GENERATION OF INTENSE ULTRA-SHORT X-RAY PULSES IN WAKE FIELD UNDULATOR

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The generation of X-rays based on the interaction mechanism of a relativistic charged particles with alternative wake fields induced in periodic structures are studied. The optimum wake field characteristics providing the maximum flux of wake field undulator radiation in an X-ray range are obtained. Using the data on beam parameters of varied photo-injector projects, the X-ray fluxes are calculated.

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1. INTRODUCTION

Because of numerous potential applications of intense X-ray beams with ultra-short duration, such ways of their generation as the self-amplifying spontaneous emission in FEL's and the inverse Compton scattering (ICS) of intense optical laser radiation on high charged bunches are powerfully developed at the present time [1,2]. The ICS-based X-ray sources with photons energies 10...40 keV (for use in medicine and material characterization) are received a large development due to compactness and relatively low cost [2]. The X-ray beam fluxes of $10^9 ... 10^{10} photons/sec$ in a 1% bandwidth are usually required for biological and medical imaging. But until now, the intensity of the most available ICS-based X-ray sources is still less than $10^8\, photons/sec$. Therefore, the search for methods that be able to provide the needed photon yields remains a relevant topic. In this connection, designing the compact wake field undulator (WFU) with sub-millimeter period [3,4], due to advanced accelerator technology, can open a new opportunities to obtain ultra-short high-brightness X-ray beams.

The goal of this work consists in obtaining optimal conditions needed to generate high-flux X-ray pulses with ultra-short duration by ultra-relativistic electron bunches moving through an weakly corrugated rectangular waveguide with sub-millimetre period.

2. WFU CHARACTERISTICS

Let us consider the WFU radiation mechanism. The wake forces induced by a bunch of relativistic charged particles in a periodic structure can be expressed as a Floquet's series

$$\vec{F}(\vec{r},t) = \sum_{p=-\infty}^{\infty} \vec{F}^{(p)}(\vec{r}_\perp, t - z/v_z)\, e^{i\frac{2\pi p}{D}z}\,, \qquad (1)$$

where $\vec{F}^{(p)}$ is the p^{th} space harmonic of the wake force, v_z is the bunch velocity, D is the waveguide period. The synchronous ($p = 0$) harmonic acting on the bunch results in the beam loading and beam break up effects well known in RF linear accelerators. It is usually assumed that the non-synchronous ($p \neq 0$) spatial harmonics of the fields do not contribute to change of the beam energy on the average by the period. However under certain conditions (for example for off-axis particles in the corrugated waveguide used in RF linacs) the non-synchronous spatial harmonics of transverse components of the radiation reaction force give rise to undulating particles with alternating transverse velocity

$$\vec{v}_\perp = \frac{ic}{2\gamma} \sum_{p\neq 0} \vec{K}_\perp^{(p)} e^{i\frac{2\pi p}{D}z}\,, \qquad (2)$$

where the WFU or deflection parameter is defined as follows

$$\vec{K}_\perp^{(p)} = -\frac{\vec{F}_\perp^{(p)} D}{p\pi m c v_z}\,. \qquad (3)$$

Here γ is the Lorentz factor ($\gamma^2 >> 1$), c is the velocity of light, m is the electron mass of rest.

The charged particle undulation with the alternating velocity Eq. (2) causes emission of the undulator-type radiation. In an X-ray range, wavelengths of this radiation are given as (see Ref.[5])

$$\lambda_p = \frac{D}{2p\gamma^2}\left(1 + \frac{1}{2}\sum_{p'\neq 0}\left|K_\perp^{(p')}\right|^2\right)\,. \qquad (4)$$

As it is known, the undulator parameter of a conventional magnetic undulator is proportional to the undulator period D ($K_\perp \sim D$). This property restricts the use of a magnetic undulator with very short periods. The WFU parameter, unlike one for the magnetic undulator, is independent on the period, and has inversely proportional dependence on an RF structure transverse size $l_\perp$, ($K_\perp \sim 1/l_\perp$) [4].

