PHENOMENOLOGICAL DESCRIPTION OF NEUTRON YIELDS FROM ACTINIDE FISSION

A.I. Lengyel [1], O.O. Parlag [1*], V.T. Maslyuk [1], Yu.V. Kibkalo [2]

[1] *Institute of Electron Physics, 88000, Uzhhorod, Ukraine*

[2] *Institute for Nuclear Research, National Academy of Sciences of Ukraine, 03680, Kiev, Ukraine*

(Received November 4, 2010)

The phenomenological analysis of the neutron yields from the fission of ^{233}U, ^{235}U, ^{239}Pu and ^{252}Cf has been performed. The parameterization used allows the peculiarities of the 'sawtooth' curve of neutron multiplicity for light and heavy fragments to be described and predicted.

PACS: 24.75.+1, 25.85.-w, 25.85. Ec, 25.85. Ca

Studying characteristics of actinide nucleus fission products, such as mass and charge distribution, average number of prompt neutrons and excitation energies of fragments, is essential for understanding basic mechanisms of the fission process [1]. These characteristics are widely used in various application fields, e.g. in nuclear energy, non-destructive analysis, active and passive detection of special nuclear materials in context of their non-proliferation, etc. Information on primary fission fragments (with respect to neutron emission) is necessary for precise theoretical analysis of the experimental data. Direct measuring of primary fragments' properties relative to neutron emission is a complicated task. At the same time knowledge of neutron yield dependence on mass of fission fragments is important for understanding dynamics of the fission process. Variation of average neutron multiplicity pattern reflects change of different fission mode dynamic effects [2]-[4]. Number of emitted neutrons depends on mass number and charge of the nucleus, total excitation energy of fission fragments, redistribution of total excitation energy between light and heavy fragments and angular momenta of primary fission fragments. Therefore to give quantitative description of average multiplicity of fission neutrons [5]-[7] we need multi-parametric experimental data. According to Terrel calculating method system of the integral equations [8],[9].

$$\int_0^{A_i-\nu_F(A_i)} Y(A)dA = \tag{1}$$

$$= \int_0^{A_i} y(A)dA + \frac{1}{2}\frac{dy}{dA} < \sigma^2(\nu_F, A) > + ...,$$

can be solved only on the basis of experimental data on total fission yields in the final state, $y(A)$ - initial fragment yields with the mass number A before neutron emission; $\sigma^2(\nu_F, A)$ - variation, ν_F - total neutron yield.

As it was shown in [8] higher-order terms make small contribution. For example, contribution $\frac{1}{2}\frac{dy}{dA} < \sigma^2(\nu_F(A) >$ into $\nu_F(A)$ is less than 0,1. Essence of Terrell calculating method (see Fig.5 [8]) is illustrated in Fig.1, where initial cumulative mass distributions of light and heavy fragments

$$S_y^L = \int_0^{A_L} y(A)dA,$$

$$S_y^H = \int_{A_F-A_L}^{A_F} y(A)dA$$

and final cumulative mass distributions of light and heavy fragments

$$S_Y^L = \int_0^{A_L} Y(A)dA,$$

$$S_Y^H = \int_{A_F-A_L}^{A_F} Y(A)dA$$

are plotted with circles and interpolated curves [10].

Values of total neutron yields $F(A)$ are taken to be the absolute difference between point values and complementary function values, illustrated in Fig.1 by the horizontal line , so that

$$\nu_F(A) = \nu_L(A) + \nu_H(A), \tag{2}$$

where ν_L and ν_H are neutron yields from light and heavy fragments, respectively. To calculate $\nu_{L,H}$ according to (1) it is necessary to set their optimal parameterization type in advance. On one hand, parameterization should contain as few free parameters as

*Corresponding author E-mail address: VFJAP<nuclear@email.uz.ua>

possible; on the other hand, it should fully reflect expected local peculiarities. For this purpose we employ Wahl parameterization [10] by introducing function

$$R(A) = \nu_{L,H}(A)/\nu_F(A)\,. \quad (3)$$

Generalized shape of R(A) function contains only 2 x 4 segments for both light and heavy fragments (dotted line on Fig.2), which is sufficient for qualitative description of existing experimental data on yields of neutrons emitted by fission fragments (see Fig.4.2.42 [10]).

