TENSIONLESS SUPER p-BRANES IN EXTENDED SUPERSPACES

D.V. Uvarov and A.A. Zheltukhin

National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine;
e-mail: uvarov@kipt.kharkov.ua

Analyzed is the symmetry structure of tensionless super p-branes in N=1 superspace enlarged by commuting antisymmetric tensor coordinates $z^{m_1 \cdots m_p}$ associated to tensorial central charge generators of N=1 extended superalgebra. Using the conversion method we find the constrained Hamiltonian, classical BRST generator and the generators of global and local symmetries of the p-brane model. Quantum realizations of the BRST generator and symmetry generators of super p-brane action are discussed.

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1. INTRODUCTION

The physical interpretation of the central charges in supersymmetry algebra as topological charges carried by branes [1] advanced understanding of the phenomenon of partial spontaneous breaking of supersymmetry [2].

Because branes are constituents of M-theory, spontaneously breaking supersymmetry, their global and local symmetries correlate with the symmetries of M-theory. Studying these symmetries resulted in the model independent classical analysis of BPS states preserving ¼, ½ or ¾ fractions of the partially spontaneously broken D=4 N=1 supersymmetry [3]. A special interest to construction of a physical model with domain wall configurations preserving ¾ fraction of the supersymmetry against spontaneous breaking was analyzed there. That configurations were earlier studied in superparticle dynamics [4] and algebraically realized as the brane intersections in [5].

Then the twistor-like action for tensionless strings and branes preserving ¾ fraction of the D=4 N=1 supersymmetry and generating static solutions for these extended objects was proposed in [6]. The Hamiltonian structure of the model was studied and classical symmetries were identified in [7]. It seems to be interesting to investigate the quantum structure of the string/brane model [6] and the problem of preservation of its rich classical symmetries on the quantum level.

The study of this problem would give answer for more general question: whether quantum exotic BPS states saturated by the p-brane states protect the same high $(M-1)/M$ fraction of N=1 global supersymmetry against spontaneous breaking as in the classical case?

We have started studying the question in [8] on the example of the p-branes preserving ¾ fraction of the partially spontaneously broken D=4 N=1 supersymmetry and found some obstacles for the quantization in the $\hat{Q}\hat{P}$-ordering previously observed in [9].

Here we analyze the quantization problem applying the BFV approach [10, 11] and construct quantum Hermitian BRST operator and the generators of gauge Weyl, Virasoro and global OSp(1|8) symmetries extended by the ghost contributions. We prove the nilpotency of the quantum Hermitian BRST charge, its (anti)commutativity with the quantum Hermitian generators of the OSp(1|8) superalgebra and the closure of this quantized superalgebra.

2. CONSTRAINTS CONVERSION FOR TENSIONLESS SUPER P-BRANE IN EXTENDED N=1 SUPERSPACE

New models of tensionless string and p-branes evolving in the symplectic superspace Σ_m^{susy} and preserving all but one fractions of N=1 supersymmetry were recently studied in [6, 12, 13]. For $m = 2^{[D/2]}$ (D=2,3,4 mod8) the space Σ_m^{susy} extends standard D-dimensional super space-time (x_{ab}, θ_a), where a=1,2,…, $2^{[D/2]}$, by tensor central charge (TCC) coordinates z_{ab}. Coordinates $x_{ab} = x^m(\gamma_m C^{-1})_{ab}$ and $z_{ab} = iz^{mn}(\gamma_{mn}C^{-1})_{ab} + z^{mnl}(\gamma_{mnl}C^{-1})_{ab} + \dots$ constitute components of the symmetric spin-tensor Y_{ab}. In terms of Y_{ab} and the Majorana spinor θ_a the action [6], invariant under the spontaneously broken N=1 supersymmetry and world-volume reparametrizations, is given by

$$S_p = \frac{1}{2}\int d\tau d^p\sigma \rho^\mu U^a W_{\mu ab} U^b, \quad (1)$$

where $W_{ab} = W_{\mu ab} d\xi^\mu$ is the supersymmetric Cartan differential 1-form

$$W_{\mu ab} = \partial_\mu Y_{ab} - 2i(\partial_\mu \theta_a \theta_b + \partial_\mu \theta_b \theta_a),$$

and $\partial_\mu = \frac{\partial}{\partial \xi^\mu}$ with $\xi^\mu = (\tau, \sigma^M)$, (M=1,2,…,p) parametrizing p-brane world volume. The local auxiliary Majorana spinor $U^a(\tau, \sigma^M)$ parametrizes the generalized momentum $P^{ab} = \frac{1}{2}\rho^\tau U^a U^b$ of tensionless p-brane and $\rho^\mu(\tau, \sigma^M)$ is the world-volume vector density providing reparametrization invariance of S_p similarly to the null branes [14, 15].