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This peculiarity allows to use the WFU with sub-millimeter periods. In this case the electron energy can be saved more than one order comparably with the conventional centimeter-period magnetic undulator. In Ref. [4] the rectangular waveguide with periodically perturbed walls is considered as the prototype of a wake field undulator. The wake forces induced by an ultra-relativistic electron bunch with 3D Gaussian charge density distribution are calculated by a perturbation method. So, using the results presented in Ref.[4] for Gaussian bunch moving along a planar waveguide with weakly-corrugated metallic surfaces (see Fig.1), we can express an absolute value of the WFU parameter in the following form

$$\left|K_x^{(p)}\right| = 8\pi N \left|\varepsilon C_p\right| \frac{r_e}{w} e^{-\frac{(v_z\tau)^2}{2\sigma_z{}^2}} \left|\sum_{m=1}^{\infty} e^{-\frac{(\pi m)^2(\sigma_x{}^2+\sigma_y{}^2)}{2w^2}} \times \right.$$
$$\times \sin\left[\frac{\pi m}{w}\left(y+\frac{w}{2}\right)\right] \sum_{n=0}^{\infty} \frac{(-1)^n X_{n,m}(x)}{1+\delta_{0,n}} \times$$
$$\left. \times W\left(\frac{\omega_{m,p,n}\sigma_z}{v_z\sqrt{2}} - i\frac{v_z\tau}{\sqrt{2}\sigma_z}\right)\right| , \tag{5}$$

where

$$X_{n,m} = \frac{\sin\left(\frac{\pi m}{2}+\frac{\pi m y_0}{w}\right)}{\mathrm{sh}\left(\frac{2\pi m}{w}b_0\right)} \left\{ \mathrm{sh}\left(\frac{\pi m(x_0+b_0)}{w}\right) \times \right.$$
$$\times \cos\left(\frac{\pi n}{2b_0}(x+b_0)\right) + \mathrm{sh}\left(\frac{\pi m(x_0-b_0)}{w}\right) \times$$
$$\left. \times \cos\left(\frac{\pi n}{2b_0}(x-b_0)\right)\right\} , \quad W(z) = e^{-z^2} erfc(-iz) ,$$

r_e is the classical radius of the electron , $\omega_{m,p,n}$ are the eigenfrequencies, $v_z\tau = v_z t - z$ the relative longitudinal coordinate of a particle, N is the number of electrons in the bunch, σ_x, σ_y, σ_z are the *rms* bunch dimensions, x and y are transverse coordinates of the particle; x_0 and y_0 are the transverse coordinates of the bunch crest. The other geometry sizes can be seen in Fig.1.

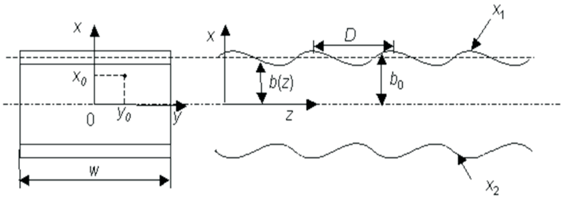

Fig.1. *A rectangular waveguide with periodic surfaces;* $b(z) = b_0[1+\varepsilon\sum_p C_p cos(2\pi pz/D)]$ *is the surface contour with the small parameter* ε*,* $(0 < \varepsilon << 1)$*,* D *is the period,* z *is a longitudinal coordinate,* w *is the waveguide width*

3. WFU OPTIMIZATION

Let us obtain the maximum photon flux from the WFU. For this we must find relations between dimension of the corrugated waveguide and the bunch at which the absolute value of the WFU parameter reaches the maximum. As it is shown in Ref [4] the wake field exited by a bunch with $\sigma_z \sim D$, is localized within the bunch and moves synchronously with it without storing into the waveguide. In addition the transverse component of the alternating wake force reaches the maximum at the maximum of the charge density. So, such the wake force distribution is most suitable for using it as a pump field.