Our recent analysis [11] of existing experimental data on neutron yields from fragments of ^{252}Cf, ^{244}Cm and ^{248}Cm spontaneous fission [12] has shown that to consider dependence of fission neutron yields on mass more accurately, range of the fragment masses should be divided into more than 4 segments (solid broken line on Fig.2). To find consistent patterns in fission neutrons distribution depending on fragment mass, we have chosen smoothed and normalized empiric values of fission neutrons yields $\nu_{L,H}(A)$ for ^{233}U(n,f), ^{235}U(n,f), ^{239}Pu(n,f) and ^{252}Cf(s,f), obtained in [13], which are in reasonable agreement with the experimental data (see Fig.1 there). $R(A)$ function for light and heavy fragments is parameterized as

$$R_i^L(A) = A_i^L + B_i^L(A - A_{L)}). \quad (4)$$

$$R_i^H(A) = 1 - R_i^L(A_H), \quad (5)$$

where i - segment number, a_i^L , b_i^L - parameters. Value of A_L is determined from condition: $R(A_L)$ = 0,5. We calculated $R(A)$ for ^{233}U(n,f), ^{235}U(n,f), ^{239}Pu(n,f) and ^{252}Cf(s,f) fission in accordance with (4) and (5), dividing the whole data range into 6 segments, as it is seen on Fig.2. Taking into account the following relations can substantially decrease number of the free parameters:

$$a_4 = 0,5; b_2 = 0; A_0 = (A_F - 4)/2\,; \quad (6)$$

$$a_1 + b_1(A_0 - A_L) = 0,5\,; \quad (7)$$

$$a_1 + b_1(A_1 - A_L) = a_3 + b_3(A_2 - A_L) = a_2\,; \quad (8)$$

$$a_{i+1} + b_{i+1}(A_{i+1} - A_L) = a_i + b_i(A_i - A_L)\,. \quad (9)$$

Values of parameters a_i and b_i are determined from the calculated $R(A)$ function and relations (6) - (9). Results of the calculation are given on Fig. 3 and Table 1. Shape of $\nu_{L,H}(A)$ can be obtained from (3). As it is seen on Fig. 5, calculated values of fission neutron yields practically everywhere coincide with initial values, obtained by Wahl [13].

Empiric $R(A)$ function possesses number of characteristic features:

- $R(A)$ = 0,5 in the point of symmetric fission, which is determined as $A_0 = (A_f - 4)/2$ [13], and also in the point A_L, determined by adjustment;
- maximal value of $R(A)$ function is restricted by 0,83 - 0,90 range;
- number of points at A_1A_2 plateau is 3 - 4 a.m.u.;
- position of A_1 point linearly depends on actinide mass (see Fig. 4.) and is determined from the condition of $R(A)$ function minimum value in the complenetary point, corresponding to fission product mass A_N =129-130 a.m.u.;
- value of A_L and parameters a_i, b_i, also obey linear dependence in the probed range of actinide masses;
- A_0A_5 segment is almost constant and slightly varies within 39 - 41 a.m.u. range.

The performed calculation allows to estimate possible values of fission neutron yields from light and heavy fragments based on the known total average yields, which, in its turn, can be obtained only from mass distribution of fission fragments with help of modified Terrell method [8],[9].
Using graph on Fig.3 and the abovementioned regularities we can estimate neutron yields for $^{237}Np(n,f)$ fission: A_0 = 117; A_L = 94; A_1= 106; A_2= 103; A_3= 97; A_4 = 90; A_5= 78; R_{max}=0,9; R(A_5)= R(A_6)= 0,2; b_3=0,05.

Anticipated behaviour of $R(A)$ function for $^{237}Np(n,f)$ fission is presented by dotted curve on Fig.3.

In the present paper phenomenological analysis of neutron yields of ^{235}U, ^{235}U, ^{239}Pu and ^{252}Cf fission fragments is performed. Chosen parameterization and established regularities allow to describe observed changes of 'sawtooth' behaviour of fission neutron yields for light and heavy fission fragments.

