 2^{2}\right)\right) \nu\left(O^{16} ; d_{5 / 2}\left(1 / 2^{2}, 3 / 2^{2}, 5 / 2^{1}\right)\right)$,
and corresponds to the prolate deformation as found for the neighboring nuclei ${ }^{23} \mathrm{Na}$ and ${ }^{27} \mathrm{Al}$. The unbinding parameter for the band built on the first excited state is found to be $a=0.2$. If one takes this state with $\eta_{i}+6$ then the orbit 9 appears lower than the orbit 5 . Ignoring the strength of the spin-orbit coupling k within the calculations of this kind can reduce this discrepancy.

Nilsson's calculations are mostly suitable for this case because they contain results for the eigenfunctions for $\mathrm{A}=25$. The energy states of the model for $\mathrm{A}=25$, are found by the extension of Nilsson's calculations of the single-particle energies in the spheroid potential towards the higher values of $\eta$. With this aim Nilsson's equation is rewritten as

$$
\begin{align*}
E_{j} / 0.75 \eta \omega_{0}= & \left(N_{j}+3 / 2\right)\left(1-1 / 3 \epsilon^{2}-2 / 27 \epsilon^{3}\right)^{-1 / 3}+ \\
& k r_{j}(\epsilon)-\left(1 / 3 \eta \omega_{0}\right)\left\langle U_{j}\right\rangle, \tag{9}
\end{align*}
$$

where the following relations between $\epsilon, \eta$ and $\delta$ are used

$$
\begin{equation*}
k \eta=\epsilon\left(1-1 / 3 \epsilon^{2}-2 / 27 \epsilon^{3}\right)^{-1 / 3}, \tag{10}
\end{equation*}
$$

and

$$
\begin{equation*}
\epsilon=3 \frac{\sqrt{1+2 / 3 \delta}-\sqrt{1-4 / 3 \delta}}{\sqrt{1-4 / 3 \delta}-2 \sqrt{1+2 / 3 \delta}} . \tag{11}
\end{equation*}
$$

The value of k which defines the weight of spin-orbit coupling used in calculations is taken as 0.08 . This appears the better value for the light nuclei than 0.05. In this connection it seems valuable to note that the $\mathrm{d}_{5 / 2^{-}} \mathrm{d}_{3 / 2}$ splitting at 5.08 MeV in ${ }^{17} \mathrm{O}$ shall require $\mathrm{k}=0.13$. Due to the indefiniteness in the values of k
for the case $\mathrm{A}=25$ all the respective curves are given as functions of $\eta$ because this conditions the necessity to consider the alteration of the weight of the spin-orbit interaction.

The values of $\eta$ used in the respective comparisons alter from +3 to +5 . It is senseless a priori to expect that $\eta$ shall be constant for each rotational band and the very fact that different values of $\eta$ give better fits to the data probably means that the neglect of the RPC effect is important.

## 4. THE BAND MIXING MODEL (RPC)

For the odd masses the Hamiltonian of the axial symmetric nuclei which describes the single particle motion, the nucleus rotation and the particle-rotation coupling is given in the form [7]

$$
\begin{align*}
H= & H_{\text {intr }}+\frac{\eta^{2}}{2 J^{\prime \prime}}\left(I^{2}-2 I_{0}^{\prime} J_{0}^{\prime}\right)+ \\
& \frac{\eta^{2}}{J^{\prime \prime}}\left(I_{1}^{\prime} J_{-1}^{\prime}+I_{-1}^{\prime} J_{1}^{\prime}\right), \tag{12}
\end{align*}
$$

and if one neglects the last term in (12) the major set of wave functions are

$$
\begin{align*}
I M K \alpha= & \left\{\frac{2 I+1}{16 \pi^{2}}\right\}^{1 / 2}\left(D_{M K}^{I} X_{K}^{\alpha}+\right. \\
& \left.(-1)^{I-J} D_{M-K}^{I} X_{-K}^{\alpha}\right) \tag{13}
\end{align*}
$$

where $J^{\prime \prime}$ is the momentum of inertia, I is the total angular momentum of nucleus, J is the internal angular momentum related to the motion of the odd nucleon, $\mathrm{I}_{0, \mp 1}^{\prime}$ and $\mathrm{J}^{\prime}{ }_{0, \mp 1}$ are their spherical components in the rest frame, K is the projection of I or J on the symmetry axis of the nucleus, M is the projection of I on the fixed axis z, $X_{K}$ are Nilsson's wave functions $\mathrm{D}_{M K}^{I}$ and $\alpha$ denote different eigenstates with the same value of K .

