STABILIZATION OF BEAM INSTABILITY AS A RESULT OF DEVELOPMENT OF LOCAL INSTABILITY IN WAVE-WAVE INTERACTION

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The new mechanism of stabilization of beam instability is proposed. The considered mechanism plays a special role at stabilization of beam instability in plasma systems with small size of interaction area of a beam of particles with field of exited waves. The basis of this mechanism is the process of three-wave decay with participation of a wave which easily abandons the field of interaction and also the process of chaotization of the fields at nonlinear interaction of waves.

1. INTRODUCTION

The essence of the mechanism can be explained by the fact that in short electrodynamic systems the known mechanisms of instability stabilization such as capture of beam particles by the field of exited waves and stochastic instability of movement of beam particles occur at mush bigger intensities of exited fields than it is observed in long systems. It is caused by short time of flight of beam particles through area of interaction with a field. Local instability of wave - wave process can be essential in such conditions. At the same time dynamics of fields becomes chaotic. Efficiency of interaction of a charged beam with fluctuating field is much lower than with fields of regular waves. In addition fluctuating fields rapidly convey their energy to heating of plasma particles. Thus a new channel of rapid dissipation of energy of excited waves appears.

As a result stabilization of instability or even failure of process of excitation of waves takes place. Chaotization of wave fields exited by beam may result in fast heating both particles of a beam, and particles of plasma. The results of some experiments at which, apparently, the described mechanism of failure of plasma-beam instability is realized, are described.

It is necessary to note, that the stabilization of instabilities caused by the regular mechanism of decay has been discussed in the literature for a long time (for example [1]).

2. STOCHASTIC INSTABILITY OF DYNAMICS OF WEAK NONLINEAR INTERACTION OF WAVES

At rather high amplitudes of the waves exited in plasma it is possible that effective nonlinear interactions of these waves with other proper waves of plasma electrodynamic structure take place. Dynamics of this interaction can be both regular and chaotic. We are interested in chaotic regimes. Such regimes appear in different schemes of nonlinear wave-wave interaction.

The more simple are modified decay and also the case of three-wave interaction, when during interaction the fourth wave can participate in this interaction. The characteristics of this wave are close, for example, to a low-frequency wave participating in the interaction. The last case we shall term quasi-four-wave. The stochastic instability develops only when amplitude of a decaying wave (pump wave) exceeds some threshold value. Let's consider these two cases in details.

2.1. QUASI-FOUR-WAVE INTERACTION

Let the wave with amplitude a_1 wave number k_1 and frequency ω_1 decay into two waves a_2, k_2, ω_2 and a_3, k_3, ω_3. Besides that let us assume, that there is one more wave with the following parameters a_4, k_4, ω_4; $k_4 = k_3$, $\omega_3 - \omega_4 << \omega_1$. Let us consider that the fourth wave does not influence the process of decay. The equation, which describes dynamics of complex amplitudes at interaction of the first three waves can be presented as [3]:

$$\dot{a}_1 = iV_1^* a_2 a_3,$$
$$\dot{a}_2 = iV_1 a_1 a_3^*, \qquad (1)$$
$$\dot{a}_3 = iV_1 a_1 a_2^*,$$

where $V_1 = |V_1| \exp(i\Phi_o)$ is the matrix element of interaction, $a_j = |a_j| \exp(i\Phi_j)$. On the linear stage ($|a_1| = const, \Phi_1 = const$) of decay the amplitudes $|a_1|$ and $|a_2|$ growth exponentially with increment $G = |a_1||V_1|$. The phase change $\Phi = 2(\Phi_1 - \Phi_2 - \Phi_3 + \Phi_0)$ obeys equation of mathematical pendulum:

$$\ddot{\Phi} + (2|a_1||V_1|)^2 \sin\Phi = 0. \qquad (2)$$

It is seen from Eq.(2) that the half width of nonlinear resonance equals $4G$. If we replace the third wave by forth wave we obtain the following set of equations:

$$\dot{a}_1 = iV_2^* a_2 a_4 \exp(-i\delta\tau),$$
$$\dot{a}_2 = iV_2 a_1 a_4^* \exp(i\delta\tau), \qquad (3)$$
$$\dot{a}_3 = iV_2 a_1 a_2^* \exp(i\delta\tau).$$

