THRESHOLD OF SPONTANEOUS EMISSION AMPLIFICATION BY RELATIVISTIC ELECTRON BEAM IN UNDULATOR

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Motion of a relativistic beam of electrons through an undulator is considered, with the influence of incoherent fields its spontaneous radiation on this motion being taken into account. Interaction of electrons with these fields is shown to result in the increase of electron-momentum spread in the beam. The conditions of high-gain self-amplified spontaneous emission process realization in the ultrashort-wavelength FELs are discussed

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1. INTRODUCTION

As it is known, the mode of high-gain self-amplified spontaneous emission (SASE) by a relativistic beam of electrons, moving in undulator, allows to produce intensive coherent short-wavelength radiation in a free-electron lasers (FEL) (see, for example, [1-3]). Thus, in initially uniform beam collective interaction of electrons with the field of radiation leads to a grouping of electrons in coherently radiating bunches and to the growth of intensity of electromagnetic radiation. For realization of such a mode of amplification beams of ultrarelativistic electrons are necessary with sufficiently high average density of electrons and small energy spread at which electrons of a beam can be grouped by a total field of their spontaneous radiation in coherently radiating bunches.

However, when beam moves in undulator there can be an increase in electrons-momentum spread due to the influence of their incoherent fields of spontaneous radiation on motion of electrons [4]. This effect is not connected with the development of collective instability at amplification by an electron beam of its own spontaneous undulator radiation. It is determined by pairwise interaction of individual electrons of beam in undulator at a stage of spontaneous radiation. The spread of electrons on a longitudinal momentum has been found in [4] in a limiting case of unbounded electron beam by radius. For determination of conditions of high-gain self-amplification spontaneous emission mode realization in ultrashort-wavelength FELs it is necessary to generalize such a theory on finite values of beam radius. In the given work results of theoretical investigation of such model - the limited by radius beam of pointed electrons, moving in helical undulator are presented.

2. MOTION AND FILD OF A TEST PARTICLE

Let's consider transverse spatial periodic helical magnetic field of undulator $\mathbf{H_u}$

$$\mathbf{H}_u = H_0\left[\mathbf{e}_x \cos(k_u z) + \mathbf{e}_y \sin(k_u z)\right], \qquad (1)$$

where $k_u = 2\pi/\lambda_u$, H_0 и λ_u are the amplitude and the period of the magnetic field, $\mathbf{e}_x$, $\mathbf{e}_y$ are the unit vectors along axes OX and OY the Cartesian system of coordinates.

Let the cylindrical beam of relativistic electrons with radius r_b, the energy $mc^2\gamma_0$ of electron and uniform average density n_0 move in a positive direction of an axis z, where γ_{0s} is Lorentz factor. The trajectory of individual electron (for example, s-th) in undulator can be present in the form

$$r_s = r_{0s} - \mathbf{e}_x r_u \sin(k_u z_s) + \mathbf{e}_y r_u\left[\cos(k_u z_s) - 1\right] + z_s, \qquad (2)$$

where $z_s = \mathrm{v}_{0s}(t - t_{0s}) + \Delta_s(t)$, $r_{0s}=\{x_{0s}, y_{0s}, 0\}$ and v_{0s} are a radius-vector and longitudinal velocity of s-th electron at initial moment of time t_{0s}, when electron crosses a plane z=0, Δ_s is displacement of a trajectory of electron in undulator relative to its equilibrium trajectory, $r_u = cK/(k_u \mathrm{v}_{0s}\gamma_{0s})$, $K = |e|H_0/(mc^2 k_u)$, e, m are the charge and mass of electron, c-speed of light.