In the representation (14) only the last term in (12) is not diagonal with respect to K . It gives the diagonal contributions only in the case when $\mathrm{K}=1 / 2$. This term couples the states with one and the same parities and total angular momenta for which $\Delta \mathrm{K}=$ $\pm 1$ or $\Delta \mathrm{K}=0$ if $\mathrm{K}=\mathrm{K}^{\prime}=1 / 2$. The diagonal matrix elements of the Hamiltonian (12) are equal to

$$
\begin{array}{r}
E(N I K \alpha)=E_{K}^{0}+\frac{\eta^{2}}{2 J^{\prime \prime}}\left(\left(I(I+1)+\alpha(-1)^{I+1 / 2} \times\right.\right. \\
\left.\left.(i+1 / 2) \delta_{K_{1 / 2}}\right)-\left(K(K+1)-\alpha \delta_{K_{1 / 2}}\right)\right) \tag{14}
\end{array}
$$

where

$$
\begin{equation*}
E_{K}^{0}=E_{0}+\frac{\eta^{2}}{2 J^{\prime \prime}}\left(K-K^{2}-\alpha \delta_{K_{1 / 2}}\right) \tag{15}
\end{equation*}
$$

The non-diagonal matrix elements can be written in $J-K$ representation as

$$
\begin{align*}
& A_{\beta \beta^{\prime}}^{I}=\left\langle I M K^{\prime} \alpha^{\prime}\right| \frac{\eta^{2}}{2 J^{\prime \prime}}\left(I_{1}^{\prime} J_{-1}^{\prime}+I_{-1}^{\prime} J_{1}^{\prime}\right)|I M K \alpha\rangle= \\
& \quad-\eta^{2} / 2 J^{\prime \prime} \sqrt{\left(I+K_{1}\right)\left(I-K_{1+I}+1\right)}\left(1-\delta_{K K^{\prime}}+\right. \\
& \left.\quad(-1)^{I-1 / 2} \delta_{K^{\prime} 1 / 2} \delta_{K 1 / 2}\right) A_{\beta \beta^{\prime}} \tag{16}
\end{align*}
$$

where $\mathrm{K}_{I}>\mathrm{K}, \mathrm{K}^{\prime} ; \beta$ and $\beta^{\prime}$ are the numbers of Nilsson's states and

$$
\begin{align*}
A_{\beta \beta^{\prime}}= & \sum_{J} C_{J}^{\prime} C_{J} \sqrt{\left(J+K_{1}\right)\left(I-K_{1}+1\right)} \times  \tag{17}\\
& \left(1-\delta_{K K^{\prime}}+(-1)^{I-1 / 2} \delta_{K^{\prime} 1 / 2} \delta_{K 1 / 2}\right)
\end{align*}
$$

or in the $\mathrm{l}-\Lambda$ representation for $\mathrm{K}=\mathrm{K}^{\prime}=1 / 2$

$$
\begin{equation*}
A_{\beta \beta^{\prime}}=(-1)^{l} \sum_{l}\left(a_{l 0}^{\prime} a_{l 0}+\sqrt{l(l+1)}\left(a_{l 0}^{\prime} a_{l 1}+a_{l 1}^{\prime} a_{l 0}\right)\right), \tag{18}
\end{equation*}
$$