On the linear stage phase $\Psi = 2(\Phi_1 - \Phi_2 - \Phi_4 + \Phi_0 + \delta\tau)$ satisfies Eq.(2) too, where $G_2 = |a_1||V_2|$, $\delta = \omega_1 - \omega_2 - \omega_4$. This means that the distance between nonlinear resonances is equal to 2δ. Assuming the width of nonlinear resonance for the forth wave is small ($G >> G_2$) we obtain the condition of the nonlinear resonance overlapping and, correspondingly, the criterion of stochastic instability:

$$2G/\delta > 1. \qquad (4)$$

2.2. MODIFIED DECAY

The important case of three-wave interaction is the case of modified decay. At such decay the increment of linear stage is larger then the frequency of a low-frequency wave, which participates in three-wave interaction. As we showed before [4] the modified decay is always chaotic.

Let us consider the decay of the HF electromagnetic wave (i) with frequency ω_i, wave vector k_i and amplitude E_i, which propagates in uniform, unbounded plasma in the HF (s) electromagnetic wave (ω_s, k_s, E_s) and LF Langmuir wave (ω_{pe}, k_p, ϕ). In order to describe this process we started from Maxwell's equations for electromagnetic fields and hydrodynamic equations for plasma electrons. We neglected the movement of ions, assuming that background ions serve for compensation of electron charge. Time averaging t_o ($t_{slw} \gg t_o \gg t_{fst}$, $t_{slw} \propto 1/\omega_{pe}$, $t_{fst} \propto 1/\omega_i$ - periods of slow and fast variables, respectively) leads to the following system of coupled equations:

$$\frac{\partial \tilde{v}_e}{\partial t} = \frac{e}{m}\tilde{\vec{E}},$$

$$\nabla^2\tilde{\vec{E}} - \frac{1}{c^2}\frac{\partial^2\tilde{\vec{E}}}{\partial t^2} - \frac{\omega_{pe}^2}{c^2}\tilde{\vec{E}} = \frac{\omega_{pe}^2}{c^2}\frac{\delta n_e}{n_{eo}}\tilde{\vec{E}}, \quad (5)$$

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{pe}^2 - V_{Te}^2\nabla^2\right)\frac{\delta n_e}{n_{eo}} = \frac{1}{2}\nabla^2 < \tilde{\vec{v}}_e^2 >,$$

where $\tilde{v}_e$ - HF electron velocity (varies on the fast time-scale - $t_{fst} \propto 1/\omega_i$), $\tilde{E}$ - HF component of the electric field (varies on the fast time-scale - $t_{fst} \propto 1/\omega_i$), n_{eo} - equilibrium electron density, δn_e - electron density perturbation (varies on the slow time-scale - $t_{slw} \propto 1/\omega_{pe}$), e, m - charge and mass of the electron, respectively, c - speed of light in vacuum, $\omega_{pe}^2 = \frac{4\pi e^2 n_{eo}}{m}$ - plasma frequency, $V_{Te}^2 = \frac{T_e}{m}$ - electrons thermal velocity. $< ... >$ - time averaging:

$$< x(t) > = \frac{1}{t_o}\int_t^{t+t_o} x(\tau)d\tau .$$

Note, that we used the similar approach described in [3] to receive the set of equations (5). Assuming following form for HF electromagnetic field and LF plasma density:

$$\tilde{\vec{E}}(\vec{r},t) = \frac{1}{2}\vec{E}_i(t)\exp(-i\omega_i t + i\vec{k}_i\vec{r}) +$$
$$+ \frac{1}{2}\vec{E}_s(t)\exp(-i\omega_s t + i\vec{k}_s\vec{r}) + c.c.$$

$$\delta n_e(\vec{r},t) = \frac{1}{2}\delta n_p(t)\exp(i\vec{k}_p\vec{r}) + c.c.$$

where $E_i(t) \, and \, E_s(t)$ - slowly varying in time amplitudes of the HF pumping wave and LF scattered wave, respectively, one can obtain from (5):

$$i\cdot\frac{d\varepsilon_i}{d\tau} = \varepsilon_s\rho\exp(i\Delta\tau),$$

$$i\cdot\frac{d\varepsilon_s}{d\tau} = \varepsilon_i\rho^*\exp(-i\Delta\tau), \qquad (6)$$

$$\frac{d^2\rho}{d\tau^2} + \Omega^2\rho = -\varepsilon_i\varepsilon_s^*\exp(-i\Delta\tau),$$