Considering only a magnetic component of Lorentz's force, the equation of longitudinal motion of individual (test) electron in the field (1) and the fields of spontaneous undulator radiation of others can be written as follows:

$$\frac{d\,\mathrm{p}_{zi}}{dt} = \mathrm{F}_z[x_i(t),t] = \sum_s F_z^{(s)}[x_i(t),t;x_s], \qquad (3)$$

$$F_z^{(s)} = |e|\beta_\perp \mathrm{Re}\left[H_y(r,t;q_{0s}) + iH_x(r,t;q_{0s})\right]e^{ik_u z}, \qquad (4)$$

where $\mathbf{p}_i$ is an momentum of i-th electron; $F_z^{(s)}(x,t;x_s)$ is a longitudinal component of force of pair electron interaction via the electromagnetic field of one of them (s-th electron); $x_s(t)=\{\mathbf{r}_s(t), \mathbf{p}_s(t)\}$ is a set of coordinates and an momentum of s-th electron, $\beta_\perp = K/\gamma_{0s}$.

In approximation of a small value of undulator parameter $K^2<<1$ and $\gamma_0>>1$, expression for field **H** derived from formulas for a field of a charge, moving with acceleration (see, for example, [5]) takes the form

$$\mathbf{H} = |e|\beta_\perp (k_0/R_*)\mathrm{Re}\left\{\left(\mathbf{e}_x - i\mathbf{e}_y\right)\exp(\mathrm{i}k_u z - i\psi)\times \left[\left(1 - \frac{i}{k_0 R_*}\right)\left(\beta_{0s} + \frac{\delta z_s}{R_*}\right) - \frac{\beta_{0s}}{k_0^2 R_0^2} - \frac{\beta_{0s}\rho_s^2}{2R_*^2\gamma_{os}^2}\right]\right\}, \qquad (5)$$

$$c(t - t_{0s}) > \left(z^2 + \rho_s^2\right)^{1/2}, \qquad (6)$$

where $\delta z = z - v_0(t - t_{0s}) - \Delta_s$, $\psi = k_u\gamma_0^2(\delta z + \beta_0 R_*)$, $\rho_s = \left[(x - x_s)^2 + (y - y_s)^2\right]^{1/2}$, $R_* = \left[(\delta z)^2 + \rho_s^2/\gamma_0^2\right]^{1/2}$, $k_0 = \beta_0\gamma_0^2 k_u$.

3. ELECTRON-MOMENTUM SPREAD

The electromagnetic field produced by a uniform beam of electrons is random. According to the equation (3) the random force will act on electrons. Thus, changes in time of the average square value of a longitudinal electron-momentum can be presented in the form

$$\frac{d}{dt}\left\langle(\Delta p_{zi})^2\right\rangle = 2\int_{t_{0i}}^{t}\left\langle \delta F_z[x_i(t'),t']\,\delta F_z[x_i(t),t]\right\rangle dt', \quad (7)$$

where $\Delta p_z = p_z - \langle p_z\rangle$, $\delta F_z = F_z - \langle F_z\rangle$, brackets $\langle\ldots\rangle$ denote ensemble average.

Considering change of a momentum on a time interval smaller than time, for which motion of an electron will essentially change. Then in subintegral expression of the Eq. (7) it is possible to replace $x_i(t)$ with unperturbed coordinates and momentum of electron in undulator $x_s^{(0)} = x_s|_{\Delta=0}$. Average value of subintegral expression in the right-hand side of the Eq. (7) can be found by means of distribution function in phase space of coordinates and momentum of all electrons at initial moment of time. Neglecting correlations between electrons, and assuming also, that electrons are monoenergetic on entrance in undulator the equation (7) can be written in the form:

$$\frac{d}{dt}\left\langle(\Delta p_{zi})^2\right\rangle = 2n_0\int_{t_{0i}}^{t}dt'\int dq_{0s}F_z^{(s)}\left[\mathbf{r}_i^{(0)}(t),t;x_s(t,q_{0s})\right] \times F_z^{(s)}\left[\mathbf{r}_i^{(0)}(t'),t';x_s(t',q_{0s})\right], \quad (8)$$

where $q_{0s} = (x_{0s}, y_{0s}, \mathrm{v}_0 t_{os})$.

In the right-hand side of this equation the region of integration on initial coordinates q_{os}, according to (6), is defined by conditions

$$z_s(t') \ge \gamma_0^2(\delta z_{si} + \beta_0 R_*), \quad (9)$$
$$r_{\perp s} \equiv \left(x_{os}^2 + y_{0s}^2\right)^{1/2} \le r_b .$$

Expression in the right-hand side of Eq.(8) depends on force of pairwise interaction of electrons. When thermal motion of beam electrons is neglected this force does not depend on time, but on the difference of initial coordinates of electrons-radiators and test electron.

