

# FORMATION OF STEADY - STATE NONEQUILIBRIUM DISTRIBUTIONS OF THE COULOMB INTERACTING - PARTICLES

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The collisional particle dynamics in such a system is investigated using a spatially homogeneous nonlinear collision integral in the Landau–Fokker–Planck form, which is a model Boltzmann collision integral for arbitrary potentials of interaction accompanied by little momentum transfer between particles in collisions. It is shown that the principal condition for the existence of steady-state nonequilibrium distributions is the presence of a particle or an energy flux oriented in the proper manner in momentum space. A steady-state local distribution exists in side the momentum interval between the energy source and sink and has the form of a gradually decreasing function. The results obtained can be used to predict the behavior of semiconductors with an intrinsic or extrinsic conductivity under the action of particle fluxes.

## 1. INTRODUCTION

Nonequilibrium states of various physical systems are attracting increased interest in connection with the development and wide use of high-power particle and energy sources. In physical systems in which the interactions between particles or waves can be described by the kinetic equations for waves, quasi-particles, or particles, the problem of constructing steady-state nonequilibrium distributions reduces to that of solving the kinetic equations. In this case, the local character of the steady-state nonequilibrium distribution corresponds to the convergence of the collision integral (at each point in momentum space, the main contribution to the collision integral comes from collisions between particles with close momenta). The existence of a steady-state nonequilibrium distribution implies that there are particle source and sink in momentum space. The energy (particle) source and sink can be provided by ion beams, emission currents, fluxes of charged particles produced in fusion or fission reactions, etc. The characteristic feature of systems of charged particles interacting through the Coulomb potential is that the interaction cross section diverges in the limit of a small momentum transferred in a collision. To describe a gas or a semiconductor plasma for which the Coulomb logarithm is large (10–15), it is sufficient to expand the integrand in powers of the small momentum transferred (the diffusion approximation) and to represent the collision integral in the Landau or Fokker–Planck form [1,2]. The Landau (Fokker–Planck) kinetic equation, which is an essential element in various physical models, is widely applied in plasma physics. Then the kinetic equations mentioned above are presented and the formulation of the problem is discussed in detail. The formation of a steady-state nonequilibrium particle distribution function is investigated numerically for a wide range of parameters (different source and sink intensities, localization of the source and sink in momentum space, etc.). The calculated results are discussed, and it is shown that the Landau collision integral provides a correct description of a nonequilibrium distribution function in a plasma in

which the main process is a small-angle scattering with little momentum transfer between the particles. In report is to consider the features of the conduction and emission characteristics of semiconductors under the action of intense particle fluxes. As was mentioned in [3] the formation of a nonequilibrium distribution of electrons and holes may give anomalously high conductivity and emission. Based on an analysis of the experimental results, Aseevskaya *et al.* [4] arrived at the conclusion that the irradiation of semiconductors by  $\alpha$  -particles may substantially change their conduction properties. Some experimental data on the emission current from metals irradiated by lasers (in particular, its dependence on the retarding potential) cannot be explained on the basis of equilibrium distribution functions [5], but they were reasonably explicable in terms of the mechanism for the formation of nonequilibrium distributions [6,8-11]. In our opinion, all these circumstances confirm the above general statement about the formation of nonequilibrium distributions in the presence of external pumping. In this context, the physical effect associated with such fundamental changes in the electron distribution function are discussed.

## 2. STEADY-STATE NONEQUILIBRIUM ELECTRON DISTRIBUTION WITH AN ENERGY FLUX ALONG THE SPECTRUM

Universal steady-state nonequilibrium power-law particle distribution functions  $f = Ap^{2s}$  ( $P$  is momentum) that are exact solutions to kinetic equations with the Boltzmann collision integral were first obtained from a symmetric approach by Kats *et al.* [7]. In the equilibrium situation, the fluxes of an energy or the particles in momentum space are identically zero, because the integrand with an equilibrium distribution vanishes at each point  $p$ . However, for a nonequilibrium distribution function, there is no need in conditions and only the integral of the collision operator (rather than the operator itself) vanishes. The distribution function  $Ap^{2s}$  corresponds to a steady-state nonequilibrium situation with a constant particle (or energy) flux in momentum space. The collision integral for steady-state nonequilibrium

