COMPTON RADIATION OF AN ELECTRON IN THE FIELD OF RUNNING PLANE LINEAR POLARIZED ELECTROMAGNETIC WAVE

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Results of integration of Lorentz force equation for a relativistic electron, moving in the field of running, plane, linear polarized electromagnetic wave are presented in the paper. It is shown that electron velocities in the field of the wave are almost periodic functions of time. Expansion of the electromagnetic field in a wave zone into generalized Fourier series was used for calculations of angular spectrum of electron radiation intensity. Expressions for the radiation intensity spectrum are presented in the paper. The derived results are illustrated for electron and laser beam parameters of NSC KIPT X-ray generator NESTOR. Simultaneously, derived expressions give possibilities to investigate dependence of energy and angular Compton radiation spectrum on phase of interaction and the interacting wave intensity.

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1. INTRODUCTION

Now, the method of generation of intense, short-wave radiation under interaction of an intense electromagnetic wave with relativistic electrons moving toward the wave front (Compton backscattering) is discussed widely [1-3]. Analytical investigations of interaction of an electromagnetic wave with a relativistic electron can be carried out as well with quantum electrodynamics as with classical electrodynamics approaches [4]. According to the generally accepted quantum electrodynamics approach interaction of an electron with an electromagnetic wave is described by Klein-Nishyna formula that is applicable only for an interacting wave with low intensity [5, 6]. In other words, the model is adequate when one electron interacts simultaneously with one photon only. If external field is more intensive (the number of photons in an oscillation mode N_{ph} are >> 1) the further derivation should be produced. At the limit (at $N_{ph} \rightarrow \infty$), according to Bohr correspondence principle, scattering of an electromagnetic wave on an electron should be described as with quantum as well as with classical electrodynamics. However, it is worth mentioning, that development of theory of interaction of an intense electromagnetic wave with an electron is a very time-consuming and complicated way [see for example 7]. In the same time, as it seems to us, use of the classical electrodynamics approach allows to consider the task with simpler methods and to investigate both an electron dynamics in the field of external electromagnetic field as effect of the external field intensity on spectrum and intensity of the generated radiation. The characteristic feature of the presented work is derivation of the accurate solutions of a Lorentz force equation for a relativistic electron, moving in the field of running, plane, linear polarized electromagnetic wave that are almost periodic functions of time. As a result of this fact, the expansion in generalized Fourier series can be used instead Fourier integral in the radiation spectrum derivation.

2. ELECTRON TRAJECTORIES IN THE FIELD OF ELECTROMAGNETIC WAVE

Let us consider a relativistic electron with energy E_0, moving in laboratory coordinate frame under angles $\alpha_1, \alpha_2, \alpha_3$ toward plane, linear polarized, electromagnetic wave with field intensity E.

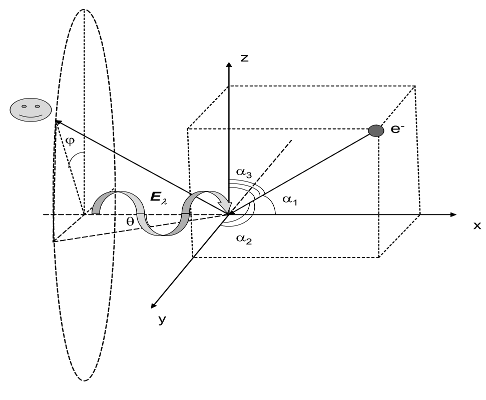

Fig.1. *The scheme of Compton scattering*

The wave is running along x axis (Fig.1), φ and θ are polar angles to an observer. Lorentz force equation for a relativistic electron moving in the external