The action (1) is invariant under (M-1)-parametric κ-symmetry transformations

$\delta_\kappa \theta_a = \kappa_a$, $\delta_\kappa Y_{ab} = -2i(\theta_a \kappa_b + \theta_b \kappa_a)$, $\delta_\kappa U^a = 0$,

that protect (M-1)/M fraction of the N=1 global supersymmetry to be spontaneously broken, because of the one real condition $U^a \kappa_a = 0$ for the transformation parameters $\kappa_a(\tau, \sigma^M)$.

For the four-dimensional space-time the action (1) takes the form

$$S_p = \int d\tau d^p \sigma \rho^\mu u^\alpha \omega_{\mu\alpha\dot\alpha} \bar u^{\dot\alpha} + \int d\tau d^p \sigma \rho^\mu (u^\alpha \omega_{\mu\alpha\beta} u^\beta + \bar u^{\dot\alpha} \omega_{\mu\dot\alpha\dot\beta} \bar u^{\dot\beta}). \tag{2}$$

This action is invariant under OSp(1|8) transformations, which is global supersymmetry of the massless fields of all spins in D=4 space-time extended by tensorial central charge (TCC) coordinates [16,17].

The Hamiltonian structure of the action (2), described in [7], is characterized by 3 fermionic and 2p+7 bosonic first-class constraints that generate its local symmetries, as well as, 1 fermionic and 8 bosonic second-class constraints taken into account by the construction of the Dirac bracket (D.B.). We found that the D.B. algebra of the first-class constraints has the rank equal two and it gives rise to the higher powers of the ghosts in the BRST generator.

To simplify transition to the quantum theory the conversion method [18-20], transforming all the primary and secondary constraints to the first class, has been applied in [8]. To this end the additional canonically conjugate pairs (P_q^α, q^α), $(\bar P_q^{\dot\alpha}, \bar q^{\dot\alpha})$, $(P_\tau^{(\varphi)}, \varphi^\tau)$ and the self-conjugate Grassmannian variable f have been introduced. As a result, all the constraints have been converted to the effective first-class constraints in the extended phase space.

The converted constraints for the auxiliary fields are

$$\tilde P_u^\alpha = P_u^\alpha + P_q^\alpha \approx 0,\ \tilde{\bar P}_u^{\dot\alpha} = \bar P_u^{\dot\alpha} + \bar P_q^{\dot\alpha} \approx 0; \tag{3}$$

$$\tilde P_\tau^{(\rho)} = P_\tau^{(\rho)} + P_\tau^{(\varphi)} \approx 0,\ P_M^{(\rho)} \approx 0 \tag{4}$$

and the converted bosonic constraints $\tilde\Phi = (\tilde\Phi^{\dot\alpha\alpha}, \tilde\Phi^{\alpha\beta}, \tilde{\bar\Phi}^{\dot\alpha\dot\beta})$ are given by

$$\begin{aligned} \tilde\Phi^{\dot\alpha\alpha} &= P^{\dot\alpha\alpha} - \tilde\rho^\tau \tilde{\bar u}^{\dot\alpha} \tilde u^\alpha \approx 0, \\ \tilde\Phi^{\alpha\beta} &= \pi^{\alpha\beta} + \frac{1}{2} \tilde\rho^\tau \tilde u^\alpha \tilde u^\beta \approx 0; \\ \tilde{\bar\Phi}^{\dot\alpha\dot\beta} &= \bar\pi^{\dot\alpha\dot\beta} + \frac{1}{2} \tilde\rho^\tau \tilde{\bar u}^{\dot\alpha} \tilde{\bar u}^{\dot\beta} \approx 0. \end{aligned} \tag{5}$$