distributions satisfying the above conditions was calculated in a straightforward way in [3,8,9]. In those papers, it was shown that the powers  $s$  of the local nonequilibrium particle distribution functions (for which the collision integral converges) lie within the intervals:  $-3/2 < s_0 < -1$ , for  $I_0 = \text{const}$ ;  $-3/2 < s_1 < -5/4$ , for  $I_1 = \text{const}$ . The powers 0 and 1 correspond to the fluxes  $I_0$  and  $I_1 = \text{const}$ , respectively. From these inequalities, we can see that, in the case of Coulomb interaction, the collision integral diverges, which corresponds to the well-known singularity of  $W$  when the momentum transferred in collisions is small. According to [3,8,9], the this divergence can be eliminated by taking into account the Debye screening. Thus, in [3,8,9], it was shown that, for the transition probability corresponding to the screened Coulomb potential and for particles with the momenta  $p < a_D$  ( $a_D$  is the Debye momentum), taking into account the Debye screening, on the one hand, eliminates the Coulomb divergence and, on the other, does not change the power of the steady-state nonequilibrium particle distribution function with a constant energy flux in momentum space. The power of the distribution function corresponds to the asymptotic behavior of  $W$  respecting to the Coulomb potential. Hence, in [3,8,9], it was shown that particles with the momenta  $p < a_D$  obey a local (in the sense of the convergence of the collision integral) power distribution function, in which case the particle density is determined by the flux intensity in momentum space. The flux conservation is ensured by the source and sink, whose positions should be consistent with the direction of the flux. Hence, the electrons may obey a universal nonequilibrium distribution even when the filling numbers are substantially (by one to two orders of magnitude) smaller than the equilibrium numbers. We have considered the formation of steady-state nonequilibrium distributions with sources and sinks localized in momentum space. Note, however, that it is often necessary to deal with systems in which the source and/or sink are nonlocalized; in particular, ionization by the wake field is described by a nonlocal source in momentum (energy) space.

### 3. NUMERICAL MODELING OF THE FORMATION OF STEADY-STATE NONEQUILIBRIUM PARTICLE DISTRIBUTIONS

In what follows, we will investigate a spatially homogeneous isotropic gas consisting of a single particle species. The particles are assumed to interact through the Coulomb potential. Our model is based on the Boltzmann equation with the Landau collision integral [12-14]. The first three tensor moments and the first four scalar moments of the model collision integral are assumed to coincide with those of the exact collision integral. The particle number and the energy are conserved, and the H-theorem for the Boltzmann equation is satisfied. The equations with the model collision integral provide a correct description of the system in the 20-moment approximation of the Grad method. Finally, an exact solution to the Landau equation for Maxwellian molecules is the exact solution to the Boltzmann equation. Note that the preliminary dynamics of the particles

interacting by means of the Coulomb potential was previously studied using both the Landau kinetic equation and the Fokker-Planck kinetic equation [15]. We introduce dimensionless variables such that the velocity is expressed in units of the thermal velocity  $V_T = (3/2)^{1/2} v_T$

( $v_T = \sqrt{\frac{2k_B T}{m}}$ ) and the time is measured in units of the electron-electron relaxation time, which has in the case

of Coulomb interaction the form  $\tau_{ee} = \frac{v_T^3 m^2}{ne^4 \ln \Lambda}$ . In

modeling the formation of a nonequilibrium distribution on the basis of kinetic equations with the Landau and Fokker-Planck modified collision integrals, respectively in astrophysical applications are usually simulated using kinetic equations with the Fokker-Planck collision integral: the particle and energy fluxes in momentum space were taken into account by supplementing the right-hand sides of the kinetic equations with the following terms  $S_+$  and  $S_-$ , which describe the energy (particle) source and sink. The source and sink functions were modeled by exponential functions localized in different intervals in momentum space  $S_{\pm} \propto I_{\pm} \times \exp(-\alpha_1 (v - v_{\pm})^2)$  and by the localized function (Dirac delta function)  $S_{\pm} \propto I_{\pm} \delta(v - v_{\pm})/v^2$  or