(if $\beta=\beta^{\prime}, \mathrm{A}_{\beta \beta}=\mathrm{a}$ equals to the unbinding parameter) and for

$$
\begin{align*}
& A_{\beta \beta^{\prime}}=\sum_{l}\left(a _ { l K _ { 1 } - 1 / 2 } ^ { \prime } \left(a_{l K_{1}-1 / 2}+\left(a _ { K _ { 1 } - 1 / 2 } ^ { \prime } \left(a_{l K_{1}-1 / 2} \times\right.\right.\right.\right. \\
& \sqrt{\left(l+K_{1}-1 / 2\right)\left(l-K_{1}+3 / 2\right)}+a_{l K_{1}-1 / 2}^{\prime}\left(a_{l K_{1}+1 / 2} \times\right. \\
& \left.\sqrt{\left(l-K_{1}+1 / 2\right)\left(l+K_{1}+1 / 2\right)}\right) . \tag{19}
\end{align*}
$$

The coefficients $a_{l \Lambda}$ are given by Nilsson [3]. The coupling parameters $\mathrm{A}_{\beta \beta^{\prime}}$ (16) are calculated using (18) and (19). By diagonalization of the energy matrix (12) the wave functions

$$
\begin{equation*}
\Phi(I M)=\sum_{K_{\alpha}} C_{K \alpha} \Psi(I M K \alpha) \tag{20}
\end{equation*}
$$

And their eigenvalues $\lambda_{I}$ can be found for each mixed state with I spin.

### 4.1. PROBABILITIES OF ELECTROMAGNETIC TRANSITIONS IN THE BAND MIXING MODEL

The formulae for the reduced probabilities M1 and E2 transitions are given in [7] for the case of two mixed rotational bands. Using the wave function (20) and the magnetic and electric multipole operators given by Nilsson one can obtain [8] the expressions for the reduced probabilities of M1 and E2 transitions when the large number of bands are mixed. When for M1 transitions

$$
\begin{align*}
& B\left(M 1, I \rightarrow I^{\prime}\right)=\frac{3}{16 \pi}\left(\frac{\eta}{2 M \omega_{0}}\right)^{2} \times \\
& \sum_{K^{\prime}, \alpha^{\prime}, K, \alpha} C_{K^{\prime}, \alpha^{\prime}}^{\prime} C_{K, \alpha}\left[\left(I 1 K K^{\prime}-K \mid I^{\prime} K^{\prime}\right)+\right. \\
& \delta_{K / 2} \delta_{K^{\prime} / 2}(-1)^{I^{\prime}-K^{\prime}} b_{\alpha \alpha^{\prime}}^{K K^{\prime}}(M 1) \times \\
& \left.\left(I 1 K-K^{\prime}-K \mid I^{\prime}-K^{\prime}\right)\right] G_{\alpha \alpha^{\prime}}^{K K^{\prime}}(M 1) . \tag{21}
\end{align*}
$$

and for E2 transitions

$$
\begin{array}{r}
B\left(E 2, I \rightarrow I^{\prime}\right)=\frac{5 e^{2}}{4 \pi}\left(\frac{\eta}{M \omega_{0}}\right)^{2} \times \\
\left(C_{K^{\prime} \alpha^{\prime}}^{\prime} C_{K \alpha} \times \sum_{K^{\prime} \alpha^{\prime} K \epsilon}\left(\left(I 2 K K^{\prime}-K \mid I^{\prime} K ;\right)+\right.\right. \\
(-1)^{I^{\prime}+K^{\prime}} b^{K K_{\alpha \alpha^{\prime}}^{\prime}}(E 2) \times\left(I 2 K-K^{\prime}-\right. \\
\left.\mid I^{\prime}-K^{\prime}\right) G^{\left.\left.K K_{\alpha \alpha^{\prime}}^{\prime}(E 2)\right)\right)^{2} .} \tag{22}
\end{array}
$$

Here the diagonal matrix elements $\mathrm{G}_{\alpha \alpha^{\prime}}^{K K^{\prime}}(\mathrm{E} 2)$ are the own quadrupole momenta $\mathrm{Q}_{0}$ for each K in the units of $2\left(\eta / \mathrm{M} \omega_{0}\right)$. The one-particle matrix elements $\mathrm{G}_{\alpha \alpha^{\prime}}^{K K^{\prime}}(\mathrm{E} 2)$ and $\mathrm{b}_{\alpha \alpha^{\prime}}^{K K^{\prime}}(\mathrm{E} 2)$ are given by Nilsson [3]. ${ }^{\alpha \alpha}$ The value $\eta / M \omega_{0}$ was determined assuming $\eta / \mathrm{M} \omega_{0}=41 \mathrm{~A}^{-1 / 3} \mathrm{MeV}$. The own quadrupole momentum is assumed one and the same for all four one-particle states which totally correspond to the known quadrupole momenta of the ground states. The mean self life of the excited states and the relation of branching are calculated using (12) and (13) and are compared with the data.