In Fig.2 the distribution of the absolute value of the WFU parameter $|K_x^{(1)}|$ is given for the bunch (with parameters $eN = 1\,nC$, $\sigma_z = 1.2D, \sigma_x = \sigma_y = 10\,\mu m$) that moves along the corrugated waveguide (with the dimensions $b_0 = D = 100\,\mu m$, $w = 5b_0$, $\varepsilon = 0.1$, and the simplest surface profile $b(z) = b_0[1+\varepsilon\cos(2\pi z/D)]$) at the transverse bunch disposition $x_0/b_0 = 0.7$. Ibidem, the Gaussian distribution of the bunch charge is imagined by the dotted line.

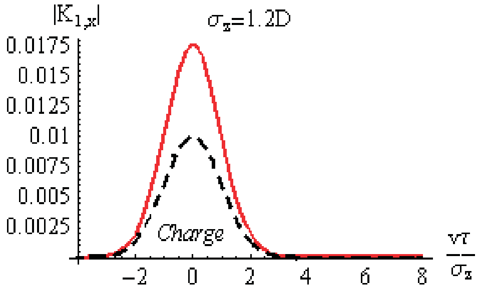

Fig.2. *The charge and* $|K_x^{(1)}|$ *distribution*

As a first step of the WFU optimization, let us build the dependence of the $|K_x^{(1)}(0)|$ taken at the crest bunch ($v_z\tau = 0$) on relative waveguide width w/b_0.

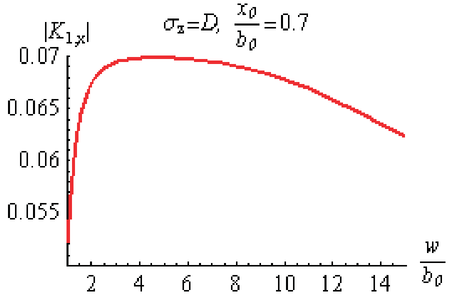

Fig.3. *The* $|K_x^{(1)}(0)|$ *v.s. relative width* w/b_0

Analysis of dependencies such as built in Fig.3 for the different relative transverse bunch positions ($x_0/b_0 = 0.8$, 0.7, 0.6, 0.4) shows that there is a simple correlation between the optimal values for which the $|K_x^{(1)}(0)|$ reaches the maximum

$$w/b_0 \approx 7.5(1.33 - x_0/b_0) . \tag{6}$$

On the other hand the maximal value of the WFU parameter is obtained if the bunch moves as close as possible to the corrugated surface of the waveguide, due to surface-wave character of the wake field [4]

$$x_{max}/b_0 = 1 - \varepsilon - 2\sigma_x/b_0 . \tag{7}$$

So, substituting Eqs. (6) and (7) into Eq. (5), we can find the $|K_x^{(1)}(0)|$ as the function of the waveguide vertical size b_0 for different transverse bunch sizes $\sigma_x = \sigma_y = 10\,\mu m$ and $\sigma_x = \sigma_y = 1\,\mu m$(see the next Fig.4).

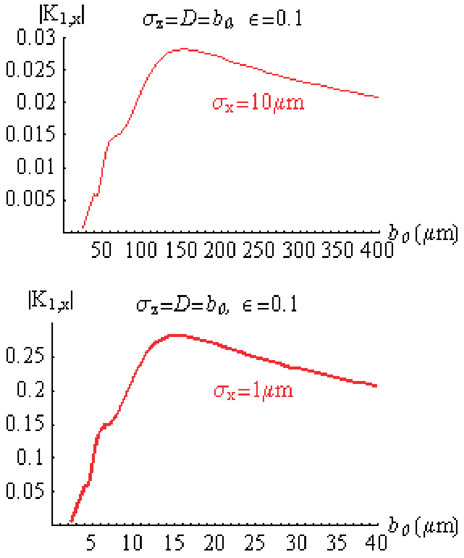

Fig.4. *The* $|K_x^{(1)}(0)|$ *v.s. a waveguide vertical size* b_0

As shown in Fig.4, the $|K_x^{(1)}(0)|$ reaches the maximum at the optimal values: $b_0 = 150\,\mu m$, $b_0 = 15\,\mu m$ for two chosen bunch sizes, accordingly. Scaling analysis of Eq. (5) indicates that the WFU parameter increases in inverse proportion with proportional reduction of all geometric dimensions of the waveguide and bunch, as it is confirmed by Figs.4.