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electromagnetic field can be written in the following form:

$$\frac{d}{dt}(m\overrightarrow{v}) = c\vec{E} + \frac{e}{c}[\vec{v}\vec{H}] \ , \qquad (1)$$

where $m = m_0/\sqrt{1-\beta^2}$, $\beta = v/c$, m_0 is the rest mass of the electron, c is velocity of light, e is electron charge, is an electron velocity vector, t is time, $\overrightarrow{E}$, $\overrightarrow{H}$ are vectors of electric and magnetic field. Electrical component of the field can be written as the following:

$$E_z = E\cos\left[2\pi\nu\left(t-\frac{x}{c}\right)+\delta\right] \ , \qquad (2)$$

where ν is the field frequency, δ is an initial value of the wave phase relative to the interacting electron, E_z is projection of electrical components of the field onto axis z. Other projections are $E_x \equiv 0, E_y \equiv 0$. Let normal to the front of the running wave $\overrightarrow{n}$ will have direction that coincides with x axis. Vector of magnetic field is:

$$\vec{H} = \left[\vec{n},\vec{E}\right] \ , \qquad (3)$$

where $\vec{n} = \vec{n}_x, \vec{n}_x$ is unit vector of x axis. Projecting (1) in to coordinate frame axes, we obtain:

$$\begin{aligned}
\frac{1}{c}\frac{d}{dt}(\varepsilon\beta_{zt}) &= e(1-\beta_{xt})E\cos\left[2\pi\nu\left(t-\frac{x}{c}\right)+\delta\right] \ , \\
\frac{1}{c}\frac{d}{dt}(\varepsilon\beta_{xt}) &= e\beta_{zt}E\cos\left[2\pi\nu\left(t-\frac{x}{c}\right)+\delta\right] \ , \\
\frac{1}{c}\frac{d}{dt}(\varepsilon\beta_{yt}) &= 0 \ , \qquad (4)
\end{aligned}$$

where ε is electron energy, $\beta_{xt} = \frac{1}{c}\frac{dx}{dt}$, $\beta_{zt} = \frac{1}{c}\frac{dz}{dt}$, $\beta_{yt} = \frac{1}{c}\frac{dy}{dt}$. Each out of 4 equations can be integrated at least one time and as a result we obtain:

$$\begin{aligned}
\beta_{xt} &= \frac{1}{c}\frac{F^2+1-B^2}{F^2+1+B^2}, \beta_{zt} = \frac{1}{c}\frac{2FB}{F^2+1+B^2}, \\
\beta_{yt} &= \frac{1}{c}\frac{2B}{F^2+1+B^2}, \qquad (5)
\end{aligned}$$

where:

$$\begin{aligned}
F &= P\sin\left[2\pi\nu\left(t-\frac{x}{c}\right)+\delta\right]+N, \\
P &= \frac{eE/2\pi\nu}{\sqrt{m_0^2c^2+a^2}}, \\
a &= \frac{m_0c\beta_{yt}(t_0)}{1-\beta_{t_0}^2}, \\
B &= \frac{1-\beta_{xt}(t_0)}{\sqrt{1-\beta_{xt}^2(t_0)-\beta_{zt}^2(t_0)}} = Const, \\
N &= \frac{m_0c\beta_{zt}(t_0)}{\sqrt{1-\beta^2(t_0)}\sqrt{m_0^2c^2+a^2}} \\
&- P\sin\left[2\pi\nu s_0+\delta\right], \\
s_0 &= t_0 - \frac{x(t_0)}{c} \ .
\end{aligned}$$

In integration of the equation (5) the following formula for energy variation was used:

$$\frac{d\varepsilon}{dt} = e\frac{dz}{dt}Ecos\left[2\pi\nu\left(t-\frac{x}{c}\right)+\delta\right] \ . \qquad (6)$$

For integration of the equations (7-9) we introduce a new variable:

$$g = 2\pi\nu\left(t-\frac{x}{c}\right)+\delta \ . \qquad (7)$$

Introducing notations:

$$\beta_{xt}(g) = \frac{1}{c}\frac{dx}{dg}, \beta_{zt}(g) = \frac{1}{c}\frac{dz}{dg}, \beta_{yt}(g) = \frac{1}{c}\frac{dy}{dg} \ . \qquad (8)$$

And substituting 7 into 8, we obtain:

$$\begin{aligned}
\beta_{xt}(g) &= \frac{1}{2B^2(2\pi\nu)}(1+F^2(g)-B^2), \\
\beta_{zt}(g) &= \frac{1}{2\pi\nu B}F(g), \qquad (9) \\
\beta_{yt}(g) &= \frac{1}{2\pi\nu B}\frac{a}{\sqrt{(m_0c)^2+a^2}}.
\end{aligned}$$