## 5. MODIFIED NILSSON'S MODEL

One of the most important problems is the investigation of a shape of a nucleus in the excited states. However, this task is not well tested, especially in the range of light nuclei. Up to now, the existence of deformation of light nuclei in both ground and excited states is experimentally proven. At the same time, it is usually assumed that the deformation does not change with the excitation energy. In our opinion, this assumption conditioned the failure of attempts to explain the probabilities of electromagnetic transitions in the framework of generalized model. Bearing this in mind, we use the new approach developed in [2]. Analyzing the probabilities of electromagnetic transitions, we treat the nuclear deformation as a variation parameter, meaning that the initial and final states are assumed to have different deformations. Thus, during the transition, the state of core nucleons changes alongside the state of the odd nucleon. Using the 1 d 2 s -shell nuclei, we have studied the influence of the changes in nucleus deformation on the probabilities of electromagnetic transitions. The initial and final states of a nucleus have been considered for different deformations and the contribution of one-particle part of wave function to the probabilities of electromagnetic transitions in light nuclei have been analyzed.

### 5.1. PROBABILITIES OF <br> ELECTROMAGNETIC TRANSITIONS IN THE MODIFIED NILSSON'S MODEL

In order to determine the matrix element of the oneparticle multipole transition operator

$$
\begin{equation*}
\mathbf{M}=\sum_{i=1} \hat{t}(i) \tag{23}
\end{equation*}
$$

we consider the systems of one-particle wave functions for the initial and the final states: $\varphi_{1}, \varphi_{2}, \ldots$.. $\varphi_{A} ; \psi_{1}, \psi_{2}, \ldots \psi_{A}$. Here A denotes the number of nucleons in the nucleus; indices 1,2 , A denote the numbers of occupied orbits in the initial and the final states. The formula takes place:

$$
\begin{equation*}
\left(\psi_{f}, \mathbf{M} \psi_{i}\right)=\sum_{s=1}^{A} \operatorname{det} B_{j}^{s} \tag{24}
\end{equation*}
$$

where the determinant elements $\mathrm{B}_{j}^{s}$ are as follows:

$$
\begin{array}{ll}
B_{j}^{s}=\left(\psi_{i}, \hat{t} \varphi_{j}\right), & i=s \\
B_{j}^{s}=\left(\psi_{i}, \varphi_{j}\right), & i \neq s \tag{25}
\end{array}
$$

In the case under study, the operator $\hat{t}$ is:

$$
\begin{equation*}
\hat{t}=e\left[1+(-1)^{\lambda} \frac{Z}{A^{\lambda}}\right] r^{\lambda} Y_{\lambda \mu}(\varphi, \psi) . \tag{26}
\end{equation*}
$$

If $\lambda<\mathrm{K}+K^{\prime}$, then the reduced probability of electric multipole transitions between the initial and the final state with IK and $I^{\prime} K^{\prime}$ taken at different deformations $\eta$ and $\eta^{\prime}$ and is equal to:

$$
\begin{align*}
& |\mathbf{M}(E \lambda)|^{2}=  \tag{27}\\
& =e^{2}\left[1+(-1)^{\lambda} \frac{Z}{A^{\lambda}}\right]^{2}\left(\frac{\hbar}{M \omega_{0}}\right)^{\lambda} \frac{2 \lambda+1}{4 \pi} \times \\
& \times\left|\left\langle I \lambda K K^{\prime}-K \mid I^{\prime} K^{\prime}\right\rangle \sum_{s=1}^{Z} \operatorname{det}\left(\psi_{i}^{s}, \varphi_{j}\right)\right|^{2}
\end{align*}
$$