Fixing the all obtained optimal values: $b_0 = 150\,\mu m$, $\sigma_x = \sigma_y = 10\,\mu m$, $b_0 = 15\,\mu m$, $\sigma_x = \sigma_y = 1\,\mu m$, one can correct the small parameter ε, the relative height of the corrugations (Fig.5). As it follows from the last figures, the optimal value of the small parameter is $\varepsilon = 0.12$.

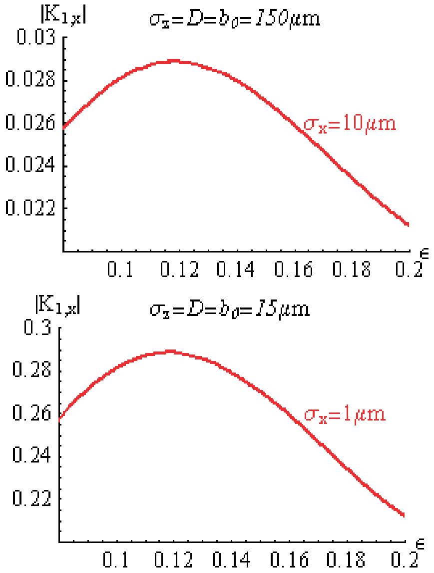

Fig.5. *The* $|K_x^{(1)}(0)|$ *v.s. a small parameter* ε

4. CALCULATION OF X-RAY FLUXES

On the analogy with the conventional magnetic undulator [6] consisting of N_u periods, the WFU radiation flux near the p^{th} harmonic with bandwidth $\Delta\omega/\omega \cong 1/pN_u$ containing in the central cone $\theta_p \approx 1/\gamma\sqrt{pN_u}$, can be obtained in the form [3]

$$F^{(p)} \approx \alpha \frac{\pi}{2} f_{rep} N \left\langle \left| K_x^{(1)} \right|^2 \right\rangle , \qquad (8)$$

where α is the fine-structure constant, f_{rep} is the bunch repetition frequency, N is the number of electrons in the bunch, $\langle ... \rangle = (v_z/eN) \int_{-\infty}^{\infty} d\tau \int \int_{S_\perp} d^2\vec{r}_\perp \rho(\vec{r}_\perp, \tau)...$ is the bunch averaging,$\rho(\vec{r}_\perp, \tau)$ is the charge density of the bunch. After averaging over the bunch in Eq. (8), the WFU radiation flux can be written as

$$F^{(p)} = \alpha \frac{\pi}{2} f_{rep} N^3 \frac{(8\pi)^2}{\sqrt{2\pi}} |\varepsilon C_p|^2 \left(\frac{r_e}{w}\right) \int_{-\infty}^{\infty} ds\, e^{-\frac{3s^2}{2}} u_p(s) , \qquad (9)$$

where the function $u_p(s)$ is defined as

$$u_p(s) \equiv \left| \sum_{m=1}^{\infty} e^{-\frac{(\pi m)^2(\sigma_x^2+\sigma_y^2)}{2w^2}} \sin\left[\frac{\pi m}{w}\left(y_0 + \frac{w}{2}\right)\right] \times \right.$$
$$\left. \times \sum_{n=0}^{\infty} \frac{(-1)^n X_{m,n}(x_0)}{1+\delta_{0,n}} W\left(\frac{\omega_{m,p,n}\sigma_z}{v_z\sqrt{2}} - i\frac{s}{\sqrt{2}}\right) \right|^2 . \qquad (10)$$