Integrating, we obtain the following:

$$\begin{aligned}
\frac{1}{c}x(g) &= \frac{1}{4\pi\nu B^2}(1-B^2+\frac{1}{2}P^2+N^2)g^2 \qquad (10) \\
&-\frac{1}{4\pi\nu B^2}(2PN-\frac{1}{2}P^2\sin g)\cos g \ , \\
\frac{1}{c}z(g) &= \frac{1}{2\pi\nu B}Ng - \frac{1}{2\pi\nu B}P\cos g \ , \qquad (11) \\
\frac{1}{c}y(g) &= \frac{a}{2\pi\nu B\sqrt{m_0^2c^2+a^2}} \ . \qquad (12)
\end{aligned}$$

It is seen from expressions 9 that components of electron velocity at its motion in a wave field are rational functions of function F, that satisfies a wave equation. Expressions (10-12) describe completely an electron motion in the field of electromagnetic wave. As one can see, an electron trajectories are not just "SIN" or "COS" like trajectories as it was supposed for example in [8,9], but have a strong dependence on intensity of an interacting wave, phase of interacting and angles of interaction.

3. PARAMETERS OF GENERATED RADIATION

On the base of the derived expressions for electron trajectories, let consider characteristics of the generated radiation and compare them with results produced with quantum electrodynamics formulas. According to results of [10] functions satisfying a wave equation are almost periodical functions of t in Hilbert space. So, we can make obvious supposition that rational functions of almost periodical function are almost periodical functions of time too. And, as it is known from Fourier series theory, almost periodical functions can be expanded in generalized Fourier series. According to the mentioned above, magnetic component of the field radiated by the electron on far distance (distance is longer than radiation wavelength) can be expressed in the following form [10, 11]:

$$\overrightarrow{H} = \sum_f \vec{H}_{\omega f}e^{-i\omega t} \ , \qquad (13)$$

where:

$$\vec{H}_{\omega f} = \frac{2ei\omega_f e^{ikR}}{c^2 R_0} \lim_{T\to\infty} \frac{1}{T} \int\limits_0^T e^{i\omega(t-\vec{n}\vec{r}/c)} \left[\vec{n} d\vec{r}(t)\right] ,$$

R_0 is distance from point of origin to point of observation, $\vec{n}$ is unit vector in the same direction, ω_f is frequency of the radiation harmonic, $\vec{k} = \frac{\omega_f}{c}\vec{n}$ is the wave vector. For $\vec{H}_{\omega f}$ calculating it is necessary to change over from time t in expression (13) to parameter (11). After change of variables we obtain:

$$\vec{H}_{\omega f} = A\frac{ei\omega_f e^{ikR}}{c^2 R_0} \lim_{g\to\infty} \frac{1}{g} \int_0^T \exp\left(i\omega\Lambda g\right) \times \exp\left(\Psi - \chi \sin g \cos g\right) \left[\vec{n} d\vec{r}(t)\right] , \quad (14)$$

where:

$$\Lambda = \frac{1}{4\pi\nu} + \frac{1}{4\pi\nu B^2} + \frac{P^2}{8\pi\nu B^2} + \frac{\widetilde{N}^2}{4\pi\nu B^2} + \left(\frac{1}{4\pi\nu} + \frac{1}{4\pi\nu B^2} + \frac{P^2}{8\pi\nu B^2} + \frac{\widetilde{N}^2}{4\pi\nu B^2}\right)\cos\gamma_1 - \frac{\widetilde{N}^2}{2\pi\nu B}\cos\gamma_2 - \frac{a\cos\gamma_3}{2\pi\nu B\sqrt{a^2+m^2c^2}} ,$$

$$\Psi = \left(\frac{\cos\gamma_2}{2\pi\nu B} - \frac{\widetilde{N}}{2\pi\nu B^2} + \frac{\widetilde{N}\cos\gamma_1}{2\pi\nu B^2}\right)P ,$$

$$\chi = \frac{P^2}{8\pi\nu B^2}(1-\cos\gamma_1) .$$

Expanding vector production and oscillator part of exponent $\exp\left(i\omega\Lambda g + (\Psi - \chi\sin g\cos g)\right)$ in 14 one can find harmonic frequency $\omega_f = f/\Lambda$, where f is integer. The harmonic radiation energy one can find from the following expression:

