RESONANT TWO-PHOTON EMISSION OF AN ELECTRON IN THE FIELD OF AN ELECTROMAGNETIC WAVE

O.I. Voroshilo and S.P. Roshchupkin

Institute of Applied Physics, National Academy of Sciences of Ukraine, Sumy, Ukraine
e-mail: al-war@ukr.net

We present, for general relativistic case, a theoretical investigation of the resonant two-photon emission of an electron in the circularly-polarized electromagnetic wave. Resonances are related to a virtual intermediate particle that falls within mass shell. We find condition when resonances occur and we derive the expressions for the resonant amplitude and the differential probability when the invariant intensity of the wave is small ($\eta << 1$). It is demonstrated that the resonant two-photon emission probability may be several orders magnitude greater than the probability of the corresponding process out of the resonance.

PACS: 3450.Rk, 12.20.-m

1. INTRODUCTION

The theoretical study of the quantum processes of the first order in the fine structure constant in the presence of the field of a plane electromagnetic wave dates back to the 1960s and is connected with the creation of lasers. Experimental testing of this study becomes possible with production of ultrahigh-power femtosecond lasers. The results of series of experiments at SLAC are found to be in general in agreement with the theoretical predictions [1-3]. For interpretation of these experiments becomes necessary to take into account the quantum processes of the second order in the fine structure constant in the wave field. The analysis of these processes in the wave field is complicated by computational difficulties and a cumbersome form of results and in works [2,3] the estimation result is only used. Characteristic feature of the second order process in the wave field is the appearance of the resonances which are related to a virtual intermediate particle that falls within the mass shell (see the work [4] and review [5]).

The purposes of the present work are clarification of a condition of resonant two-photon emission and calculation of a resonant probability.

The relativistic system of units, where $\hbar = c = 1$, and standard metric $(ab) = a_0 b_0 - \mathbf{ab}$ will be used throughout this paper.

2. AMPLITUDE

Let us choose the 4-potential of an external circularly polarized electromagnetic wave as

$$A(\varphi) = a\,(e_x \cos\varphi + \delta e_y \sin\varphi), \qquad (1)$$

where $a = F/\omega$; F and ω are the amplitude of the electric field strength and the frequency of the wave; $\delta = \pm 1$; $\varphi = (kx) = \omega t - \mathbf{kx}$ is the phase; $k = (\omega, \mathbf{k})$ and $e_x = (0, \mathbf{e}_x)$, $e_y = (0, \mathbf{e}_y)$ are the four-momentum and the polarization four-vectors of the wave meeting the standard condition: $(e_x e_y) = (e_x k) = (e_y k) = 0$, $e_x^2 = e_y^2 = -1$.

The amplitude for the two-photon emission of an electron in the field of a wave is represented by two Feynman diagrams (Fig. 1).

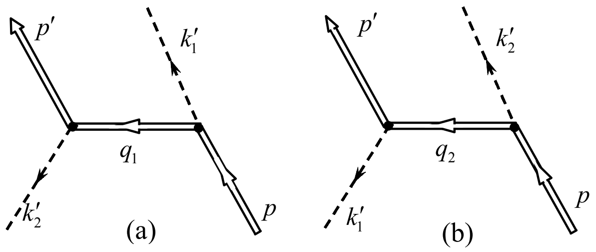

***Fig. 1.** Feynman's diagrams of two-photon emission of an electron in the field of a plane electromagnetic wave. The double lines correspond to the wave function of an electron in the field of the wave (the Volkov functions), and the dashed lines represent a photon*