They have zero Poisson brackets (P.B.) with the constraints (3), (4) and among themselves. The converted fermionic constraints $\tilde\Psi = (\tilde\Psi^\alpha, \tilde{\bar\Psi}^{\dot\alpha})$ originating from the primary Ψ-constraints and generating four κ-symmetries take the form

$$\tilde\Psi^\alpha = \pi^\alpha - 2i\bar\theta_{\dot\alpha} P^{\dot\alpha\alpha} - 4i\pi^{\alpha\beta}\theta_\beta + 2(\tilde\rho^\tau)^{1/2} \tilde u^\alpha f \approx 0; \tag{6}$$

$$\tilde{\bar\Psi}^{\dot\alpha} = -(\tilde\Psi^\alpha)^* = \bar\pi^{\dot\alpha} - 2iP^{\dot\alpha\alpha}\theta_\alpha - 4i\bar\pi^{\dot\alpha\dot\beta}\bar\theta_{\dot\beta} - 2(\tilde\rho^\tau)^{1/2} \tilde{\bar u}^{\dot\alpha} f \approx 0,$$

where $f^* = f$ is an auxiliary Grassmannian variable characterized by the P.B.

$$\{f(\vec\sigma), f(\vec\sigma')\}_{P.B.} = -i\delta^p(\vec\sigma - \vec\sigma').$$

The addition of the field $f(\tau, \vec\sigma)$ restores the forth κ-symmetry and transforms all $\tilde\Psi$-constraints to the first-class ones. The Weyl symmetry constraint $\tilde\Delta_W$ in the extended phase space is

$$\tilde\Delta_W = (\tilde{\tilde P}_u^\alpha \tilde u_\alpha + \tilde{\tilde{\bar P}}_u^{\dot\alpha} \tilde{\bar u}_{\dot\alpha}) - 2\tilde\rho^\tau \tilde{\tilde P}_\tau^{(\rho)} - 2\rho^M P_M^{(\rho)} \approx 0, \tag{7}$$

where the variables ($\tilde u^\alpha = u^\alpha - q^\alpha$, $\tilde{\tilde P}_u^\alpha = \frac{1}{2}(P_u^\alpha - P_q^\alpha)$) and ($\tilde\rho^\tau = \rho^\tau - \varphi^\tau$, $\tilde{\tilde P}_\tau^{(\rho)} = \frac{1}{2}(P_\tau^{(\rho)} - P_\tau^{(\varphi)})$) form canonically conjugate pairs. Finally, the converted constraints $\tilde L_M$ of the world-volume $\vec\sigma$-reparametrizations are

$$\begin{aligned} \tilde L_M &= \partial_M x_{\alpha\dot\alpha} P^{\dot\alpha\alpha} + \partial_M z_{\alpha\beta}\pi^{\alpha\beta} + \partial_M \bar z_{\dot\alpha\dot\beta}\bar\pi^{\dot\alpha\dot\beta} \\ &+ \partial_M \theta_\alpha \pi^\alpha + \partial_M \bar\theta_{\dot\alpha} \bar\pi^{\dot\alpha} + \partial_M \tilde u_\alpha \tilde{\tilde P}_u^\alpha + \partial_M \tilde{\bar u}_{\dot\alpha} \tilde{\tilde{\bar P}}_u^{\dot\alpha} \\ &- \tilde\rho^\tau \partial_M \tilde{\tilde P}_\tau^{(\rho)} - \rho^N \partial_M P_N^{(\rho)} - \frac{i}{2} f\partial_M f \approx 0. \end{aligned} \tag{8}$$

The P.B. superalgebra of the converted first-class constraints (3)-(8) is described by the following non zero relations

$$\left\{ \tilde\Psi^\alpha(\vec\sigma), \tilde\Psi^\beta(\vec\sigma') \right\}_{P.B.} = -8i\tilde\Phi^{\alpha\beta}\delta^p(\vec\sigma - \vec\sigma'); \tag{9}$$

$$\left\{ \tilde\Psi^\alpha(\vec\sigma), \tilde{\bar\Psi}^{\dot\beta}(\vec\sigma') \right\}_{P.B.} = -4i\tilde\Phi^{\dot\beta\alpha}\delta^p(\vec\sigma - \vec\sigma'); \tag{10}$$

$$\left\{ \tilde\Delta_W(\vec\sigma), P_M^{(\rho)}(\vec\sigma') \right\}_{P.B.} = 2P_M^{(\rho)}\delta^p(\vec\sigma - \vec\sigma'); \tag{11}$$

$$\left\{ \tilde L_M(\vec\sigma), P_N^{(\rho)}(\vec\sigma') \right\}_{P.B.} = \partial_M P_N^{(\rho)}\delta^p(\vec\sigma - \vec\sigma'); \tag{12}$$