$$\varepsilon_{ph} = h\omega_f . \quad (15)$$

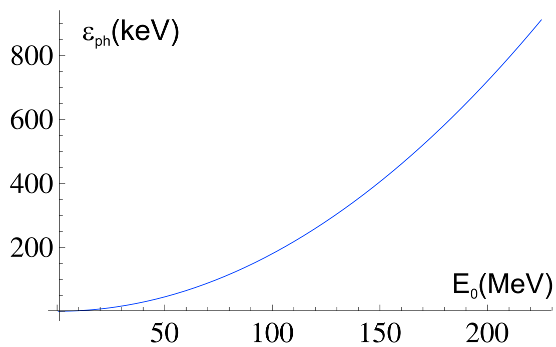

Fig.2. *The energy of the first harmonic of scattered radiation depending on initial energy of electron*

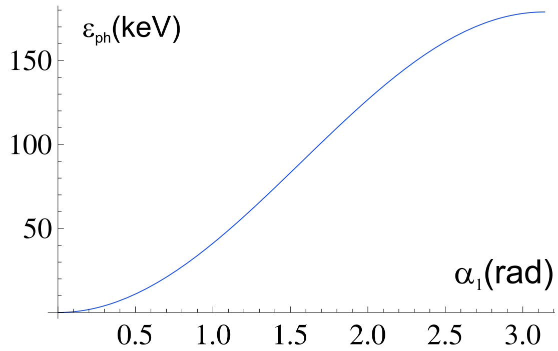

Fig.3. *The energy of the first harmonic of scattered radiation depending on interaction angle* $\alpha_1 = \alpha_2 = 0$ $(E_0 = 100\,MeV)$

Let us compare obtained results with quantum electrodynamics formulas for the first radiation harmonic $(f = 1)$ [12]. Fig.2 shows the energy of the first harmonic of scattered radiation depending on initial electron energy for operation energy range of NSC KIPT X-ray generator based on Compton scattering NESTOR [13] for both approaches. Fig.3 shows energy of the first harmonic of scattered radiation depending on interaction angle for initial electron beam energy equal to 100 MeV. As one can see from the figures the coincidence of the results of the obtained classical electrodynamics formulas and quantum electrodynamics formulas is practically ideal. But under small interaction angles (electron and wave move in the same direction) there is a difference in radiation energy value that was caused by the fact that formulas 9 did not take into account relativistic factor β. Such consideration was carried out in [14]. The natural result is that under zero interaction angle the energy of scattered radiation is equal to initial wave energy.

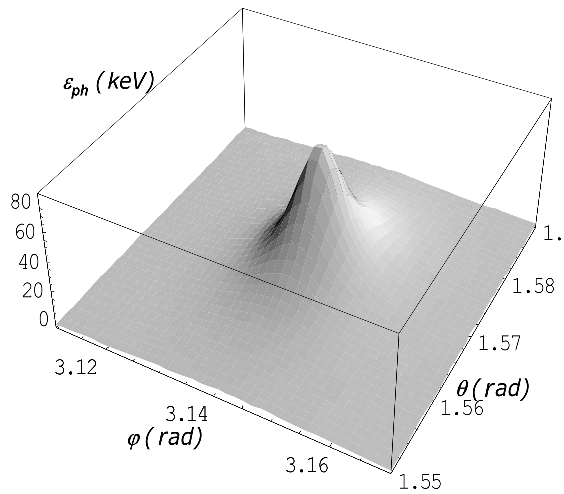

Fig.4. *The energy of the first harmonic of scattered radiation depending on observation angles (interaction angle equal to* $\pi/2$, $E_0 = 100$ *MeV)*

Fig.4 shows the energy of the first harmonic of scattered radiation depending on observation angles θ, φ at interaction angle equal to $\pi/2$ and electron beam equal to 100 MeV. As it is clear from expression (22) there is dependence of the energy of scattered radiation on initial energy of interacting electromagnetic wave and phase of interaction.

