

# ON THE MICROSCOPIC THEORY OF NON-DISSIPATIVE FLOWS IN BOSE SYSTEMS

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The states with non-dissipative flows are explored within a microscopic approach to a problem of spatially inhomogeneous multiparticle Bose-systems. Density of superfluid component was evaluated and its correlation with one- and two-particle Bose condensate density was analyzed also.

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## 1. INTRODUCTION

One of the most prominent features of new phase in liquid 4He discovered by Kapitsa [1] was the existence of non-dissipative mass fluxes. The underlying phenomenology, proposed by Landau [2], is tight into two-liquids hydrodynamics framework. Later on, in Ginzburg and Pitaevsky approach [3], which was developed by analogy with Ginzburg and Landau superconductivity theory [4], the existence of non-dissipative fluxes took place due to system phase symmetry breakdown. Relation of symmetry breakdown with non-dissipative fluxes was considered in [5,6] (see also [7,8]). Though it is not clear up to now how the superfluidity phenomenon is related to Bose condensation.

In present paper we study the states with superfluid fluxes on the basis of microscopic model for spatially-inhomogeneous systems [9]. Density of normal and superfluid components were found, and the relation of that to Bose condensate density was analyzed.

## 2. SELF-CONSISTENT FIELD EQUATIONS SET FOR BOSE SYSTEMS WITH BROKEN PHASE SYMMETRY

Consider many-particle system of Bose particles with delta-like potential  $U(\mathbf{x}, \mathbf{x}') = U_0 \delta(\mathbf{x} - \mathbf{x}')$ . In that case equations set for self-consistent field reads [9]

$$\left( \frac{\hbar^2 \Delta}{2m} + \mu + \varepsilon_i - 2U_0 \tilde{\rho} \right) u_i(\mathbf{x}) - U_0 \tilde{\tau} v_i(\mathbf{x}) = 0, \quad (1)$$

$$\left( \frac{\hbar^2 \Delta}{2m} + \mu - \varepsilon_i - 2U_0 \tilde{\rho} \right) v_i(\mathbf{x}) - U_0 \tilde{\tau}^* u_i(\mathbf{x}) = 0, \quad (2)$$

$$\left( \frac{\hbar^2 \Delta}{2m} + \mu - 2U_0 \tilde{\rho} + 2U_0 |\chi(\mathbf{x})|^2 \right) \chi(\mathbf{x}) + U_0 \tilde{\tau} \chi^*(\mathbf{x}) = 0, \quad (3)$$

where  $u_i(\mathbf{x}), v_i(\mathbf{x})$  are the quasiparticle wave function components in state  $i$ ,  $\chi(\mathbf{x})$  is Bose condensate wave function,  $\mu$  is the chemical potential. Equations (1)-(3) contain normal  $\tilde{\rho}$  and anomalous  $\tilde{\tau}$  implicit one-particle density matrices:  $\tilde{\rho} = \rho + |\chi|^2$ ,  $\tilde{\tau} = \tau + \chi^2$ . One-particle overcondensate density matrices  $\rho, \tau$  are written out below.

Solutions for equations set (1)-(3) are to be found in the form

$$u_i(\mathbf{x}) = \frac{u_k}{\sqrt{V}} e^{i(\mathbf{k}+\mathbf{q})\mathbf{x}}, v_i(\mathbf{x}) = \frac{v_k}{\sqrt{V}} e^{i(\mathbf{k}+\mathbf{q})\mathbf{x}}, \quad (4)$$

$$\chi(\mathbf{x}) = \frac{\chi}{\sqrt{V}} e^{i(\mathbf{q}')\mathbf{x}},$$

where  $V$  is the system volume. By inserting the solutions (4) into (1)-(3), one finds the equations to be fulfilled provided that  $\mathbf{q}'' = (\mathbf{q} - \mathbf{q}')/2$ . In that case equations (1)-(3) take the form