$$S_{\pm} \propto I_{\pm} \delta(v - v_{\pm})/v^2 f(v, t) \quad (2)$$

When  $I_+ = I_-$ , we deal with the energy flux from the source toward the sink, and, when  $I_+ = I_- \frac{v_-^2}{v_+^2}$ , we deal with an analogous particle flux. The direction of the fluxes depends on the ratio of the velocities  $v_+$  and  $v_-$ . The initial distribution function was assumed to be either a Maxwellian function or a delta function. The simulations were carried out on the basis of completely conservative implicit difference schemes [12-14] for which discrete analogues of the conservation laws are satisfied and which make it possible to perform long-run simulations without accumulating computational errors. An infinite velocity interval was bounded from above by the highest possible velocities  $10 V_T - 14 V_T$ , at which the distribution function was equal to a computer zero. The initial distribution in the form of a delta function was modeled by the function, which was zero over the entire velocity range used in simulations, except for a single velocity value (usually,  $v=1$ ). Since the relaxation problem is, in a sense, a test problem, we earlier began by considering the Cauchy problem for kinetic equations

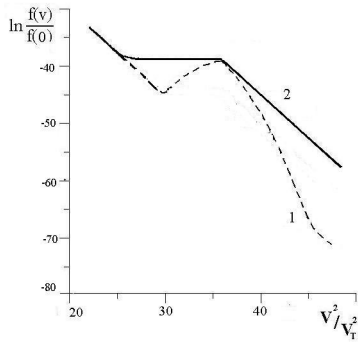


Fig. 1. Logarithm of the steady-state nonequilibrium distribution function normalized to its value at  $v=0$  vs. squared velocity. The computations were carried out for the initial Maxwellian distribution function and the source function with  $I=10^{-16}$ ,  $v_- =5$ , and  $v_+ =6$ . The solid and dashed curves were calculated from the Fokker–Planck equation at the times  $t_1=5$  and  $t_2=100$ , respectively

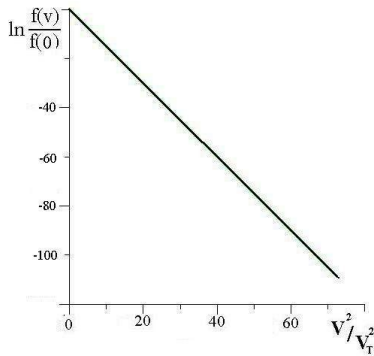


Fig. 2. Steady-state (equilibrium) distribution functions calculated for reversed positions of the source and sink ( $v_+ =5$ ,  $v_- =7$ ) from the Fokker–Planck equation and from the Landau equation

with the initial distribution function. The corresponding simulations were carried out [15] for kinetic equations with the Landau and Fokker–Planck collision integrals. Now, we proceed to a discussion of the results obtained from simulations of the formation of steady-state nonequilibrium distributions in the presence of energy or particle fluxes in momentum space. The right-hand sides of kinetic equations were supplemented with the source and sink terms  $S_+$  and  $S_-$ . First, we consider the results obtained for the case in which the source and sink were consistent with the direction of the collision-related flux in momentum space. Note that an analytic solution of the kinetic equations with a localized source and sink yields a correct flux direction, namely, the flux is directed downward along the velocity axis. From Fig. 1, we can see that, in the presence of an energy flux in momentum space, the initial particle distribution in the interval between the source and sink relaxes to a Kolmogorov steady-state nonequilibrium distribution, while the distribution function outside this interval is in thermodynamic equilibrium. Recall that the positions of the source

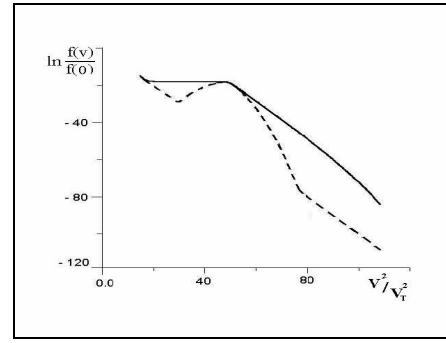


Fig. 3. Distribution function calculated from the Fokker–Planck equation with the source and sink functions  $S_{\pm} \propto I_{\pm} \exp[-\alpha_1(v - v_{\pm})^2]$  for  $\alpha_1 =100$ ,  $v_- =4$ , and  $v_+ =7$ . The dashed and solid curves refers to the times  $t=25$  and  $100$ , respectively