One can show that for bunches with $\sigma_z \geq D$, the function $u_p(s)$ is interpolated by $u_p(s) = u_p(0) - [u_p(0) - u_p(-1)]s^2$. So, the flux Eq. (9) can be easily calculated by the formula

$$F^{(p)} \approx \alpha f_{rep} N^3 \pi^3 \frac{\sqrt{3}}{2} \left(\frac{8}{3}\right)^2 |\varepsilon C_p|^2 \left(\frac{r_e}{w}\right)^2 \times$$
$$\times \left[2u_p(0) + u_p(-1)\right] . \qquad (11)$$

As an instance, let us calculate flows of X-rays with the sub-picosecond duration ($\sigma_\tau \approx 0.5\,ps$) and $30\,keV$ photon energy generated by an electron beam in the sub-millimetre WFU consisting of $N_u = 100$ periods. The optimal sizes of the WFU and beam obtained by Eqs. (6) and (7) are given in Table 1.

Table 1. *The optimal parameters of the WFU and bunch*

$b_0 = D$ μm	w μm	ε	x_{max} μm	σ_z μm	σ_x, σ_y μm	θ_1 rad
150	656	0.12	112	150	10	7×10^{-5}

Substituting the beam parameters of the varied photo injector projects [7] into Eq. (11), we calculate the fluxes given in Table 2. Here it is supposed that the electron beams from the selected photo injectors be able to accelerate to the energy $690\,MeV$ and the to compress the bunches to required sizes given in Table 1.

Table 2. *The X-ray fluxes from varied photo injectors*

Lab	$q = eN$ nC	f_{rep} MHz	$F^{(1)}$ $ph/s/1\%bw$
$JLab$	0.112	75	4.45×10^{9}
$JLab$	0.133	748.5	5.75×10^{10}
$Cornel$	0.077	1300	1.94×10^{10}
$Cornel$	1	10	3.267×10^{11}
$Boeing$	7	6.7	2.2×10^{11}
FZR	1	1	3.267×10^{10}
BNL	1.42	357.87	3.29×10^{13}
BNL	10	10	3.267×10^{14}
$LANL$	1	100	3.267×10^{12}
CEA	3	0.1	8.82×10^{10}
$DESY$	1	0.072	2.35×10^{9}
$DESY$	1	0.0325	1.06×10^{9}
$BESSY$	2.5	0.025	1.276×10^{10}

5. CONCLUSIONS

In this paper, the optimal relations between sizes of the WFU and bunches at which the $|K_x^{(1)}|$ reaches the maximum providing the maximum WFU radiation fluxes of in the X-ray range are obtained. It is shown that the average X-ray fluxes can attain $10^{9}...10^{14}\, photons/sec/1\%bw$, that one can open opportunities to use this X-rays source in medicine imaging and in many other applications.

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ГЕНЕРАЦИЯ ИНТЕНСИВНОГО РЕНТГЕНОВСКОГО ИЗЛУЧЕНИЯ УЛЬТРАКОРОТКОЙ ДЛИТЕЛЬНОСТИ В КИЛЬВАТЕРНО-ПОЛЕВОМ ОНДУЛЯТОРЕ

Анатолий Опанасенко

Представлены результаты исследования процесса генерации рентгеновского излучения, основанного на механизме взаимодействия заряженных частиц со знакопеременными кильватерными полями, индуцируемыми в периодических структурах. Найдены оптимальные соотношения между размерами гофрированного волновода и размерами электронного сгустка, при которых достигаются максимальные потоки излучения.

ГЕНЕРАЦІЯ ІНТЕНСИВНОГО РЕНТГЕНІВСЬКОГО ВИПРОМІНЮВАННЯ УЛЬТРАКОРОТКОЇ ТРИВАЛОСТІ В КІЛЬВАТЕРНО-ПОЛЬОВОМУ ОНДУЛЯТОРІ

Анатолій Опанасенко

Представлено результати дослідження процесу генерації рентгенівського випромінювання, заснованого на механізмі взаємодії заряджених частинок зі знакозмінними кільватерними полями, що індуковані у періодичних структурах. Знайдено оптимальні співвідношення між розмірами гофрованого хвилеводу та електронного згустка, при яких досягаються максимальні потоки випромінювання.