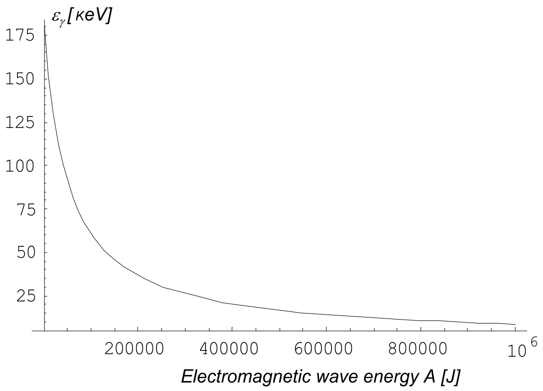

Fig.5. *The energy of the first harmonic of scattered radiation depending on intensity of initial wave (interaction angle equal to $\pi, E_0 = 100MeV$)*

Fig.5 and Fig.6 show these dependences for the following parameters of interaction: photon beam radius $R = 10^{-4}\,m$, laser flash length $L = 10^{-2}\,m$, interaction angle equal to π, electron energy $E_0 = 100\,MeV$. With such parameters of laser flash the flash energy equal to 1J will correspond to electric field strength $1.89 \times 10^{10}\,V/m$. As it is shown in Fig.5-6 dependence of scattered radiation energy on initial wave intensity becomes essential only for very intense initial wave but it has to be taken into account at consideration of interaction with new type of terawatt lasers. To determine intensity of the radiation we will use the following well known expression [12]:

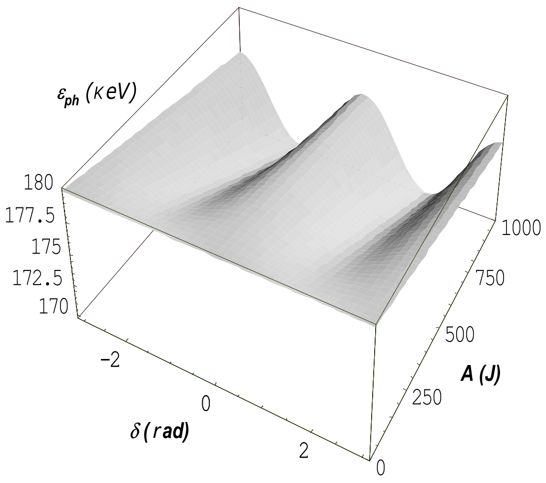

Fig.6. *The energy of the first harmonic of scattered radiation depending on intensity of initial wave and phase of interaction (interaction angle equal to $\pi, E_0 = 100MeV$)*

$$\frac{dI}{d\Omega} = \frac{c}{4\pi}|H^2|R_0^2\,. \qquad (16)$$

It is necessary to make integration in expression (17), then:

$$\begin{aligned}\vec{H}_{\omega f} = e\frac{\omega E^{ikr}}{cR_0}\left[\cos\gamma_3\frac{P}{2B} - \frac{\omega}{2B\pi\nu}\left(\cos\gamma_3 + \frac{\widetilde{N}}{B}(\cos\gamma_3 - 1)\right)\left(\cos\gamma_1\frac{1}{2B}\frac{a}{\sqrt{a^2+m^2c^2}} - \cos\gamma_3\frac{\widetilde{N}}{2B}\right)\right]\vec{n}_x + \\ e\frac{\omega e^{ikr}}{cR_0}\left[-\frac{\omega P}{2B\pi\nu}\left(\frac{\widetilde{N}}{B}\cos\gamma_1 - 1\right)\left(\cos\gamma_3\frac{1}{2B^2}(1 - B^2 + \widetilde{N}^2) - \cos\gamma_1\frac{1}{2B}\frac{a}{\sqrt{a^2+m^2c^2}}\right) - \cos\gamma_3\frac{P\widetilde{N}}{2B^2}\right]\vec{n}_z + \\ e\frac{\omega e^{ikr}}{cR_0}\left[\cos\gamma_2\frac{P\widetilde{N}}{2B^2} - \frac{\omega P}{2B\pi\nu}\left(\cos\gamma_1 + \frac{\widetilde{N}}{B}(\cos\gamma_1 - 1)\right)\left(\cos\gamma_1\frac{\widetilde{N}}{2B} - \cos\gamma_2\frac{1}{4B^2}(1 - B^2 + \widetilde{N}^2)\right) - \cos\gamma_1\frac{P}{2B}\right]\vec{n}_y.\end{aligned} \qquad (17)$$