Let's consider a test particle, moving near the beam axis: $r_{\perp i} << r_b$. Since force of interaction of electron-radiators and test electron is axially symmetric, let's substitute variables x_{0s}, y_{0s}, t_{0s} for ξ and θ, defined by formulas

$$\xi = k_0 R_*(\delta z_{si}, \rho_{si}/\gamma_0), \quad \theta = arctg\left(\frac{\rho_{si}}{\delta z_{si}\gamma_0}\right),$$

where $\rho_{si} = \left[(x_{0s} - x_{0i})^2 + (y_{0s} - y_{0i})^2\right]^{1/2}$, $\delta z_{si} = \mathrm{v}_0(t_{0s} - t_{0i})$.

Taking into account the fields of only those electrons, which move behind considered (test) electron ($z \ge z_s(t)$) the following expression for the rate of change of the average square-value deviation from the mean value of a longitudinal momentum of test electron is obtained

$$\frac{d}{dz}\left\langle(\Delta p_z)^2\right\rangle = \frac{4\pi}{c^2}e^4\mathrm{K}^4 k_u n_0\int_0^z dz'\int_0^{\pi/2}d\theta\,\sin\theta \int_0^{\xi_m}d\xi\left[\left(\beta_0 + \cos\theta - \frac{\beta_0}{\xi^2} - \frac{\beta_0}{2}\sin^2\theta\right)\sin\psi + (\beta_0 + \cos\theta)\frac{\cos\psi}{\xi}\right]^2, \quad (10)$$

where $\psi = (1 + \cos\theta/\beta_0)\xi$.

Here we have passed to an independent variable z a distance from an undulator entrance.

The values of the upper limit of integration $\xi_m(\theta)$ are found from expressions (9). Hence, at $z'<z_r$ from (9) it follows for $\xi_m(\theta)$

$$\xi_m = \xi_{\inf}(\theta) \equiv \frac{\beta_0 k_u z'}{(\cos\theta + \beta_0)}, \quad (11)$$

where $z_r = \gamma_0\beta_0 r_b$.

At $z'>z_r$ the limit of integration is equal to

$$\xi_m = \xi_{\inf} \text{ at } \theta \le \theta_*, \quad \xi_m = \frac{k_u z_r}{\sin\theta} \text{ at } \theta > \theta_*, \quad (12)$$

where $\theta_* = \arccos\left(\frac{1-\alpha\beta_{0z}}{1+\alpha}\right)$, $\alpha = (r_b\gamma_{0z}/z')^2$.

At $z<z_r$, integrating the right-hand side of the Eq.(10) in view of (11), is obtained [4]

$$\left\langle(\Delta p_z)^2\right\rangle = \frac{5\pi}{16}e^4\mathrm{K}^4 k_u^2 n_0\frac{z^3}{c^2}. \quad (13)$$

At greater distances from the undulator entrance at $z>>z_r$, integrating (10) in view of (12), the following estimation of square value of the longitudinal electron-momentum spread is obtained

$$\left\langle(\Delta p_z)^2\right\rangle = 3\pi e^4\mathrm{K}^4 k_u^2 n_0\frac{z^2}{c^2}z_r. \quad (14)$$

4. DISCUSSION

Formulas (13) and (14) describe dependence of longitudinal electron-momentum spread from the distance passed in undulator and from parameters of an electron beam and undulator. According to (13) and (14) the root-mean-square value of a longitudinal electron-momentum may be written as follows:

$$\left|\left\langle(\Delta p_z)^2\right\rangle\right|^{1/2} = \left|\langle\Delta p_z\rangle_R\right|\sqrt{\mathrm{N}_{eff}}, \quad (15)$$

where $\langle\Delta p_z\rangle_R \equiv \frac{z}{v_{0z}}F_R$, $F_R = -(2/3)(r_0 H_0\gamma_0)^2\beta_0^3$ is the force of radiative deceleration of electron, $r_0=e^2/mc^2$ is the classical electron radius,

$N_{eff} = \frac{3}{2\pi}N z/\lambda_u$ at $z<z_r$ and $N_{eff} = \frac{9}{2}N z_r/\lambda_u$ at $z>>z_r$, $N = n_0\lambda_u^3/8\gamma_0^4$.