Using the established regularities (solid curves on Fig.3) behaviour of $R(A)$ function for $^{237}Np(n,f)$ fission is predicted (dotted curve on Fig.3). It is obvious from Fig.5 that we obtained good quantitative agreement with initial fission neutron yields for the whole range of the fragment masses.

This parameterization can be used for solving integral equations of type (1) to determine fission fragments' yields before neutron emission.

Table 1. *Parameters of R(A) function for fission fragments of* $^{233}U(n,f)$*,* $^{235}U(n,f)$*,* $^{239}Pu(n,f)$ *and* $^{252}Cf(s,f)$

^{252}Cf (s.f)	A_L=105					
	a_1	a_2	a_3	a_4	a_5	a_6
	2,4	0,9	0,51	0,50	0,47	0,20
	b_1	b_2	b_3	b_4	b_5	b_6
	-0,10	0,0	0,038	0,020	0,013	0,0
A_0	A_1	A_2	A_3	A_4	A_5	A_6
124	120	117	107	100	83	67

^{239}Pu (n,f)	A_L=95					
	a_1	a_2	a_3	a_4	a_5	a_6
	1,4	0,83	0,32	0,50	0,49	0,20
	b_1	b_2	b_3	b_4	b_5	b_6
	-0,04	0,0	0,050	0,006	0,016	0,0
A_0	A_1	A_2	A_3	A_4	A_5	A_6
118	110	105	100	94	88	80

^{235}U (n,f)	A_L=92					
	a_1	a_2	a_3	a_4	a_5	a_6
	1,2	0,85	0,42	0,50	0,50	0,20
	b_1	b_2	b_3	b_4	b_5	b_6
	-0,03	0,0	0,050	0,020	0,021	0,0
A_0	A_1	A_2	A_3	A_4	A_5	A_6
124	104	101	97	86	77	65

^{233}U (n,f)	A_L=90					
	a_1	a_2	a_3	a_4	a_5	a_6
	1,2	0,85	0,35	0,50	0,55	0,20
	b_1	b_2	b_3	b_4	b_5	b_6
	-0,03	0,0	0,050	0,020	0,025	0,0
A_0	A_1	A_2	A_3	A_4	A_5	A_6
115	102	99	85	840	76	65

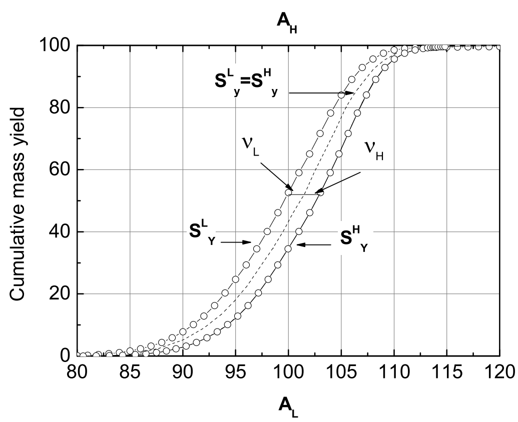

Fig.1. *Typical curves of cumulative mass yields* S^L_y*,* S^H_y*,* S^L_Y *and* S^L_Y *used to find neutron yields* ν_L*,* ν_H *depending on mass of fission fragments*

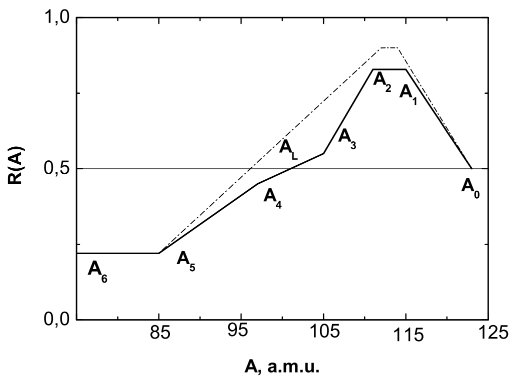

Fig.2. *R(A) - functions. Solid line represents our parameterisation; dotted curve is Wahl parameterization [10]*