where we neglected thermal term $\propto V_{Te}^2$,

$$\varepsilon_i = \frac{E_i}{E_{io}}, \, \varepsilon_s = \frac{E_s}{E_{io}}\sqrt{\frac{\omega_s}{\omega_i}}$$

$$\rho = \frac{\delta n_p}{n_{eo}}\left[\frac{\omega_{pe}^2\cos(\alpha)\pi \, m \, n_{eo}}{2k_p^2\sqrt{\omega_i/\omega_s}E_{io}^2}\right]^{1/3},$$

$$\tau = t\left[E_{io}^2\cos^2(\alpha)\frac{e^2 k_p^2\omega_{pe}^2}{8m^2\omega_i\omega_s^2}\right]^{1/3},$$

$$\Omega^2 = \left[\frac{8m^2\omega_{pe}\omega_i\omega_s^2}{e^2 E_{io}^2\cos^2(\alpha)k_p^2}\right]^{2/3},$$

$$\Delta = (\omega_i - \omega_s)\left[\frac{8m^2\omega_i\omega_s^2}{e^2 E_{io}^2\cos^2(\alpha)k_p^2\omega_{pe}^2}\right]^{1/3},$$

where $\cos(\alpha)$ - angle between E_i and E_s, $E_{io} = E_i(t = 0)$. In order to obtain the set of equations (6) we assume that there is a spatial synchronism between coupling waves: $k_i - k_s = k_p$.

On the linear stage of the decay when $|\varepsilon_i| = const$ we can obtain from (6) the dispersion relation:

$$(\omega^2 - \Omega^2)(\omega + \Delta) = 1, \qquad (7)$$

and following expressions for maximum values of the growth rates:

$$G = Im\omega = 1/\sqrt{2\Omega}, \, \Omega^2 \gg 1;$$
$$G = Im\omega = \sqrt{3}/2, \, \Omega^2 \ll 1.$$

In the first case, when the amplitude of the pumping wave is small, parameter $K \ll 1$, $(\Delta \propto \Omega)$ and dynamics of the decay according to (4) must be regular. At large amplitudes of the incident wave $K \gg 1$, $(\Delta \propto 0)$ and the decay must be chaotic. Note that the region of parameters where $K \gg 1$ is related to the modified decay.

3. EFFICIENCY OF TRANSMISSION OF ELECTRONIC BEAM ENERGY TO FLUCTUATING FIELD

It was noted earlier that as a result of wave - wave interaction dynamics of field becomes chaotic. It could cause diminution of effectiveness of interaction of parti-

cles with field in electrodynamic system and respectively stabilization of beam instability. For the study of process of energy interchange with fluctuating field in restricted area of space let's use the set of equations given in [5]:

$$\frac{d\varepsilon}{d\xi} = \mu\left[\cos\Phi + \cos\left(\Phi + \frac{2\xi}{V_{ph}}\right)\right],$$

$$\frac{d\Phi}{d\xi} = \frac{1}{\sqrt{\varepsilon}} - \frac{1}{V_{ph}}, \qquad (8)$$

Where $\varepsilon = V^2/V_0^2$ –dimensionless energy of particle, V, V_0 –current and initial velocity of particle, $V_{ph} = v_{ph}/V_0$, v_{ph}– phase velocity of wave, $\mu = (2E)/(m\omega V_0) \ll 1$ – dimensionless velocity of wave, $\Phi = \omega t - kz$ –phase of wave, L –length of a system, $\xi = z\omega/V_0$– normalized coordinate of a particle.

The set of equations (8) describes moving of a charged particles in a field of a standing wave and can be solved by the method of successive approximations using the small parameter μ. Suspecting that the phase of a wave has fluctuation parts ($\Delta\Phi$), the system (8) can be reduced to:

$$\frac{d\varepsilon}{d\xi} = \mu\left[\cos\Phi + \cos\left(\Phi + \frac{2\xi}{V_{ph}}\right)\right]\cos\Delta\Phi -$$

$$- \mu\left[\sin\Phi + \sin\left(\Phi + \frac{2\xi}{V_{ph}}\right)\right]\sin\Delta\Phi, \qquad (9)$$