For $\mathrm{i}=\mathrm{s}$ :

$$
\begin{align*}
\left(\psi_{i}, \hat{t} \phi_{j}\right) & =\sum_{l^{\prime}, l}\left\langle N^{\prime} l^{\prime}\right| r^{\lambda}|N l\rangle \sqrt{\frac{2 l+1}{2 l^{\prime}+1}}\left\langle l \lambda 00 \mid l^{\prime} 0\right\rangle \times \\
& \times \sum_{\Lambda^{\prime}, \Lambda, \Sigma^{\prime}, \Sigma} \delta_{\Sigma^{\prime} \Sigma} a_{l^{\prime} \Lambda^{\prime}}^{\prime} a_{l \Lambda}\left\langle l \lambda \Lambda K^{\prime}-K \mid l^{\prime} \Lambda^{\prime}\right\rangle \tag{28}
\end{align*}
$$

while when $\mathrm{l} \neq \mathrm{s}$ we have:

$$
\begin{equation*}
\left(\psi_{i}, \varphi_{j}\right)=\delta_{N^{\prime} N} \sum_{l, \Lambda} a_{l^{\prime} \Lambda^{\prime}}^{\prime} a_{l \Lambda} . \tag{29}
\end{equation*}
$$

For the case of magnetic multipole transitions, the sum of N determinants over all orbits occupied by nucleons can be divided on two sums - over N orbits occupied by neutrons and Z ones taken by protons. Thus for $\mathrm{L}<\mathrm{K}+\mathrm{K}^{\prime}$ we find:

$$
\begin{align*}
& |\mathbf{M}(M \lambda)|^{2}=\left(\frac{e \hbar}{2 M c}\right)^{2}\left(\frac{\hbar}{M \omega_{0}}\right)^{\lambda} \frac{2 \lambda+1}{16 \pi} \times \\
& \mid\left\langle I \lambda K K^{\prime}-K \mid I^{\prime} K^{\prime}\right\rangle\left\{\operatorname{det}\left(\psi_{i_{p}} \varphi_{j_{p}}\right) \sum_{s_{n}=1}^{Z} \operatorname{det}\left(\psi_{i_{n}}^{s_{n}}, \varphi_{j_{n}}\right)\right. \\
& \left.\quad-\operatorname{det}\left(\psi_{i_{n}} \varphi_{j_{n}}\right) \sum_{s_{p}=1}^{Z} \operatorname{det}\left(\psi_{i_{p}}^{s_{p}}, \varphi_{j_{p}}\right)\right\}\left.\right|^{2} \tag{30}
\end{align*}
$$

The determinant elements for $\mathrm{i}_{n}=\mathrm{s}_{n}$ and $\mathrm{i}_{p}=\mathrm{s}_{p}$ correspond to the values $\mathrm{G}_{M L}$ for the neutron and the proton from ref. [3].

## 6. APPLICATION OF THE MODIFIED NILSSON'S MODEL TO THE ODD NUCLEI OF sd SHELL

In refs. [9-12], using M1 and E2 transitions, the influence of the changes in nucleus deformation on the probabilities of electromagnetic transitions in ${ }^{21,23} \mathrm{Na},{ }^{25,27} \mathrm{Al}$ nuclei have investigated (table). The calculated value of the matrix element depends on
two deformation parameters corresponding to the initial and the final states of the nucleus. Therefore, performing the comparison between the theory and the experiment, in the two-dimensional space of the deformation parameters we obtain the regions in which the theoretical and experimental matrix elements co-
incide. Analyzing the bands of transitions from one and the same level or onto one and the same state, we are able to reduce the regions of possible values of deformation parameters for some levels and sometimes even give exact values.