The amplitude of the two-photon emission of an electron is given by the expression

$$S = -ie^2 \int dx dx' \overline{\Psi}_{p'}(x) \gamma^{\mu} G(x, x') \gamma^{\nu} \Psi_p(x')$$
$$\times \left(A^*_{\mu}(k'_1 x) A^*_{\nu}(k'_2 x') + A^*_{\nu}(k'_1 x') A^*_{\mu}(k'_2 x) \right), \qquad (2)$$

where $p = (\varepsilon, \mathbf{p})$, $p' = (\varepsilon', \mathbf{p}')$ are four-momentums for initial and final electrons; $k_1 = (\omega_1, \mathbf{k}_1)$, $k_2 = (\omega, \mathbf{k}_2)$ are four-momentums of emitted photons; γ^{μ} ($\mu = 0,1,2,3$) are Dirac matrixes; $A_{\mu}(k'_{1,2} x)$, $\Psi_p(x)$ and $G(x, x')$ are the wave function of the photon, the wave function and the Green-function electron in the field (1) [1,6]:

$$A_{\mu}(k'_1 x) = \sqrt{\frac{2\pi}{\omega}} e_{\mu}\, \mathrm{e}^{-i(k'_1 x)}, \qquad (3)$$

$$\Psi_p(x) = B_p e^{iS_p(x)} \frac{u_p}{\sqrt{2\tilde{\varepsilon}}}, \quad B_p(x) = 1 + \frac{e}{(kp)} \hat{k}\hat{A}, \qquad (4)$$

$$G(x, x') = \frac{1}{(2\pi)^4} \int \mathrm{d}^4 q\, B_q(x) \frac{\hat{q} + m}{q^2 - m^2} \overline{B}_q(x')$$
$$\times \exp\left(iS_q(x) - iS_q(x') \right), \qquad (5)$$

where e, m are the charge and the mass of an electron; quantities with caps represent scalar products of a four-vectors by Dirac matrixes; u_p is Dirac bispinor; S_p is the classical action of an electron in the field (1):

$$S_p(x) = -(px) - \frac{e}{(kp)} \int_0^{(kx)} \left[(pA(\varphi)) - \frac{e}{2} A^2(\varphi) \right] \mathrm{d}\varphi \,. \quad (6)$$

The geometry of a two-photon emission of an electron is depicted on Fig. 2.

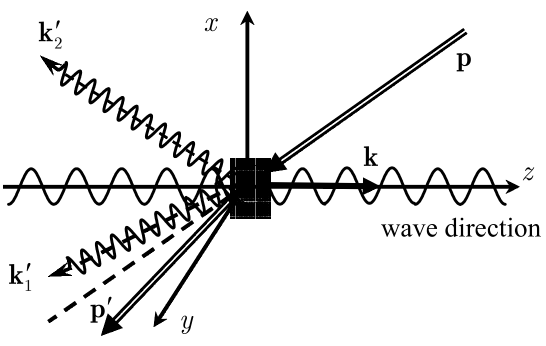

***Fig. 2**. The geometry for study of two-photon emission in the field of a plane electromagnetic wave*

The analysis of quantum-electrodynamics processes of the second order in the fine structure constant in the wave field is complicated by computational difficulties and a cumbersome form of results. Therefore we restrict our consideration to the case when the intensity of the wave meets the condition:

$$\eta = e\sqrt{-A^2} / m = eF/(m\omega) \ll 1 \,. \quad (7)$$

With accuracy to $\sim\eta^2$ the amplitude (2) can be written as

$$S = D\Big(\eta \cdot \left(T_{\nu\mu}^{(0,-1)} + T_{\nu\mu}^{(-1,0)}\right)\delta^{(4)}(\tilde{p} + \mathrm{k} - \tilde{p}' - k_1' - k_2')$$
$$+\eta^2 (T_{\nu\mu}^{(0,-2)} + T_{\nu\mu}^{(-1,-1)} + T_{\nu\mu}^{(-2,0)})$$
$$\times (2\pi)^4 \delta^{(4)}(p + 2\mathrm{k} - p' - k_1' - k_2')\Big), \quad (8)$$

where $D = -ie^2\pi / \sqrt{\varepsilon\varepsilon'\omega_1'\omega_2'}$ is normalization constant. In Ex. (8) we introduce the notations:

$$T_{\nu\mu}^{(\lambda',\lambda)} = e_1^{*\nu} e_2^{*\mu} \overline{u}_{p'} \Bigg(M_v^{(\lambda')}(p', q_1) \frac{\hat{q}_1 + m}{q_1^2 - m^2} M_\mu^{(\lambda)}(q_1, p)$$
$$+ M_v^{(\lambda')}(p', q_2) \frac{\hat{q}_2 + m}{q_2^2 - m^2} M_\mu^{(\lambda)}(q_2, p) \Bigg) u_p \,. \quad (9)$$