$$\left[ \frac{\hbar^2 (\mathbf{k} + \mathbf{q})^2}{2m} - \tilde{\mu} - \varepsilon_k \right] u_k + \Delta v_k = 0, \quad (5)$$

$$\Delta^* u_k + \left[ \frac{\hbar^2 (\mathbf{k} + \mathbf{q}')^2}{2m} - \tilde{\mu} + \varepsilon_k \right] v_k = 0, \quad (6)$$

$$\left[ \frac{\hbar^2 (\mathbf{q} - \mathbf{q}')^2}{8m} - \tilde{\mu} - 2U_0 \frac{|\chi|^2}{V} \right] \chi + \Delta \chi^* = 0, \quad (7)$$

with  $\Delta = U_0 (\tau + \chi^2/V)$ ,  $\tilde{\mu} = \mu - 2U_0 \tilde{\rho}$  is the effective chemical potential. The condition  $|u_k|^2 - |v_k|^2 = 1$  has to be fulfilled. Normal and anomalous density matrices are determined by formulae

$$\rho = V^{-1} \sum_k \left[ |u_k|^2 f_k + |v_k|^2 (1 + f_k) \right], \quad (8)$$

$$\tau = V^{-1} \sum_k u_k v_k^* (1 + 2f_k), \quad (9)$$

where  $f_k = (\exp(\varepsilon_k/T) - 1)^{-1}$ ,  $T$  being the temperature. From equations (5) and (6) dispersion relation for quasiparticles follows:

$$\varepsilon_k = \frac{1}{2} \left[ \xi_k(\mathbf{q}) - \xi_k(\mathbf{q}') + \sqrt{[\xi_k(\mathbf{q}) + \xi_k(\mathbf{q}')]^2 - 4|\Delta|^2} \right], \quad (10)$$

where  $\xi_k(\mathbf{q}) = \hbar^2 (\mathbf{k} + \mathbf{q})^2 / 2m - \tilde{\mu}$ . The solutions have the form

$$|u_k|^2 = \frac{1}{2} \left[ 1 + \frac{\xi_k(\mathbf{q}) + \xi_k(\mathbf{q}')}{D_k} \right],$$

$$|v_k|^2 = \frac{1}{2} \left[ 1 - \frac{\xi_k(\mathbf{q}) + \xi_k(\mathbf{q}')}{D_k} \right],$$

where  $D_k = \sqrt{[\xi_k(\mathbf{q}) + \xi_k(\mathbf{q}')]^2 - 4|\Delta|^2}$ ,  $\Delta$  is defined by (7)-(9). Note, that the quantities  $\varepsilon_k, u_k, v_k, D_k$  also

depend on two vectors  $\mathbf{q}, \mathbf{q}'$ , though we omit explicit notation.

### 3. PARTICLE FLUX AND NORMAL COMPONENT DENSITY

It is possible to find the superfluid component flux in the system. Let us take advantage of the formula for the particle total flux

$$\mathbf{j} = \frac{\hbar}{2mi} \{ \psi^+(x) \nabla \psi(x) - \nabla \psi^+(x) \psi(x) \}$$

where  $\psi(x), \psi^+(x)$  are the field operators.

Calculation of the particle total flux leads to

$$\mathbf{j} = \frac{\hbar(\mathbf{q}-\mathbf{q}')}{2mV} |\chi|^2 + \frac{\hbar}{mV} \sum_k \left[ \mathbf{k} (f_k - |\nu_k|^2) + \mathbf{q} |\nu_k|^2 f_k - \mathbf{q}' |\nu_k|^2 (1 + f_k) \right]. \quad (11)$$