The nucleus deformation parameters in the region $21<A<27$, extracted via comparing the experimental and theoretical values of $B\left(\sigma_{l}\right)$
for the case $L<K+K^{\prime}$

M1 transition

| Nucleus | $E_{i} \rightarrow E_{f}$, <br> $M e V$ | $J_{i}^{\pi} \rightarrow J_{f}^{\pi}$ | $B\left(\sigma_{l}\right)^{e x p}$, <br> $W . u$ | $B\left(\sigma_{l}\right)^{c m}$, <br> $W . u$ | $\eta_{i}$ | $\eta_{f}$ |
| :---: | ---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{21} N a$ | $2.432 \rightarrow 0$ | $1 / 2^{+} \rightarrow 3 / 2^{+}$ | $0.043(5)$ | 0.038 | 2 | 4 |
| ${ }^{23} N a$ | $2.982 \rightarrow 0$ | $3 / 2^{+} \rightarrow 3 / 2^{+}$ | $0.09(2)$ | 0.014 | 2 | 4 |
|  | $\rightarrow 0.44$ | $3 / 2^{+} \rightarrow 5 / 2^{+}$ | $0.014(4)$ | 0.022 | 2 | 4 |

$E 2$ transition

| Nucleus | $E_{i} \rightarrow E_{f}$, <br> $M e V$ | $J_{i}^{\pi} \rightarrow J_{f}^{\pi}$ | $B\left(\sigma_{l}\right)^{\text {exp }}$, <br> $W . u$ | $B\left(\sigma_{l}\right)^{c m}$, <br> $W . u$ | $\eta_{i}$ | $\eta_{f}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{23} \mathrm{Na}$ | $2.982 \rightarrow 0$ | $3 / 2^{+} \rightarrow 3 / 2^{+}$ | $1.3(2)$ | 0.082 | 2 | 4 |
| ${ }^{25} \mathrm{Al}$ | $0.451 \rightarrow 0$ | $1 / 2^{+} \rightarrow 5 / 2^{+}$ | $3.0(5)$ | 2.82 | 0 | 4 |
| ${ }^{25} \mathrm{Al}$ | $2.486 \rightarrow 0$ | $1 / 2^{+} \rightarrow 5 / 2^{+}$ | $0.8(3)$ | 0.07 | 0 | 4 |
| ${ }^{27} \mathrm{Al}$ | $0.843 \rightarrow 0$ | $1 / 2^{+} \rightarrow 5 / 2^{+}$ | $7.5(5)$ | 5.75 | 2 | 4 |
| ${ }^{27} \mathrm{Al}$ | $3.673 \rightarrow 0$ | $1 / 2^{+} \rightarrow 5 / 2^{+}$ | $(\sim 0)$ | 0 | -2 | 4 |

The joint analysis of the transition matrix elements allowing for deformations in the initial and final states and the other data on the low-lying levels depending on the deformation (the position, the quadrupole momenta etc.) also help to determine the values of deformation for these levels more exactly. The deformation parameters of the nuclei in the ground and excited states are usually extracted either from the data or from the theoretical calculations. In both cases the nucleus is treated as a deformed object so that the extracted information is model dependent. The statements about the deformation parameters made due to the calculated probabilities of the transitions between the levels lying in the rotational bands witness that the deformation parameters are different not only for the transitions between one-particle states but also between the rotational levels in the band.

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## ПАРАМЕТРЫ ДЕФОРМАЦИИ ВОЗБУЖДДННЫХ СОСТОЯНИЙ sd-ОБОЛОЧКИ В НЕЧЕТНЫХ ЯДРАХ

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Дан обзор основных моделей, используемых для описания деформированных ядер. Прослеживаются трудности, связанные с однозначностью выбора параметра деформации ядра во вращательных полосах. Расчеты приведенных вероятностей в рамках модифицированной модели Нильсона показывают, что использование разной деформации для возбужденных состояний улучшает согласие с экспериментом. Рассмотрение ведется для ядер sd-оболочки.

# ПАРАМЕТРИ ДЕФОРМАЦІЇ ЗБУДЖЕНИХ СТАНІВ sd-ОБОЛОНКИ В НЕПАРНИХ ЯДРАХ 

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Приведено огляд основних моделей, використовуваних для опису деформованих ядер. Простежуються труднощі, пов'язані з однозначністю вибору параметра деформації ядра в обертальних смугах. Розрахунки наведених імовірностей в рамках модифікованої моделі Нільсона показують, що використання різної деформації для збуджених станів поліпшує згоду з експериментом. Розгляд ведеться для ядер sd-оболонки.


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