Here

$$\tilde{q}_1 = \tilde{p} - k_1' - \lambda k = \tilde{p}' + k_2' + \lambda' k \,; \quad (10)$$

$$\tilde{q}_2 = \tilde{p} - k_2' - \lambda k = \tilde{p}' + k_2' + \lambda' k \quad (11)$$

are 4-quasimomentums of the intermediate electrons. Quasimomentum is equal to the momentum with addition which is caused by influence of external field [1,6]: $\tilde{q} = q + \eta^2 \left(2(kq)\right)^{-1} k$. Taking into account the condition (7) in the expression (8) we may neglect these additions in the item which is proportional to $\sim\eta^2$. In the expression (9) $M_\mu^{(l')}$ are the invariant amplitudes:

$$M_\mu^{(0)}(p_1, p_2) = \left(1 - \frac{y^2(p_1, p_2)}{2}\right)\gamma_\mu + \frac{m^2\eta^2}{2(kp_1)(kp_2)} \hat{\mathrm{k}}\mathrm{k}_\mu$$
$$- \frac{y(p_1, p_2)}{2} \left(e^{i\chi} a_\mu^{(-)}(p_1, p_2) - e^{-i\chi} a_\mu^{(+)}(p_1, p_2) \right); \quad (12)$$

$$M_\mu^{(\pm 1)}(p_1, p_2) = \pm \frac{1}{2} y(p_1, p_2) e^{\mp i\chi} \gamma_\mu + a_\mu^{(\mp)}(p_1, p_2) \,; \quad (13)$$

$$M_\mu^{(\pm 2)}(p_1, p_2) = \frac{y^2(p_1, p_2) e^{\mp 2i\chi}}{8} \gamma_\mu$$
$$+ \frac{m^2\eta^2}{4(kp_1)(kp_2)} \hat{\mathrm{k}}\mathrm{k}_\mu \pm \frac{1}{2} y(p_1, p_2) e^{\mp i\chi} a_\mu^{(\mp)}(p_1, p_2), \quad (14)$$

where

$$a_\mu^{(\mp)}(p_1, p_2) = \frac{m}{2(kp_2)} \left(\hat{k} e_\mu^{(\mp)} - \hat{e}^{(\mp)} k_\mu \right)$$
$$+ \frac{m}{4} \left(\frac{1}{(kp_2)} - \frac{1}{(kp_1)} \right) \hat{e}^{(\mp)} \hat{k} \gamma_\mu \,; \quad (15)$$

$$y(p_1, p_2) = m\sqrt{-g^2(p_1, p_2)} \,, \ \tan\chi = \frac{\delta(ge_y)}{(ge_x)} \,; \quad (16)$$

$$g = \frac{p_1}{(\mathrm{k}p_1)} - \frac{p_2}{(\mathrm{k}p_2)} \,. \quad (17)$$

3. CONDITIONS OF RESONANT TWO-PHOTON EMISSION

Hence we can conclude that probability ($\sim |S|^2$) of two-photon emission of an electron in the wave field is proportional to η^2 and in comparison with one photon emission [1] it contains additional factor $e^2 = 1/137$ and therefore it is small. Still the situation changes when virtual intermediate electrons fall within the mass shell:

$$q_1^2 = m^2 \,, \quad q_2^2 = m^2 \,. \quad (18)$$

Rigorously, the divergence of the amplitude of scattering in the resonance range indicates that expansion into a perturbation series is inapplicable in the situation under study. Correct calculation of the scattering amplitude requires an approach that would fall beyond the framework of the perturbation theory. Specifically, we can perform summation of a principal sequence of Feynman diagrams. In practice, such summation is reduced to a consideration of radiative corrections to the masses of particles involved in the process under investigation. This procedure leads to a finite width of a resonance.