$$\frac{d\Phi}{d\xi} = \frac{1}{\sqrt{\varepsilon}} - \frac{1}{V_{ph}}.$$

Initial conditions for dimensionless energy e, m. We guess that $\varepsilon = 1 + \mu\varepsilon^{(1)} + \mu^2\varepsilon^{(2)} + \mu^3\varepsilon^{(3)} + ...$ Then in zero-order approximation from the second equation of the system (9) for a phase of a wave in which there is a particle we shall receive the following expression:

$$\Phi = \frac{V_{ph} - 1}{V_{ph}}\xi + \Phi_0, \qquad (10)$$

Where Φ_0 - phase of field at the moment of entrance of a particle in a cavity. Substituting expression for Φ as (10) in the first equation of the system (9) and integrating it we shall receive expression for $\varepsilon^{(1)}$:

$$\varepsilon^{(1)} = 2\int_0^\xi \cos(\xi' + \Phi_0)\cos\frac{\xi'}{V_{ph}}\cos\Delta\Phi(\xi')d\xi' -$$

$$- 2\int_0^\xi \sin(\xi' + \Phi_0)\cos\frac{\xi'}{V_{ph}}\sin\Delta\Phi(\xi')d\xi' \qquad (11)$$

By analogy with ε the phase can be submitted as:

$$\Phi = \frac{V_{ph} - 1}{V_{ph}}\xi + \Phi_0 + \mu\Phi^{(1)} + \mu^2\Phi^{(2)} + ...$$

From the second equation of the system (9) it is possible to receive the expression for correction of the first degree to the phase:

$$\Phi^{(1)} = -\frac{1}{2}\int_0^\xi \varepsilon^{(1)}(\xi')d\xi'.$$

Prolonging iterative procedure we shall receive the correction of the second degree to dimensionless energy:

$$\varepsilon^{(2)} = \int_0^\xi d\xi'\int_0^{\xi'} d\xi_1\int_0^{\xi_1} d\xi_2 \cos\frac{\xi'}{V_{ph}}\cos\frac{\xi_2}{V_{ph}}\chi$$

$$\{[\sin(\xi' + \xi_2 + 2\Phi_0) + \sin(\xi' - \xi_2)][\cos\Delta\Phi(\xi')\cos\Delta\Phi(\xi_2)] +$$

$$[\cos(\xi' + \xi_2 + 2\Phi_0) - \cos(\xi' - \xi_2)][\cos\Delta\Phi(\xi')\sin\Delta\Phi(\xi_2)] +$$

$$[\cos(\xi' + \xi_2 + 2\Phi_0) + \cos(\xi' - \xi_2)][\sin\Delta\Phi(\xi')\cos\Delta\Phi(\xi_2)] +$$

$$[\sin(\xi' + \xi_2 + 2\Phi_0) - \sin(\xi' - \xi_2)][\sin\Delta\Phi(\xi')\sin\Delta\Phi(\xi_2)]\}$$

$$(12)$$

First of all, we are interested in the corrections of the first and the second degree in the expression for dimensionless energy, which are defined in relations (11) and (12). From (11), in particular, it follows that at injection of continuous homogeneous monoenergetic beam, an addend $\mu\varepsilon^{(1)}$ (i.e. linear with respect to the amplitude of the field) does not give any contribution in the expression for interchange of energy of beam with a field. It will influence only on modulated beam. The influence of fluctuations of a phase $\Delta\Phi$ on interchange of energy is of interest. For this purpose it is necessary to average the expressions (11) and (12) at realization of random function $\Delta\Phi(\xi)$. Analytically it can be made in the elementary case by guessing that the density of probabilities of distribution $\Delta\Phi(\xi)$ is uniform. Let us guess that the peak value of fluctuations of phase is equal $\Delta\Phi_m$ and average value is $<\Delta\Phi(\xi)> = 0$.

It is possible to show, that $<\sin\Delta\Phi(\xi)> = 0$ and $<\cos\Delta\Phi(\xi)> = \frac{\sin\Delta\Phi_m}{\Delta\Phi_m}$. Using these relations for (11), we shall receive for average value $<\varepsilon^{(1)}>$:

$$<\varepsilon^{(1)}> = \varepsilon^{(1)}\frac{\sin\Delta\Phi_m}{\Delta\Phi_m}. \qquad (13)$$

In the case when δ - correlated fluctuations, i.e. at realization of requirement

$$<\Delta\Phi(\xi)\Delta\Phi(\xi')> = N\delta(\xi - \xi')$$

for average value of the correction of the second degree to dimensionless energy we shall receive:

$$<\varepsilon^{(2)}> = \varepsilon^{(2)}\frac{\sin^2\Delta\Phi_m}{\Delta\Phi_m^2}. \qquad (14)$$

Thus, from expressions (13) and (14) it follows that at presence of fluctuations of the phase of electromagnetic field in restricted area, contribution of addends (linear and square-law in amplitude of a field) in the expression ε decreases. Therefore, the efficiency of interaction of the electron beam with fluctuating electromagnetic fields reduces.