In the first approximation on  $\mathbf{q} - \mathbf{q}'$  dispersion rule (10) for quasiparticle is determined by expression

$$\varepsilon_k = \frac{\hbar^2 \mathbf{k}(\mathbf{q}-\mathbf{q}')}{2m} + \varepsilon_k^{(0)}, \quad (12)$$

where [7]

$$\varepsilon_k^{(0)} = \sqrt{\left( \frac{\hbar^2 k^2}{2m} + \frac{2U_0 |\chi|^2}{V} \right) \left( \frac{\hbar^2 k^2}{2m} + \frac{2(U_0 - \Theta) |\chi|^2}{V} \right)} \quad (13)$$

is the quasiparticle dispersion rule at the state without fluxes,

$$\Theta = U_0 / (1 + U_0 J), \quad J = 2V^{-1} \sum_k (1 + 2f_k) / \varepsilon_k^{(0)}.$$

We see that the energy of a quasiparticle at  $\mathbf{k} = 0$  does not go to zero but takes a finite value

$$\varepsilon_0 = 2n_B \sqrt{U_0(U_0 - \Theta)} = 2U_0 n_B \sqrt{\frac{U_0 R}{1 + U_0 R}},$$

where  $n_B \equiv |\chi|^2 / V$ .

Thus the spectrum has an energy gap.

Let the reference frame be introduced as one where the normal component stays at rest. We suppose the relative motion velocity of superfluid and normal components is  $\mathbf{v}_s$ . Superfluid component velocity is assumed to be small in comparison with any characteristic velocity in the system (with sound velocity, for instance).

Let us consider the particular case when  $\mathbf{q} = -\mathbf{q}'$ . If we define the superfluid velocity by relation

$$\mathbf{v}_s = \frac{\hbar \mathbf{q}}{m}$$

the dispersion rule (12) becomes  $\varepsilon_k = \varepsilon_k^{(0)} + \hbar \mathbf{k} \mathbf{v}_s$ . Flux density (11) takes the form

$$\mathbf{j} = n \mathbf{v}_s + \frac{\hbar}{mV} \sum_k \mathbf{k} (f_k - |\nu_k|^2), \quad (14)$$

where the total particle number density can be written as [9]

$$n = n_B + n_q + n_p. \quad (15)$$

The first term in (15)

$$n_B = |\chi|^2 / V$$

is the particle number density in the Bose condensate

The second term

$$n_q = V^{-1} \sum_k f_k$$

is the number density of the quasiparticle excitations and the last term

$$n_p = V^{-1} \sum_k |\nu_k|^2 (1 + 2f_k)$$

is the number density of the particle correlated in Cooper pairs.

In the case when the normal component stays at rest and superfluid component has velocity  $\mathbf{v}_s$ , the total flux will be [10]

$$\mathbf{j} = \mathbf{j}_s = n_s \mathbf{v}_s.$$

Further, taking this into account, from the formula (14) it follows

$$n_s \mathbf{v}_s = - \frac{\hbar}{mV} \sum_k \mathbf{k} (f_k - |\nu_k|^2),$$

and the normal density expression is obtained as a result

$$n_n = - \frac{\hbar}{mV} \sum_k \frac{\mathbf{k} \mathbf{v}_s}{v_s^2} (f_k - |\nu_k|^2). \quad (16)$$

Preserving a good accuracy in equations (16), one can neglect the second term in parentheses. Taking into account (15) the expression for the particle density of superfluid component takes on form

$$n_s = n_B + n_q + n_p - n_n.$$

Note, that the particle density of superfluid component is not equal to that of one-particle and pair condensate  $n_B + n_p$ . Superfluid component density is equal to net density of one-particle and pair condensate only at  $T = 0^0 K$

$$n_s = n_B + n_p.$$

Let velocity  $\mathbf{v}_s$  be small. Then one can expand the distribution function. Such an expansion leads to the formula

$$n_n = - \frac{\hbar^2}{mV} \sum_k \frac{(\mathbf{k} \mathbf{v}_s)^2}{v_s^2} \frac{\partial f_k^{(0)}}{\partial \varepsilon_k^{(0)}}, \quad f_k^{(0)} = (\exp \varepsilon_k^{(0)} / T - 1)^{-1}.$$