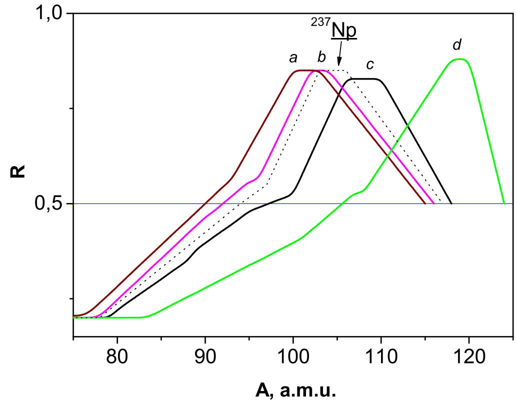

Fig.3. *Calculated R(A) for fission of a)* $^{233}U(n,f)$*, b)* $^{235}U(n,f)$*, c)* $^{239}Pu(n,f)$*, d)* $^{252}Cf(s,f)$*; and predicted R(A) for* $^{237}Np(n,f)$ *fission reaction (dotted line)*

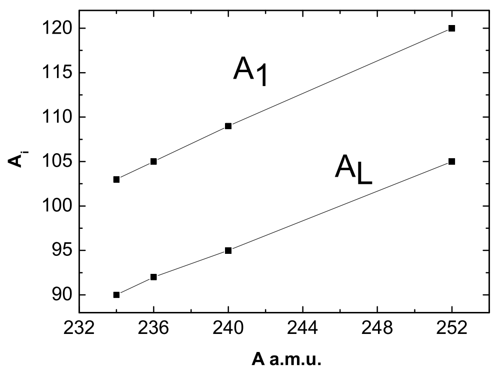

Fig.4. *Behavior of the outermost points of R(A) segments*

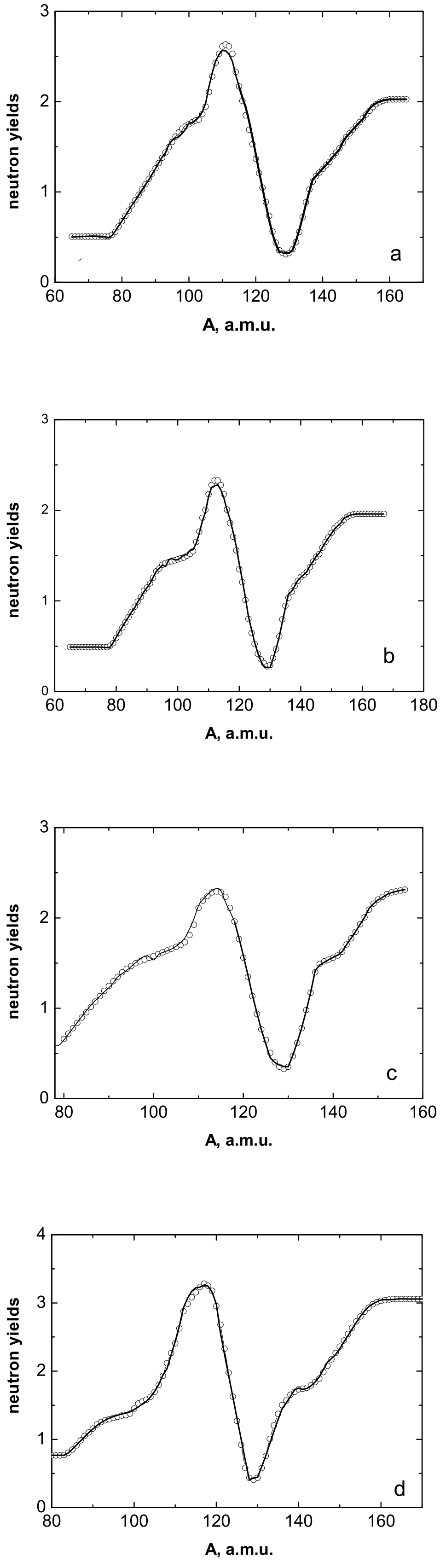

Fig.5. *Fission neutron yields $\nu_{L,H}(A)$ for light and heavy fragments: a) $^{233}U(n,f)$, b) $^{235}U(n,f)$, c) $^{239}Pu(n,f)$, d) $^{252}Cf(s.f)$; solid curve is the result of $\nu_{L,H}(A)$ calculation; circles are smoothed and normalized values of fission neutron yields [13]*

References

1. G.M. Ter-Akopjan, Yu.C. Ohanesjan, V.A.Daniel, et al. Experimental methods for the study of spontaneous and low-energy nuclear fission // *Physics of Elementary Particles and Atomic Nuclei.* 1997, 28, p.1357-1388 (in Russian).