$$\left\{ \tilde L_M(\vec\sigma), \chi(\vec\sigma') \right\}_{P.B.} = -\chi(\vec\sigma)\partial_M\delta^p(\vec\sigma - \vec\sigma'); \tag{13}$$

$$\left\{ \tilde L_M(\vec\sigma), \tilde L_N(\vec\sigma') \right\}_{P.B.} = (\tilde L_M(\vec\sigma')\partial_{N'} - \tilde L_N(\vec\sigma)\partial_M)\delta^p(\vec\sigma - \vec\sigma'), \tag{14}$$

where χ are $\tilde\Phi$, $\tilde\Psi$ and $\tilde\Delta_W$ constraints. The c.c. relations have to be added to (9)-(14). The P.B.'s of the remaining constraints vanish in the strong sense. Having the algebra (9)-(14) one can construct the BRST charge of the tensionless super p-brane.

3. BRST CHARGE AND OSP(1|8) SYMMETRY GENERATORS

The algebra(9)-(14) has the rank equal unity and may be presented in the generalized canonical form

$$\left\{ \Upsilon^A(\vec\sigma), \Upsilon^B(\vec\sigma') \right\}_{P.B.} = \int d^p\sigma'' f^{AB}{}_C(\vec\sigma, \vec\sigma' | \vec\sigma'')\Upsilon^C(\vec\sigma''), \tag{15}$$

where $f^{AB}{}_C$ are structure functions. Let us note that the algebra (15) generalizes the original algebra [10, 11] by taking into account $\partial_M \delta^p(\vec{\sigma}-\vec{\sigma}')$ in the structure functions following from the P.B.'s including the Virasoro constraints $\widetilde{L}_M(\vec{\sigma})$ such as

$$f^{\widetilde{L}_M \widetilde{\Phi}^{\dot{\alpha}\beta}}{}_{\widetilde{\Phi}^{\dot{\gamma}\delta}} = -\delta^{\dot{\alpha}}_{\dot{\gamma}}\delta^{\beta}_{\delta}\partial_M \delta^p(\vec{\sigma}-\vec{\sigma}')\delta^p(\vec{\sigma}-\vec{\sigma}'');$$

$$f^{\widetilde{L}_M \widetilde{L}_N}{}_{\widetilde{L}_Q} = -\delta^Q_N \partial_M \delta^p(\vec{\sigma}-\vec{\sigma}')\delta^p(\vec{\sigma}-\vec{\sigma}'')$$
$$+\delta^Q_M \partial_{N'}\delta^p(\vec{\sigma}-\vec{\sigma}')\delta^p(\vec{\sigma}'-\vec{\sigma}'')$$

and other ones.

The canonically conjugate ghost pairs of the minimal sector corresponding to the first-class constraints may be introduced forming the following triads

$$(\widetilde{\Phi}^{\alpha\beta}, C_{\alpha\beta}, \breve{P}^{\alpha\beta}), (\overline{\widetilde{\Phi}}^{\dot{\alpha}\dot{\beta}}, \overline{C}_{\dot{\alpha}\dot{\beta}}, \overline{\breve{P}}^{\dot{\alpha}\dot{\beta}});$$
$$(\widetilde{\Phi}^{\dot{\alpha}\beta}, C_{\beta\dot{\alpha}}, \breve{P}^{\dot{\alpha}\beta}); (\widetilde{\Psi}^{\alpha}, C_{\alpha}, \breve{P}^{\alpha}), (\overline{\widetilde{\Psi}}^{\dot{\alpha}}, \overline{C}_{\dot{\alpha}}, \overline{\breve{P}}^{\dot{\alpha}});$$
$$(\widetilde{P}_u^{\alpha}, C_{u\alpha}, \breve{P}_u^{\alpha}), (\overline{\widetilde{P}}_u^{\dot{\alpha}}, \overline{C}_{u\dot{\alpha}}, \overline{\breve{P}}_u^{\dot{\alpha}});$$
$$(\widetilde{P}_\tau^{(\rho)}, C^{(\rho)\tau}, \breve{P}_\tau^{(\rho)}), (P_M^{(\rho)}, C^{(\rho)M}, \breve{P}_M^{(\rho)});$$
$$(\widetilde{\Delta}_W, C^{(W)}, \breve{P}^{(W)}), (\widetilde{L}_M, C^M, \breve{P}_M).$$