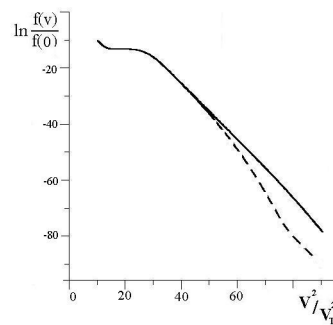


Fig. 4. Distribution function calculated from the Fokker–Planck equation with the source and sink functions  $S_{\pm} \propto I_{\pm} \exp[-\alpha_1(v - v_{\pm})^2]$  for  $\alpha_1 =10$ ,  $v_- =3$  and  $v_+ =5$ . The dashed and solid curves refers to the times  $t = 25$  and  $100$ , respectively

and sink should be consistent with the flux direction in momentum space. In order to convince ourselves that this condition is very important, we interchanged the source and sink in energy space and performed the corresponding simulations. The results calculated for reversed positions of the source and sink are illustrated in Fig. 2, which shows the logarithm of the distribution function versus the square of the dimensionless velocity. In this case, a change in the flux intensity by several orders of magnitude produces no change in the equilibrium particle distribution. Figure 3 was calculated for the source and sink terms described by the exponential functions of velocity. The source  $S_+$  is localized in a “narrow” interval of energies corresponding to seven thermal velocities, and the sink  $S_-$  is localized in the energy interval corresponding to four thermal velocities. The lengths of the intervals in which the source and sink are localized is controlled by the coefficient  $\alpha_1$  in the exponential functions. In the case at hand, the coefficient is equal to 100; this very large value ensures a strong localization of the source and sink.

In order to investigate how the electron distribution function depends on the extent to which the source and sink are localized in energy space, we carried out a

series of computations with a smaller (10 instead of 100) value of the above coefficient in the exponential functions. A comparison between the results presented in Figs. 3 and 4 shows that the character of the steady-state nonequilibrium distribution in the main region between the source and sink is independent of the extent to which they are localized, thereby providing evidence of the local (universal) character of this solution to the kinetic equation.

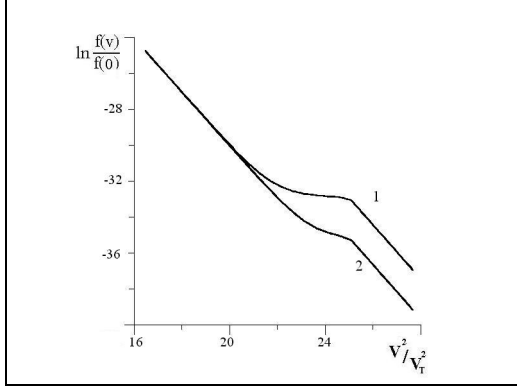


Fig. 5. Steady-state distribution function calculated from the Landau equation with  $\nu = 2$  for the source and sink functions in the form of  $\delta$ -functions at different flux intensities  $I_1=0.01$  and  $I_2=0.001$  ( $\nu_- = 4$ ,  $\nu_+ = 5$ )

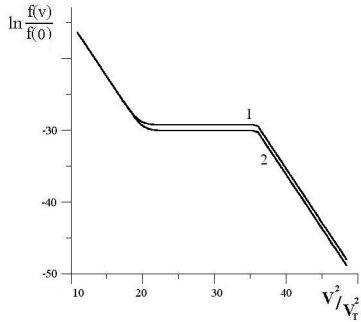


Fig. 6. Steady-state distribution function calculated from the Landau equation for the source and sink functions in the form of  $\delta$ -functions at  $I = 0.01$ ,  $\nu_- = 4$ , and  $\nu_+ = 6$ . Curve 1 and 2 correspond to a constant energy flux ( $dN = 0$ ) and a constant particle flux ( $dE = 0$ ), respectively