One can see from (13)-(15), that the electron-momentum spread increases in an electron beam moving through an undulator. It follows from the formula (15), that the root-mean-square value of a longitudinal electron-momentum is proportional to $z^{3/2}$ at $z<z_r$ [4]. Such a dependence of the spread from z is connected with the increase in a deviation of electron-momentum from the average value under the action of forces of pairwise interaction, that are independent on time, on the one hand, and with the increase in the number of electrons (N_{eff}) in the region of whose fields the test electron is located on other. Thus such an increase of N_{eff} occurs through the increase in volume of this region.

The spread on a longitudinal electron-momentum increases mainly due to the action of forces of pairwise interaction of electrons at $z>z_r$, since the number of electrons effectively interacting with the test electron practically does not increase because of the finite cross-section of a beam.

In the formula (15) we have expressed momentum spread through the average losses of a momentum caused by radiation deceleration individual electron, and N - number of electrons in the cube characteristic length on edge which is equal to the radiation wavelength in the system of a beam rest frame. Such representation of spread shows, that root-mean-square momentum spread grows more quickly at N>1, than the change of momentum caused by radiative deceleration. It is necessary to note, that the collective radiative instability of an electron beam in an undulator for the self-amplification mode of FEL is only realized when the strong inequality N>>1 is satisfied [6].

The increase in spread of electron-momentum considered above can be neglected, if at a characteristic length of $z_{sat} = \lambda_u / \rho$ where exponential growth of SASE saturates, the momentum spread will be small enough: $\Delta p_z / p_{0z} < \rho$, where ρ is the dimensionless spatial growth rate [1, 2, 6, 7] (for example, in one-dimensional model $\rho = \left(K^2 n_0 r_0 \lambda_u^2 / 16\pi\right)^{1/3} / \gamma_0$). Hence, condition

$$\rho > 2\pi\, K\left(\frac{r_0}{\lambda_u}\gamma_0\right)^{1/2}\left(2N\frac{z_r}{\lambda_u}\right)^{1/4} \qquad (16)$$

is necessary to fulfill for realization of high-gain self-amplified spontaneous emission process.

Here, $z_{sat} >> z_r$ is considered for parameters of electron beams and undulators in the range of ultrashort-wavelenght FELs.

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ПОРОГ УСИЛЕНИЯ СПОНТАННОГО ИЗЛУЧЕНИЯ РЕЛЯТИВИСТСКОГО ПУЧКА ЭЛЕКТРОНОВ В ОНДУЛЯТОРЕ

В.В. Огнивенко

Рассмотрено движение релятивистского пучка электронов в ондуляторе с учетом влияния на это движение некогерентных полей их спонтанного излучения. Показано, что взаимодействие электронов с этими полями приводит к увеличению разброса электронов по импульсам в пучке. Обсуждаются условия реализации процесса интенсивного самопроизвольного усиления спонтанного излучения в ультракоротковолновых ЛСЭ.

ПОРІГ ПІДСИЛЕННЯ СПОНТАННОГО ВИПРОМІНЮВАННЯ РЕЛЯТИВІСТСЬКОГО ПУЧКА ЕЛЕКТРОНІВ В ОНДУЛЯТОРІ

В.В. Огнівенко

Розглянуто рух релятивістського пучка електронів в ондуляторі з урахуванням впливу на цей рух некогерентних полів їхнього спонтанного випромінювання. Показано, що взаємодія електронів із цими полями призводить до збільшення розкиду електронів по імпульсах у пучку. Обговорюються умови реалізації процесу інтенсивного самочинного підсилення спонтанного випромінювання в ультракороткохвильових ЛВЕ.