Utilizing nonzero structure functions of the superalgebra (9)-(14) one can present corresponding BRST generator Ω of the minimal sector [10,11]

$$\Omega = \int d^p\sigma (C_A \Upsilon^A + \frac{1}{2}(-)^b C_B C_A f^{AB}{}_C \breve{P}^C)(\vec{\sigma}),$$

by the following integral along the hypersurface of the closed super p-brane

$$\Omega = \int d^p\sigma (C_{\alpha\beta}\widetilde{\Phi}^{\alpha\beta} + \overline{C}_{\dot{\alpha}\dot{\beta}}\overline{\widetilde{\Phi}}^{\dot{\alpha}\dot{\beta}} + C_\alpha \widetilde{\Psi}^\alpha - \overline{C}_{\dot{\alpha}}\overline{\widetilde{\Psi}}^{\dot{\alpha}}$$
$$+ C_{u\alpha}\widetilde{P}_u^\alpha + \overline{C}_{u\dot{\alpha}}\overline{\widetilde{P}}_u^{\dot{\alpha}} + C_{\alpha\dot{\beta}}\widetilde{\Phi}^{\dot{\beta}\alpha} + C^{(\rho)\tau}\widetilde{P}_\tau^{(\rho)} \qquad (16)$$
$$+ C^{(\rho)M}P_M^{(\rho)} + C^{(W)}\widetilde{\Delta}_W^{ext} + C^M \widetilde{L}_M^{ext}$$
$$+ 4i(C_\alpha C_\beta \breve{P}^{\alpha\beta} + \overline{C}_{\dot{\alpha}}\overline{C}_{\dot{\beta}}\overline{\breve{P}}^{\dot{\alpha}\dot{\beta}} - C_\alpha \overline{C}_{\dot{\beta}}\breve{P}^{\dot{\beta}\alpha})).$$

$\widetilde{\Delta}_W^{ext}$ in Eq.(16) is the generator of the gauge worldvolume Weyl symmetry

$$\widetilde{\Delta}_W^{ext} = \widetilde{\Delta}_W - 2C^{(\rho)M}\breve{P}_M^{(\rho)},$$

and $\widetilde{L}_M^{ext}$ is the generalized Virasoro generator

$$\widetilde{L}_M^{ext} = \widetilde{L}_M + \partial_M C_{\alpha\beta}\breve{P}^{\alpha\beta} + \partial_M \overline{C}_{\dot{\alpha}\dot{\beta}}\overline{\breve{P}}^{\dot{\alpha}\dot{\beta}}$$
$$+ \partial_M C_{\alpha\dot{\beta}}\breve{P}^{\dot{\beta}\alpha} + \partial_M C_\alpha \breve{P}^\alpha + \partial_M \overline{C}_{\dot{\alpha}}\overline{\breve{P}}^{\dot{\alpha}}$$
$$+ \partial_M C^{(W)}\breve{P}^{(W)} - C^{(\rho)N}\partial_M \breve{P}_N^{(\rho)} + \partial_M C^N \breve{P}_N$$

extended by the ghost contributions.

Using the P.B.'s of the superalgebra (9)-(14) one can show that the P.B. of the BRST generator density $\Omega(\tau,\vec{\sigma})$, defined by the integrand (16), with itself is equal to the total derivative

$$\{\Omega(\vec{\sigma}), \Omega(\vec{\sigma}')\}_{P.B.} = \partial_M (C^M (C_{\alpha\beta}\widetilde{\Phi}^{\alpha\beta} + \overline{C}_{\dot{\alpha}\dot{\beta}}\overline{\widetilde{\Phi}}^{\dot{\alpha}\dot{\beta}}$$
$$+ C_{\alpha\dot{\beta}}\widetilde{\Phi}^{\dot{\beta}\alpha} + C_\alpha \widetilde{\Psi}^\alpha - \overline{C}_{\dot{\alpha}}\overline{\widetilde{\Psi}}^{\dot{\alpha}} + C^{(\rho)N}P_N^{(\rho)}$$
$$+ C^{(W)}\widetilde{\Delta}_W^{ext} + C^N \widetilde{L}_N^{ext} + 4i(C_\alpha C_\beta \breve{P}^{\alpha\beta} \qquad (17)$$
$$+ \overline{C}_{\dot{\alpha}}\overline{C}_{\dot{\beta}}\overline{\breve{P}}^{\dot{\alpha}\dot{\beta}} - C_\alpha \overline{C}_{\dot{\beta}}\breve{P}^{\dot{\beta}\alpha})))\delta^p(\vec{\sigma}-\vec{\sigma}'),$$