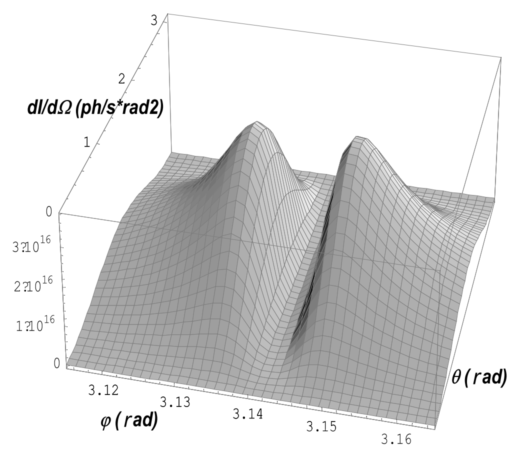

Fig.7. *The scattered radiation intensity depending on observer angles (interaction angle equal to π, $E_0 = 100\,MeV$, $A = 0.001\,J$, $I = 0.01\,A$)*

And then, we change over direction to observer angles $\gamma_1, \gamma_2, \gamma_3$, and in the laboratory coordinate frame to angles in spherical coordinate frame θ and φ. Thus, substituting square of module from expression (20) and using expression (23), we obtain radiation intensity. Dependence of scattered radiation intensity on observer angles is shown on Fig. 7 for the interacting beam parameters mentioned above and laser beam intensity equal to $A = 0.001J$.

4. CONCLUSION

Thus, using classical electrodynamics approach we succeeded to obtain trajectories of an electron motion in the field of electromagnetic wave. Then, using a fact that the obtained solutions are almost periodical functions we used generalized Fourier series and derived expressions for frequencies of the scattered radiation harmonics and spectrum of the radiation intensity. The obtained expressions allow to investigate electron beam dynamics in laser electron X-ray generators as well as dependences of generated radiation parameters.

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КОМПТОНОВСКОЕ ИЗЛУЧЕНИЕ ЭЛЕКТРОНА В ПОЛЕ БЕГУЩЕЙ ПЛОСКОЙ ЛИНЕЙНО ПОЛЯРИЗОВАННОЙ ЭЛЕКТРОМАГНИТНОЙ ВОЛНЫ

И. Дребот, Ю. Григорьев, А. Зелинский

Приведены результаты интегрирования уравнения Лоренца для релятивистского электрона, движущегося в поле бегущей плоской линейно поляризованной электромагнитной волны. Показано, что скорость электрона в поле такой волны является почти периодической функцией времени. Для расчета углового спектра интенсивности излучения электрона было использовано разложение электромагнитного поля в волновой зоне в обобщенный ряд Фурье. Выражения для интенсивности спектра излучения представлены в работе. Полученные результаты иллюстрируются для параметров электронного и фотонного пучков источника рентгеновского излучения ННЦ ХФТИ "НЕСТОР". В то же время полученные выражения дают возможность исследовать зависимость энергетического и углового спектров комптоновского излучения от фазы взаимодействия и интенсивности взаимодействующей волны.

КОМПТОНІВСЬКЕ ВИПРОМІНЮВАННЯ ЕЛЕКТРОНА В ПОЛІ БІГУЧОЇ ПЛОСКОЇ ЛІНІЙНО ПОЛЯРИЗОВАНОЇ ЕЛЕКТРОМАГНІТНОЇ ХВИЛІ

І. Дребот, Ю. Григорьєв, А. Зелінський

Приведено результати інтегрування рівняння Лоренца для релятивістського електрона який рухається в полі бігучої плоскої лінійно поляризованої електромагнітної хвилі. Показано, що швидкість електрона в полі такої хвилі є практично періодична функція часу. Для розрахунку кутового спектра інтенсивності випромінювання електрона було використано розклад електромагнітного поля в хвильовій зоні в узагальнений ряд Фур'є. Вирази для інтенсивності спектра випромінювання представлені в роботі. Отримані результати ілюструються для параметрів електронного та фотонного пучків джерела рентгенівського випромінювання ННЦ ХФТІ "НЕСТОР". У той же час отримані вирази дають можливість досліджувати залежність енергетичного та кутового спектрів комптонівського випромінювання від фази взаємодії та інтенсивності хвилі що взаємодіє.