4. HEATING OF PLASMA PARTICLES BY A FIELD OF NOISE WAVES

As it was mentioned above stochastic instability results in chaotization of excited fields. The field energy

with randomly varying parameters is transmitted rather effectively into thermal energy of charged particles, which move in this field. The transmission of energy from the field to particles is powerful mechanism of wave attenuation. The presence of such mechanism of energy sink, together with mechanism reducing efficiency of energy transmission from beam to exited waves (at development of stochastic instability) can break the process of excitation of waves by electronic beam. Let us estimate the efficiency of energy transmission from random field to the particles. For this purpose we shall choose the most prime model. Let us consider that the charged particles move in random field where there are no correlations, i.e. $\langle E(t_1) \cdot E(t_2) \rangle = A^2 \cdot \delta (t_1 - t_2)$. From general equations of motion of charged particles in field of electromagnetic waves it is possible to receive the following equation for definition of variation of particle energy $\gamma(\tau)$ in time:

$$\dot\gamma = v \cdot F \qquad (15)$$

In equation (15) the following designations are used:

$$\dot\gamma = d\gamma / d\tau, \quad \tau = \omega \cdot t, \quad v \to v/c,$$

$F = q \cdot E / m \cdot c \cdot \omega$ - parameter of wave force, ω - some average frequency of spectral distribution of electromagnetic field.

Taking into account that there is no field correlation from the equation (15) it is easy to determine the following estimation for energy which can be gained by the particles in such field:

$$\langle (\Delta \gamma)^2 \rangle = \langle (\gamma(\tau) - \gamma(0)) \rangle = v^2 \cdot A^2 \cdot \tau \qquad (16)$$

For nonrelativistic moving it is convenient to rewrite equation (13) in dimensional unities:

$$\langle W - W_0 \rangle = 2W_0 \cdot A \cdot \omega \cdot \frac{c}{v_0} \cdot t, \qquad (17)$$

where W - kinetic energy of particles, v_0 - initial velocity of particles.

As it is seen from (17) the energy of plasma electrons can vary from several eV (electronvolt) up to keV (kiloelectronvolt) in a time about hundreds of periods of high-frequency field, electric field amplitude of which is equal ~100 V/cm.

5. CONCLUSIONS

Above we have considered the processes which develop in time. In beam amplifiers the processes pass in space. Many of described above peculiarities of interaction of beam particles and plasma with electromagnetic waves will take place in this case too. Thus, the equations of three-wave interaction will differ, for example, from the equations (1) only by the fact that time derivative will be replaced by coordinate derivative and the coupling coefficients will gain a multiplier v_g equal to group velocity of the appropriate wave in a denominator.

In general case the processes take place both in time and in space in plasma-beam experiments. Therefore it is difficult to expect good quantitative agreement of the results of the analysis of the simplified theoretical models with the results of experiments. Only in the specially posed experiments it is possible to rely on such coincidence. Below we shall shortly describe the results of the experiment, in which above described mechanism of stabilization of level of waves exited by beam was probably observed.

In the experiment the electronic beam with current 1-10 A and energy 10-40 keV was injected in the interacting region. As a result beam-plasma discharge development the plasma was created. Density of plasma varied from $5 \cdot 10^{11}$ up to $1 \cdot 10^{13}$ e/sm^{-3}. All system was located in a constant external homogeneous magnetic field. The strength of this field was made ~0.2T. During creation of plasma it got the form of the tubular cylinder.

As a result of development beam-plasma instability the proper wave of plasma electrodynamic structure were exited. The pulse radiation of electromagnetic waves, which were directed practically as perpendicular to the axis of plasma electrodynamic structure, were observed in experiment. The duration of pulses varied from several µs (3-10) down to 0.5 µs.