because of the presence of $\partial_M \delta^p(\vec{\sigma}-\vec{\sigma}')$ in the structure functions of the superalgebra (9)-(14). But, the contribution of the total derivative in the r.h.s. of (17) vanishes after integration in $\vec{\sigma}$ and $\vec{\sigma}'$ due to the periodical boundary conditions for the closed p-brane. It results in the P.B.-anticommutativity of the BRST charge $\Omega = \int d^p\sigma \Omega(\tau,\vec{\sigma})$ (16) with itself

$$\{\Omega, \Omega\}_{P.B.} = 0.$$

The introduction of the ghost variables leads to the extension of the OSp(1|8) symmetry generators providing the P.B.-(anti)commutativity of the OSp(1|8) generators with Ω (16). For instance, the ghost extended "square roots" $\widetilde{S}_\gamma(\tau,\vec{\sigma})$ and $\overline{\widetilde{S}}_{\dot{\gamma}}(\tau,\vec{\sigma})$ of the ghost extended conformal boost densities $\breve{K}_{\gamma\dot{\gamma}}(\tau,\vec{\sigma})$ and $\breve{K}_{\gamma\lambda}(\tau,\vec{\sigma})$ are given by

$$\widetilde{S}_\gamma(\tau,\vec{\sigma}) = (z_{\gamma\delta} - 2i\theta_\gamma\theta_\delta)Q^\delta + (x_{\gamma\dot{\delta}} - 2i\theta_\gamma\overline{\theta}_{\dot{\delta}})\overline{Q}^{\dot{\delta}}$$
$$+ 4i(\widetilde{u}^\delta\theta_\delta - \overline{\widetilde{u}}^{\dot{\delta}}\overline{\theta}_{\dot{\delta}})\widetilde{\widetilde{P}}_{u\gamma} + \frac{2}{(\widetilde{\rho}^\tau)^{1/2}}\widetilde{\widetilde{P}}_{u\gamma}f + C_\gamma{}^\delta \breve{P}_\delta$$
$$+ C_{\gamma\dot{\delta}}\overline{\breve{P}}^{\dot{\delta}} + 4iC_\gamma(\theta_\delta\breve{P}^\delta - \overline{\theta}_{\dot{\delta}}\overline{\breve{P}}^{\dot{\delta}}) - 8i\theta_\delta C_\gamma{}^\beta \breve{P}_\beta{}^\delta$$
$$+ 4i\overline{\theta}^{\dot{\delta}}C_\gamma{}^\beta \breve{P}_{\beta\dot{\delta}} + 4i\theta_\delta C_{\gamma\dot{\beta}}\breve{P}^{\dot{\beta}\delta} - 8i\overline{\theta}^{\dot{\delta}}C_{\gamma\dot{\beta}}\overline{\breve{P}}^{\dot{\beta}}{}_{\dot{\delta}};$$

$$\overline{\widetilde{S}}_{\dot{\gamma}}(\tau,\vec{\sigma}) = (\overline{z}_{\dot{\gamma}\dot{\delta}} - 2i\overline{\theta}_{\dot{\gamma}}\overline{\theta}_{\dot{\delta}})\overline{Q}^{\dot{\delta}} + (x_{\delta\dot{\gamma}} + 2i\theta_\delta\overline{\theta}_{\dot{\gamma}})Q^\delta$$
$$- 4i(\widetilde{u}^\delta\theta_\delta - \overline{\widetilde{u}}^{\dot{\delta}}\overline{\theta}_{\dot{\delta}})\overline{\widetilde{\widetilde{P}}}_{u\dot{\gamma}} - \frac{2}{(\widetilde{\rho}^\tau)^{1/2}}\overline{\widetilde{\widetilde{P}}}_{u\dot{\gamma}}f - C_{\delta\dot{\gamma}}\breve{P}^\delta$$
$$- \overline{C}^{\dot{\delta}}{}_{\dot{\gamma}}\overline{\breve{P}}_{\dot{\delta}} - 4i\overline{C}_{\dot{\gamma}}(\theta_\delta\breve{P}^\delta - \overline{\theta}_{\dot{\delta}}\overline{\breve{P}}^{\dot{\delta}}) - 8i\theta^\delta C_{\beta\dot{\gamma}}\breve{P}_\delta{}^\beta$$
$$+ 4i\overline{\theta}_{\dot{\delta}}C_{\beta\dot{\gamma}}\breve{P}^{\dot{\delta}\beta} + 4i\theta^\delta\overline{C}^{\dot{\beta}}{}_{\dot{\gamma}}\breve{P}_{\delta\dot{\beta}} - 8i\overline{\theta}_{\dot{\delta}}\overline{C}^{\dot{\beta}}{}_{\dot{\gamma}}\overline{\breve{P}}^{\dot{\delta}}{}_{\dot{\beta}}.$$