2. U. Brosa. Sawtooth curve of neutron multiplicity // *Phys. Rev.* 1985, 32, p.1438-1441.

3. N.V. Kornilov, F.-J. Hambsch, A.S. Vorobyev. Neutron emission in fission // *Nucl. Phys.* 2007, A789, p.55-72.

4. Yu.V. Kibkalo. Phenomenological model for fragment mass and charge distribution in actinide nuclei fission // *Fission product yield data for the transmutation of minor actinide nuclear waste. IAEA. Vienna. 2008.* p.157-182.

5. H. Marten, A. Ruben, D. Seeliger. Fission neutron multiplicity calculations // *Proceedings of a consultants meeting "Nuclear data for neutron emission in the fission process"*. Vienna, Austria. 1990, p.115-133.

6. S. Lemaire, P. Talou, T. Kawano, et al. Monte Carlo approach to sequential neutron emission from fission fragments // *Phys. Rev. C.* 2005, 72, 024601.

7. A. Tudora. Experimental prompt fission neutron "sawtooth" data described by "point by point" model // *Annals of Nuclear Energy.* 2006, 33, p.1030-1038.

8. J. Terrell. Neutron Yields from Individual Fission Fragments // *Phys. Rev.* 1962, 127, p.880-904.

9. A.I. Lengyel, Yu.V. Kibkalo, O.O. Parlag, V.T. Maslyuk. Calculations of the yield of prompt neutrons from fission fragments // *Uzhhorod university scientific herald. Series Physics.* 2008, N23, p.58-63(in Ukrainian).

10. A.C. Wahl. Phenomenological model for fragment mass and charge distribution in actinide nuclei fission // *Fission product yield data for the transmutation of minor actinide nuclear waste.* Vienna, Austria. 2008, p.117-148.

11. A.I. Lengyel, O.A. Parlag, V.T. Maslyuk, Yu.V. Kibkalo. Fission neutron yields from product mass distribution // *The 3rd International Conference "Current problems in nuclear physics and atomic energy"*. Kyiv, Ukraine. 2010, p.109.

12. V. Kalinin, V. Dushin, F.-J. Hambsch, et al. Measurements of prompt neutron multiplicity distributions in correlation with mass-energy distribution of fission fragments in spontaneous fission of ^{252}Cf, ^{244}Cm and ^{248}Cm // *J. Nucl. Sc. Tech.* 2002, N2, p.250-253.

13. A.C. Wahl. Nuclear-charge distribution and delayed-neutron yields for thermal-neutron-induced fission of ^{235}U, ^{233}U, and ^{239}Pu and for spontaneous fission of ^{252}Cf // *Atomic Data and Nuclear Data Tables*. 1988, 38, p.1-156.

ФЕНОМЕНОЛОГИЧЕСКОЕ ОПИСАНИЕ ВЫХОДОВ НЕЙТРОНОВ ДЕЛЕНИЯ АКТИНИДОВ

A.I. Lengyel, O.O. Parlag, В.Т. Маслюк, Ю.В. Кибкало

Проведен феноменологический анализ выходов нейтронов из осколков деления ^{233}U, ^{235}U, ^{239}Pu и ^{252}Cf. Выбранная параметризация и установленные закономерности позволяют описать и предсказать наблюдаемые изменения "пилообразного"поведения выхода нейтронов из легкого и тяжелого осколков деления ядер.

ФЕНОМЕНОЛОГІЧНИЙ ОПИС ВИХОДІВ НЕЙТРОНІВ ПОДІЛУ АКТИНІДІВ

A.I. Lengyel, O.O. Parlag, В.Т. Маслюк, Ю.В. Кібкало

Виконано феноменологічний аналіз виходів нейтронів з уламків поділу ^{233}U, ^{235}U, ^{239}Pu та ^{252}Cf. Вибрана параметризація та встановлені закономірності дозволяють описати та передбачити зміни "пилоподібної"поведінки виходу нейтронів з легкого та важкого уламків поділу ядер, які спостерігаються.