Passing from summation to integration and doing integration one obtains

$$n_n = - \frac{\hbar^2}{6\pi^2 m_0} \int_0^\infty dk k^4 \frac{\partial f_k^{(0)}}{\partial \varepsilon_k^{(0)}}. \quad (17)$$

The main contribution to integral (17) is from small  $k$  region, thus the rather complicated expression (13) can be replaced by the expansion of that, up to  $k^2$  order:

$$\varepsilon_k^{(0)} \approx \varepsilon_0 + \frac{\hbar^2 k^2}{2m_*},$$

where  $\varepsilon_0 = 2n_B \sqrt{U_0(U_0 - \Theta)}$  is the energy gap,  $m_* = 2m \sqrt{U_0(U_0 - \Theta)} / (2U_0 - \Theta)$  is the effective mass.

The expansion of the function  $f_k^{(0)}$  in power series and integration in the formula (17) give the temperature dependence of the normal density  $n_n$

$$n_n = \frac{T^{3/2}}{m(2\pi)^{3/2} (\hbar)^3} m_*^{5/2} g_{3/2} \left( \frac{\varepsilon_0}{T} \right),$$

where the function  $g_{3/2}(\varepsilon_0)$  is defined by

$$g_{3/2}\left(\frac{\varepsilon_0}{T}\right) = \sum_{l=1}^{\infty} \frac{\exp(-l \frac{\varepsilon_0}{T})}{l^{3/2}}.$$

Thus we also obtain the temperature dependence of superfluid component density since  $n_s = n - n_n$ . Note, in addition to the contribution from one-particle excitations in superfluid and normal component density, there exists a noticeable contribution from long-wave excitations (phonons).

In conclusion, we would like to write out the flux state constancy condition in the system considered. Expression (10) implies that the constancy may be achieved at  $\varepsilon_k \geq 0$ . As the minimum of  $\varepsilon_k$  is equal (see (12)) to

$$\varepsilon_k^{(\min)} \approx -|\mathbf{p}_k \|\mathbf{v}_s| + \sqrt{\varepsilon_k^2 - |\Delta|^2}, \quad \mathbf{p}_k = \hbar \mathbf{k}$$

the constancy takes place at

$$\sqrt{\varepsilon_k^2 - |\Delta|^2} \geq |\mathbf{p}_k \|\mathbf{v}_s|$$

or

$$\sqrt{\frac{1}{\hbar^2 k^2} \left( \frac{\hbar^2 k^2}{2m} + 2(U_0 - \Theta)n_B \right) \left( \frac{\hbar^2 k^2}{2m} + 2U_0 n_B \right)} \geq |\mathbf{v}_s|.$$

Thus, present paper introduces the calculation of one-particle excitations contribution to normal and superfluid component density. It takes place for Bose systems with one-particle and paired condensate. It is shown, that superfluid component density is not equal

to total number density of particles, forming Bose condensate.

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## К МИКРОСКОПИЧЕСКОЙ ТЕОРИИ НЕЗАТУХАЮЩИХ ПОТОКОВ В БОЗЕ-СИСТЕМАХ

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На основе микроскопического подхода для описания пространственно-неоднородных многочастичных бозе-систем исследованы состояния со сверхтекучими потоками. Рассчитана плотность сверхтекучей компоненты и проанализирована ее связь с плотностями одночастичного и парного бозе-конденсатов.

## ДО МІКРОСКОПІЧНОЇ ТЕОРІЇ НЕЗАТУХАЮЧИХ ПОТОКІВ У БОЗЕ-СИСТЕМАХ

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На основі мікроскопічного підходу до опису просторово-неоднорідних багаточастинкових бозе-систем досліджено стан з незгасаючими потоками. Розраховано густину надплинної компоненти і проаналізовано її зв'язок з густинами одночастинкового і парного бозе-конденсатів.