Using the densities $\Omega(\tau,\vec{\sigma})$ and $\widetilde{S}_\gamma(\tau,\vec{\sigma}')$ we find their P.B.

$$\{\Omega(\vec{\sigma}), \widetilde{S}_\gamma(\vec{\sigma}')\}_{P.B.} = -(C^M\widetilde{S}_\gamma)(\vec{\sigma})\partial_M\delta^p(\vec{\sigma}-\vec{\sigma}')$$

and conclude that the contribution of the total derivative in the r.h.s. vanishes after integration with respect to $\vec{\sigma}$ and $\vec{\sigma}'$. Thus, the BRST charge Ω (16) has zero P.B. with the conformal supercharges $\widetilde{S}_\gamma$, $\overline{\widetilde{S}}_{\dot{\gamma}}$

$$\{\Omega, \widetilde{S}_\gamma\}_{P.B.} = \{\Omega, \overline{\widetilde{S}}_{\dot{\gamma}}\}_{P.B.} = 0.$$

The same P.B.-commutativity

$$\{\Omega, G\}_{P.B.} = 0$$

between Ω and other OSp(1|8) symmetry charges $G = \int d^p\sigma G(\tau,\vec{\sigma})$ extended by the ghost contributions will also be preserved, because of the general relation for the generator densities:

$$\{\widetilde{L}_M(\vec{\sigma}), G(\vec{\sigma}')\}_{P.B.} = -G(\vec{\sigma})\partial_M\delta^p(\vec{\sigma}-\vec{\sigma}').$$

Explicit expressions for other OSp(1|8) generators extended by ghost contributions can be found in [8,21].

4. QUANTIZATION: NILPOTENT BRST OPERATOR AND QUANTUM OSP(1|8) ALGEBRA

Upon transition to quantum theory all the quantities entering the converted constraints and OSp(1|8) generator densities are treated as operators that implies a choice of a certain ordering for products of noncommuting operators. At the same time canonical Poisson brackets

$$\left\{ P^{\mathrm{M}}(\vec{\sigma}), Q_{\mathrm{N}}(\vec{\sigma}') \right\}_{P.B.} = \delta^{\mathrm{M}}_{\mathrm{N}} \delta^{p}(\vec{\sigma} - \vec{\sigma}') \qquad (18)$$

used in classical mechanics transform into (anti)commutators

$$[P^{\mathrm{M}}(\vec{\sigma}), Q_{\mathrm{N}}(\vec{\sigma}') \} = -i\delta^{\mathrm{M}}_{\mathrm{N}} \delta^{p}(\vec{\sigma} - \vec{\sigma}').$$

It is necessary to provide further nilpotence of the BRST operator, fulfilment of (anti)commutation relations of the OSp(1|8) superalgebra and its generator (anti)commutativity with the BRST operator ensuring global quantum invariance of the model. In addition, the Hermiticity of the quantum BRST operator and OSp(1|8) generators has to be supported. The Hermiticity requirement may be manifestly satisfied if we start from the above constructed classical representations for the OSp(1|8) generators and BRST charge in which all coordinates are disposed from the left of momenta, i.e. in the form $\hat{Q}\hat{P}$, where $\hat{Q}$ and $\hat{P}$ are the products of the coordinates and momenta contained in Ω and the generators. Then the operator expressions for the latter are presented in the manifestly Hermitian form composed of the operator products

$$\frac{1}{2}\left(\hat{Q}\hat{P} + (-)^{\varepsilon(Q)\varepsilon(P)} (\hat{Q}\hat{P})^{+} \right),$$

where ε(Q) and ε(P) are Grassmannian gradings of these coordinate and momentum monomials.