We use a resonant approach to obtain a resonant amplitude and a differential probability. In accordance with it in the nominator $q_{1,2}^2 = m^2$ and in the dominator the mass of an electron in the wave field becomes complex:

$$\mu = m - i\Gamma / 2 \,, \quad (19)$$

where the width of resonance Γ is determined by the total probability of decay of intermediate state, i.e., the probability of the single emission [1]:

$$\Gamma = \frac{e^2 m}{4}\eta^2 F(u_1), \qquad (20)$$

where $u_1 = 2(kp)/m^2$ is the invariant parameter and we introduce the notation:

$$F(u_1) = \left(1 - \frac{4}{u_1} - \frac{8}{u_1^2}\right)\ln(1+u_1) + \frac{1}{2}$$

$$+\frac{8}{u_1} - \frac{1}{2(1+u_1)^2}. \qquad (21)$$

The width of the resonance in non relativistic case ($\omega/m << 1$ in the frame of reference where the initial electron remains at rest) is very small: $\Gamma \sim 10^{-4}\eta^2\omega$. But it becomes significant in relativistic case. For example for $u_1 = 0.5$, $\eta = 0.1$ we have $\Gamma \sim 10^{-6} m$ (~0.1 eV).

Conditions (18) satisfy $\lambda = \lambda' = -1$. In this case we can rewrite a process as a sequence of two subprocesses (see Fig. 3): an emission of the photon k_1' (k_2') by the initial electron in the field of the wave and an emission of the photon k_2' (k_1') by the intermediate electron in the field of the wave. The conservation laws of four-momentum which correspond to these subprocesses have the following forms:

$$\begin{cases} p + k = q_{1,2} + k'_{1,2}; \\ q_{1,2} + k = p' + k'_{2,1}. \end{cases} \qquad (22)$$

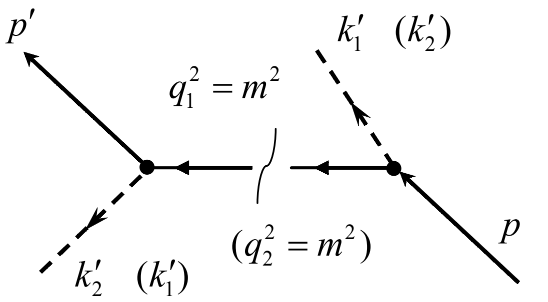

Fig. 3. *Resonance of two-photon emission in the field of a plane electromagnetic wave (four-vectors out (in) brackets relate to the first (second) diagram on Fig. 1)*

The resonance conditions (18) can be written as:

$$\omega'_{1,2;res} = \frac{(kp)}{\varepsilon + \omega - ([\mathbf{p}+\mathbf{k}]\mathbf{n}'_{1,2})}. \qquad (23)$$

Here $\mathbf{n}'_{1,2}$ are unit vectors of the directions of emitted photons.

On the other side to exclude $q_{1,2}$ from (19) we get conservation four-momentum law:

$$p + 2k = p' + k_1' + k_2'. \qquad (24)$$

As it follows from (20) the frequency of the photon k_1' lays in the interval:

$$0 \le \omega_1' \le \frac{2(kp)}{\varepsilon + 2\omega - ([\mathbf{p}+2\mathbf{k}]\mathbf{n}_1')}, \qquad (25)$$

and the frequency of the photon k_2' is connected with the frequency ω_1' by the relationship:

$$\omega_2' = \frac{2(kp) - ([p+2k]k_1')}{\varepsilon + 2\omega - \omega_1' - ([\mathbf{p}+2\mathbf{k}-\mathbf{k}_1']\mathbf{n}_2')}. \qquad (26)$$

We can consider resonances of the first and the second diagrams separately with exception of a case when it is executed an equation:

$$\frac{(kp)}{([p+k]n_2')} = \frac{2(kp) - ([p+2k]k'_{1,res})}{([p+2k-k_1']n_2')}, \qquad (27)$$

where $k'_{1,\mathrm{res}} = (\omega'_{1,res}, \omega'_{1,res}\mathbf{n}_1')$; $\omega'_{1,res}$ is the resonant frequency (20).