Figure 5 shows how the distribution function depends on velocity for different flux intensities. We established that, when the intensities of the source ( $I_+$ ) and sink ( $I_-$ ) are both low, a universal nonequilibrium distribution forms in the velocity range  $\nu_+$ . The reasons for this are twofold: first, with increasing velocity, the cross section for Coulomb scattering decreases (in proportion to  $\sim v^{-3}$ ) and, second, diffusion due to Coulomb collisions always gives rise to energy and particle fluxes toward the region of the main (“background”) equilibrium distribution function. As a result, the higher the intensities of the source and sink, the lower the relative intensity of the particle flux toward the background plasma and, accordingly, the larger the region in which a universal nonequilibrium particle distribution forms between the source and sink. It should be noted that, as

the flux becomes more intense, the values of a nonequilibrium distribution function increase. This is because the nonequilibrium distribution function is proportional to the flux intensity. These values illustrate the results of a detailed study of the dependence of the solution on the flux intensity in momentum space, which was varied over a wide range. The initial source and sink functions were taken to be functions with  $\nu_- = 4$ ,  $\nu_+ = 8$ , and  $dE = 0$ . Now, we consider the formation of a nonequilibrium particle distribution in the case of a constant particle ( $I_+ = I_- \frac{\nu_+^2}{\nu_-^2}$ ,  $dE = 0$ ) or energy ( $dN = 0$ ,  $I_+ = I_-$ ) flux.

In order to simplify comparisons of the results obtained, it is convenient to normalize the distribution function to its value at zero, as is done in all figures presented in this paper, and, in particular, in Fig. 6, from which we can see the formation of gradually decreasing distribution functions with close powers  $s$  in the cases of a particle and an energy flux. The larger the power, the shorter the interval over which the distribution function is nonequilibrium. These calculated results agree qualitatively with the above analytic predictions.

#### 4. MECHANISM FOR THE FORMATION OF THE ELECTRON DISTRIBUTION FUNCTION IN THE INTERACTION OF INTENSE PARTICLE FLUXES WITH A SOLID-STATE PLASMA

In this section, we focus on the distinctive features of the conduction and emission characteristics of a semiconductor plasma that is affected by intense particle fluxes. Note: the values of the distribution function are seen to increase with increasing intensity  $I$ : at low intensities (to 0.1), the values of the distribution function increase in proportion to  $I$  because of the large contribution of the interaction of nonequilibrium particles (i.e., the particles from the interval between the source and sink) with the background particles, whose distribution is thermodynamically equilibrium; at moderate intensities (from 0.1 to 20), the distribution function is of a universal nature over the entire interval between the source and sink and is proportional to the square root of the intensity  $I$ , in agreement with theory and at higher intensities, the distribution function is no longer proportional to the square root of the intensity because it enters into the sink function.

As an example, we consider a solid-state plasma that is affected by a flux of fast ions (with speeds higher than those of atomic electrons). A typical situation in a single ion track can be described as follows. Let the energy of a helium ion be  $\mathcal{E}_i = 5$  MeV, the excitation potential be  $\phi = 100$  eV, and the ion mean free path in the target material be  $R_{mfp} = 10^{-3}$  cm. The number of electrons produced during ionization processes along the path of a helium ion is about  $10^4 - 10^5$ . The radius of the ionization track of an ion is comparable with the mean free path of the electrons produced ( $R_e = 10^{-6}$