As the result quantum Hermitian BRST generator acquires the form

$$\hat{\Omega}_{H} = \frac{1}{2}\int d^{p}\sigma \left(\hat{\Omega}(\tau,\vec{\sigma}) + \hat{\Omega}^{+}(\tau,\vec{\sigma}) \right). \qquad (19)$$

Now we are ready to prove that this realization of $\hat{\Omega}_H$ preserves its nilpotency and (anti)commutativity with the Hermitian operators generating quantum realization of the classical OSp(1|8) superalgebra. The proof is obvious and based on the observation that $\hat{\Omega}_H$ and other considered Hermitian operators are linear in the momentum operators $\hat{P}^{\mathrm{M}}(\tau,\vec{\sigma})$ of the original coordinates and ghost fields. The remarkable property of the ordered polynomial operators composed of $\hat{Q}_{\mathrm{N}}(\tau,\vec{\sigma})$ and $\hat{P}^{\mathrm{M}}(\tau,\vec{\sigma})$, which form the Weyl-Heisenberg algebra (18), and are linear in $\hat{P}^{\mathrm{M}}$ is the preservation of the chosen ordering in the course of calculations of their (anti)commutators. Thus the transition from the P.B.'s to the (anti)commutators will preserve all classical results obtained in the P.B. realization of the extended algebra of the OSp(1|8) generators and classical BRST charge of the super p-brane. So, the quantum Hermitian BRST operator (19) occurs to be nilpotent

$$\left\{ \hat{\Omega}_{H}, \hat{\Omega}_{H} \right\} = 0.$$

However, the Hermiticity of $\hat{\Omega}_H$ and the OSp(1|8) generating operators by itself is only a necessary condition for the quantum realization of the physical operators, because the relevant vacuum and physical states have also to be constructed. So, the problem of existence of the selfconsistent quantum realization of the exotic BPS states by the states of quantum tensionless super p-brane is reduced to the proof of existence of the relevant vacuum and the corresponding physical space of quantum states. At the present time we investigate this problem.

5. CONCLUSION

The general problem of quantum brane realization of the BPS states preserving (M-1)/M fraction of partially spontaneously broken global N=1 supersymmetry at the classical level was analyzed on example of the twistor-like p-brane in four-dimensional space-time. Twistor-like brane models are characterized by a fine tuning of a large set of classical local and global symmetries caused by the absence of tension. We have constructed classical BRST charge and generators of these global and gauge symmetries and proved the closure of their unified P.B. superalgebra. The P.B. realization of the nilpotency condition for the BRST charge and its (anti)commutativity with the unified symmetry generators have been proved. After that we considered quantization of the model and proved the preservation of the above classical results using Hermitian operator realization of the symmetry generators and BRST operator. This Hermitian realization gives the relevant physical foundation for the solution of the quantization problem. The remaining problem here is the construction of the vacuum and the Hilbert space of quantum states of tensionless super p-branes which is under our investigations.

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СУПЕР p-БРАНЫ С НУЛЕВЫМ НАТЯЖЕНИЕМ В РАСШИРЕННЫХ СУПЕРПРОСТРАНСТВАХ

Д.В. Уваров, А.А. Желтухин

Представлен анализ структуры симметрий супер p-бран с нулевым натяжением в N=1 суперпространстве, расширенном коммутирующими антисимметричными тензорными координатами $z^{m_1 \cdots m_p}$, отвечающими генераторам тензорных центральных зарядов N=1 расширенной супералгебры. Используя метод конверсии, получены функция Гамильтона, классический БРСТ-генератор и генераторы глобальных и локальных симметрий моделей p-бран. Обсуждаются квантовые реализации БРСТ-генератора и OSp(1|8)-генераторов.

СУПЕР p-БРАНИ З НУЛЬОВИМ НАТЯГОМ У РОЗШИРЕНИХ СУПЕРПРОСТОРАХ

Д.В. Уваров, О.О. Желтухін

Представлений аналіз структури симетрій супер p-бран з нульовим натягом у N=1 суперпросторі, розширеному переставними антисиметричними тензорними координатами $z^{m_1 \cdots m_p}$, які відповідають генераторам тензорних центральних зарядів N=1 розширеної супералгебри. З використанням методу конверсії отримано функцію Гамільтона, класичний БРСТ-генератор та генератори глобальних та локальних симетрій моделей p-бран. Обговорюються квантові реалізації БРСТ-генератора та OSp(1|8)-генераторів.