This is the condition of the interference resonant amplitudes. The solution of Eq. (24) is the directions of an emission of photons k_2' which satisfies the equation (see Fig. 4):

$$\cos\angle(\mathbf{n}_2', \mathbf{j}) = j_0/|\mathbf{j}|. \qquad (28)$$

Here

$$j = (j_0, \mathbf{j}) = \frac{(kk'_{\mathrm{res}})}{(kp)}[p+k] + k - k'_{1,\mathrm{res}}. \qquad (29)$$

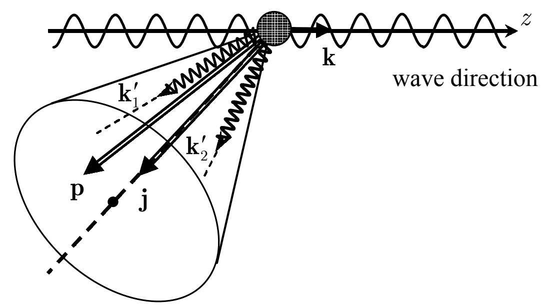

Fig. 4. *Interference of the resonant amplitudes*

4. RESONANT PROBABILITY

Let us obtain probability of the resonant two-photon emission in the absence of interference of resonant amplitudes. In the case of resonance of the first diagram (see Fig. 1) the differential transition rate (per unit time) is given by the expression:

$$\frac{d\,\mathrm{w}}{d\omega_1' d\Omega_1' d\Omega_2'} = \frac{e^4\eta^4}{32\pi^2\varepsilon}\frac{|T|^2}{|q_1^2 - \mu^2|^2}\frac{\omega_1'\omega_2'}{(E_0 - \mathbf{E}\mathbf{n}_2')}. \qquad (30)$$

Here $r_0^2 = e^2/m$ is the classical electron radius,

$$T = e_1^{*\nu} e_2^{*\mu} \bar{u}_{p'} M_\nu^{(-1)}(p', q_1)(\hat{q}_1 + m) M_\mu^{(-1)}(q_1, p) u_p; \qquad (31)$$

$$q_1 = p - k_1' + k = p' + k_2' - k; \qquad (32)$$

$$E = (E_0, \mathbf{E}) = p + 2k - k_1'. \qquad (33)$$

The resonant denominator in the expression (25) in the vicinity of a resonance has the form

$$|q_1^2 - \mu^2|^2 \approx m^4\left(u_1^2\left(1 - \frac{\omega_1'}{\omega'_{1,\mathrm{res}}}\right)^2 + \left(\frac{\Gamma}{m}\right)^2\right). \qquad (34)$$

The procedure of averaging and summation in the polarization of the initial and the final electrons and of the final photons yields

$$\overline{|T|^2} = 2m^4 \left(f(\upsilon_2,u_1) f(\upsilon_1,u_1) + g(\upsilon_2,u_1) g(\upsilon_1,u_1) \right.$$
$$- \frac{2\upsilon}{(1+\upsilon)} \frac{\upsilon_1\upsilon_2}{u_1} \frac{1+2\upsilon_1}{1+\upsilon_1}$$
$$\left. + \frac{2\upsilon_1\upsilon_2}{(1+\upsilon_1)(1+\upsilon_2)} \left(\frac{\upsilon_1+\upsilon_2}{u_1} - \frac{2\upsilon_1\upsilon_2}{u_1^{\,2}} \right) \right), \quad (35)$$

where $\upsilon_1 = (kk_1')/(kp)$, $u_1 = 2(kp)m^{-2}$, $\upsilon_2 = (kk_2)/(kp')$, $\upsilon = ([2k-k_1']k_2')/([2k-k_1']p')$ are invariant parameters;

$$f(\upsilon_1,u_1) = 2 + \frac{\upsilon_1^2}{1+\upsilon_1} - 4\frac{\upsilon_1}{u_1}\left(1 - \frac{\upsilon_1}{u_1}\right); \quad (36)$$

$$g(\upsilon_1,u_1) = \frac{(2+\upsilon_1)(u_1-2\upsilon_1)\upsilon_1}{2u_1(1+\upsilon_1)}. \quad (37)$$