The higher level of electromagnetic wave intensity, the shorter duration radiation pulses were transforming to intense single pulses. The radiation pulses from plasma follow through approximately equal time intervals. At the large levels of capacity of an electron beam the duration of a radiation pulse is reduced and the time interval between them becomes larger transforming to individual pulses. During failure of electromagnetic wave radiation the heating both of plasma electrons and electrons of beam was observed.

The details of experimental researches are in detail represented in the report [6].

At theoretical description, apparently, it is possible to consider that electrodynamic structure is tubular plasma. As it is known there can be two eigen slow surface waves in such plasma. The frequencies of these waves are of order $\omega_p / 2$. These waves are exited by beam. When amplitude of exited waves reaches some threshold level, the mechanism of three-wave decay of these waves on low-hybrid wave and on electromagnetic transverse wave takes part. Let us note, that electromagnetic transverse wave is an improper wave of plasma electrodynamic structure. The appropriate dispersion curves and scheme of three-wave decay are shown in figure 1.

We do not give the expressions for coupling coefficients of three-wave decay as they are enough unwieldy and what is more important, we have no opportunity to carry out quantitative comparison of the experimental results with the results of calculations. The values of many parameters defining the dynamics of model are unknown for us. In particular we don't know field intensities. Furthermore the plasma cylindrical stratum is nonuniform in the experiment. And, typical size of inhomogeneity is of order of wave length. In experiment the process takes place both in time and in space. All this suggests that the quantitative comparison won't be correct. It is possible to speak only about qualitative description of the process.
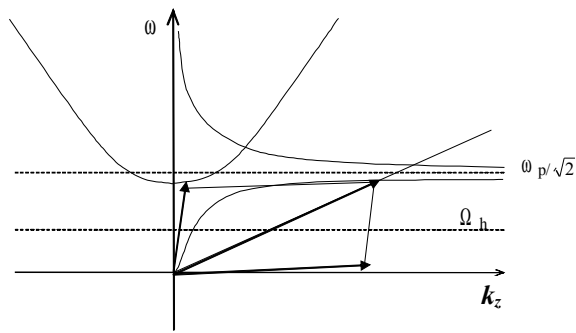
Fig.1. Dispersion curves of a tubular plasma waveguide

The qualitative situation is prime enough. Let us give its brief description. At the beginning the beam excites one of eigen surface waves. These waves do not abandon the plasma cylinder. When the level of excited waves is high enough, eigen wave decays into transverse electromagnetic wave and on low-hybrid one. As the transverse wave is not eigen, it easily abandons plasma. At this moment the effective radiation from the plasma cylinder is observed. And the radiation is practically perpendicular to the axe of the plasma cylinder. This part of the process is in good qualitative agreement with the experiment. Really in experiment the excitation of low-hybrid waves was always observed simultaneously with radiation of a transverse wave (perpendicular to the axe of a waveguide). Heating of both plasma particles and particles of a beam was also observed. It shows that the process of decay becomes chaotic. Let's note that heating of the particles of a beam could be observed as a result of local instability of dynamics of the particles of a beam in fields of excited waves. However for plasma particles this mechanism of heating is impeded. Thus, at the moment of radiation of transverse waves from plasma two channels of sink of energy appear. The first channel is outlet of energy together with improper electromagnetic waves. The second one is effective heating of particles of a beam and plasma by fields of fluctuating waves. These two channels cause depression of level of fields in plasma and as a result failure of three-wave decay. The radiation from plasma stops. Let us note one feature of the considered mechanism which can appear in such rather long system. In ordinary dynamics of plasma-beam instability the stabilization of level of exited field occurs as a result of inverse influence of the field on dynamics of particles of a beam. As a result of such influence the particles of a beam move from braking phase into accelerating phase. The electromagnetic energy of the field is returned to particles of a beam. As a result of three-wave interactions the level of the field is reduced. The inverse action of the field on particles weakens. Efficiency of energy transmission from particles to the field can be enlarged. Besides if the process of decay becomes chaotic, the phases of the field vary at random fashion and the inverse action of the field (which causes extraction of energy of the wave by the particles of a beam) is broken. The particles of the beam will prolong transmission of the energy to the wave though with smaller efficiency. This pattern of the process of excitation of waves in plasma-beam system qualitatively coincides well with a pattern, which is observed in experiment. We can not speak about quantitative coincidence yet.

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