cm). The electron density in an ionization track is  $n_{en} = 10^{19} - 10^{20} \text{ cm}^{-3}$ , the equilibrium electron density being  $n_e = 10^{22} \text{ cm}^{-3}$ . Hence, in this case (see [8,9,16]), the time of the electron energy relaxation due to electron–electron collisions is substantially shorter than that due to electron–phonon collisions. A comparison of the characteristic ionization time with the relaxation times shows that, in our case, the electron distribution function is quasi-steady and is governed primarily by electron–electron collisions. Consequently, this function can be determined from the condition for the Boltzmann (Landau–Fokker–Planck) collision integral to vanish. The earlier study [8,9,16] and the numerical modeling analysis show that, in a semiconductor plasma, the power-law distribution function corresponding to a constant energy or particle flux in momentum space can exist in the energy range  $(E - E_F) > E_F$ . This distribution is formed by both collisions with electrons in the energy range  $(E - E_F) > E_F$  and collisions with background (equilibrium) electrons. As was shown above, the nonequilibrium electron distribution is close to a universal distribution if the intensity of the flux produced by the source and sink in momentum space is sufficiently high. For such ratios of the electron densities, the source and sink are sufficiently intense for the nonequilibrium distribution function to form. Let us now consider the mechanisms by which a fast ion loses its energy in a solid-state plasma. Note that, under conditions typical of the ion beam – driven inertial confinement fusion, the ion energy loss is independent of the ion density in the beam. Taking into account this circumstance, we can calculate the energy lost by an ion from the Bethe–Bloch formula. According to [17], the energy of a fast nonrelativistic particle is transferred to a medium by the following two mechanisms. A fraction of the particle energy is expended on the excitation of collective oscillations associated with wake charge density waves, and the remaining fraction is transferred to individual electrons, with a subsequent ionization of the medium [17,18]. The former fraction corresponds to the macroscopic energy loss in long-range collisions with little momentum transfer, and the latter fraction corresponds to collisions with large momentum transferred. The fraction of the particle energy that is lost through the excitation of collective oscillations is relatively large [18]. A fast particle passing through the medium produces slow electrons by two equiprobable mechanisms: by avalanche ionization and by ionization through the excitation of plasma oscillations. The main features of the electron production through ionization by plasma oscillations are associated with long time and spatial scales of the wake charge density waves. Because of the long time scale of the waves, the secondary ionization inside the beam proceeds for a long time after the passage of a beam particle. A substantial number of slow electrons are liberated in cascade ionization caused by a high-energy secondary electron. Since the mean free path of such an electron in a medium is long, most of the slow electrons are produced in cascade ionization along its path. As a result, ionization by the wake field is the dominant process affecting the distribution of lib-

erated electrons near the axis of the particle track, whereas cascade ionization is responsible for the distribution of liberated electrons at distances from the track axis that are on the order of the electron mean free path. Since a charged particle ionizes the medium by its self-field only during its passage and since the wake charge density waves play the role of a linear source of secondary electrons for a long time after the passage of the ionizing particle, the ionization dynamics is governed completely by the secondary electrons. As was mentioned above, despite the short time required for an ion to pass along the track, the characteristic time of the avalanche ionization by wake charge density waves is fairly long (about  $10^{-13}$  s). Let us analyze some physical consequences of such a fundamental change of the electron distribution function. From the expression for a density of the electron emission current we can see [3,8,9,11] that, in a plasma with a nonequilibrium electron distribution, the electron emission current density is anomalously high, because, in the inertial interval, the distribution function decreases very gradually. The conduction characteristics of the medium are governed by the density of the current carriers, so that, in a semiconductor plasma with a nonequilibrium electron distribution, this density is very high, in contrast to the case of an exponentially decreasing equilibrium distribution function. That is why, under the action of intense fluxes of fast particles, the emission and conduction properties of a semiconductor plasma can become anomalous. Note that such anomalies have already been observed in some experiments (see, e.g., [4,5,11]).

## 5. CONCLUSION

In conclusion, we summarize the results obtained from numerical modeling of the formation of steady-state nonequilibrium distribution functions. For a distribution function computed from the have established the following:

(i) In the interval between the source and sink located at certain positions in momentum space, the particles interacting through the Coulomb potential relax to a steady-state nonequilibrium distribution. Moreover, above certain source and sink intensities, the distribution function over almost the entire interval between the source and sink is of a universal nature; i.e., it obeys the same power law.

(ii) The functional dependence of the steady-state nonequilibrium electron distribution is sensitive neither to the extent to which the source and sink are localized in momentum space nor to the length of the inertial interval (i.e., the interval between the source and sink). It is, however, necessary that the positions of the source and sink be consistent with the direction of the energy flux in momentum space. Recall that the flux direction is determined by the nature of the interaction between particles and is entirely independent of the source and sink structures. In other words, the particles relax to universal local distribution functions.

(iii) For the source and sink intensities in the interval  $0.001 \leq I \leq 0.1$ , the distribution function computed from the Fokker–Planck equation is proportional to the intensity; as the intensity increases from 1 to 10, the values of the distribution function increase only by a factor

of 3. The fundamental change in the electron distribution function under nonequilibrium conditions leads to an anomalous enhancement of the conduction and emission characteristics of the medium. The results obtained in our investigations can be used to predict the behavior of semiconductors with an intrinsic or extrinsic conductivity in the fluxes of fast particles, as well as in intense electromagnetic radiation.

This work was supported in part by the STCU on project #1862.

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