In general relativistic case probability of resonant two-photon emission of electron in the field of the low intense electromagnetic wave cannot be effectively divided into two subprocesses of the first order in the fine-structure constant. However it is possible in the non relativistic case

$$u_1 \ll 1 \quad (38)$$

(all the rest invariant parameters in Eq. (32) are less or equal to u_1). In this case for direct amplitude we can write

$$\frac{d\,\mathrm{w}}{d\omega_1' d\Omega_1' d\Omega_2'} \sim \left(u_1^2 \left(1 - \frac{\omega_1'}{\omega_{1,\mathrm{res}}'}\right)^2 + \left(\frac{\Gamma}{m}\right)^2 \right)^{-1}$$
$$\times \frac{d\mathrm{w}(\upsilon_2,u_1)}{d\Omega_2'} \cdot \frac{d\mathrm{w}(\upsilon_1,u_1)}{d\omega_1' d\Omega_1'}, \quad (39)$$

where $d\mathrm{w}(\upsilon_1,u_1)$ is the differential probability per unit time to emit the photon k_1' with absorption of one photon of the electromagnetic wave, $d\mathrm{w}(\upsilon_2,u_1)$ is the differential probability per unit time to emit the photon k_2' with absorption of one photon of the electromagnetic wave.

5. CONCLUSION

Analysis of two-photon emission of electron in the field circularly polarized wave has demonstrated that this process may occur in a resonant region. The resonance has a place when the frequency at least one of the emitted photons approaches to the frequency one-photon emission. Estimation shows that the resonant probability of two-photon emission may be several orders magnitude greater than the probability of the corresponding process out of the resonance.

REFERENCES

1. V.I. Ritus, A.I. Nikishov. Quantum effects of interaction of elementary particles with intense electromagnetic field //*Trudy FIAN.* 1979, v. 11, p. 5-151 (in Russian).
2. C. Bula et al. Observation of Nonlinear Effects in Compton Scattering //*Phys. Rev. Lett.* 1996, v. 76, p. 3116-3119.
3. D.L. Burke at al. Positron Production in Multiphoton Light-by-Light Scattering //*Phys. Rev. Lett.* 1997, v. 79, p. 1626-1629.
4. V.P. Oleinik. Resonant effects in the field of intense laser beam //*Zh. Eksp. Teor. Fiz.* 1967, v. 52, p. 1049-1067 (in Russian).
5. S.P. Roshchupkin. Resonant effects in collisions of relativistic electrons in the field of a light wave //*Laser Phys.* 1996, v. 6, p. 837-858.
6. V.B. Berestetskii, E.M. Lifshitz, L.P. Pitaevskii. *Quantum Electrodynamics.* M.: "Nauka", 1980 (in Russian).

РЕЗОНАНСНОЕ ДВУХФОТОННОЕ ИЗЛУЧЕНИЕ ЭЛЕКТРОНА В ПОЛЕ ЭЛЕКТРОМАГНИТНОЙ ВОЛНЫ

А.И. Ворошило, С.П. Рощупкин

В общем релятивистском случае теоретически исследовано резонансное двухфотонное излучение электрона в поле циркулярно-поляризованной электромагнитной волны. Установлены условия резонансного протекания процесса и получены резонансные амплитуда и дифференциальная вероятность в случае, когда инвариантный параметр интенсивности волны мал ($\eta << 1$). Показано, что резонансная вероятность на несколько порядков может превышать вероятность данного процесса в нерезонансных условиях.

РЕЗОНАНСНЕ ДВОФОТОННЕ ВИПРОМІНЮВАННЯ ЕЛЕКТРОНА В ПОЛІ ЕЛЕКТРОМАГНІТНОЇ ХВИЛІ

О.І. Ворошило, С.П. Рощупкін

В загально-релятивістському випадку теоретично досліджено резонансне двофотонне випромінювання електрона в полі циркулярно-поляризованої хвилі. Знайдені умови резонансного проходження процесу і отримані резонансні амплітуда і диференційна ймовірність у випадку, коли інваріантний параметр інтенсивності хвилі малий ($\eta << 1$). Показано, що резонансна ймовірність може на декілька порядків перевищувати ймовірність даного процесу в нерезонансних умовах.