Section B. ELEMENTARY PARTICLE THEORY

SINGLE–SPIN ASYMMETRIES IN ELECTRON–PROTON AND PHOTON-PROTON SCATTERING IN THE BETHE–HEITLER PROCESSES INDUCED BY LOOP CORRECTIONS

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The single–spin target asymmetries in the hard electroproduction process $e^- + p \to e^- + \gamma + p$ and in the e^+e^--pair photoproduction $\gamma + p \to e^+ + e^- + p$, induced by the loop radiative corrections to the vertex part of lepton interaction are considered. The physical reason to appearance such a kind of asymmetries is the nonzero imaginary part of the respective Bethe-Heitler amplitudes (on the level of radiative correction). The single–spin target asymmetries at unpolarized ingoing electron or photon beams and at arbitrary polarizations of the target proton for conditions of CLAS (Jefferson Lab, USA) and HERMES (DESY) experiments are calculated.

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1. INTRODUCTION

The parity–conserving single–spin beam and target correlations in elastic electron–proton scattering and radiative reaction are used to extract information about virtual Compton scattering (VCS) amplitude. This last is very important physical quantity which has triggered a significant experimental and theoretical activity.

In elastic scattering the VCS amplitude enters through the two–photon exchange diagram (TPE) with two off-shell photons. The cross section and parity-conserved spin–spin correlations in this case are sensitive only to the real part of this diagram and, therefore, to the real part of the double off-shell VCS amplitude. Contrary, the single–spin normal asymmetry probes only the imaginary part of TPE amplitude for both beam and target normal polarizations.

If the electron beam or the target proton is polarized in the reaction plane, the parity–conserving single–spin asymmetry for elastic scattering is strictly zero. Nevertheless, the nonzero such kind asymmetry can manifest itself in the process with three (and more) final particles provided that all the final–particle 3–momenta do not belong to single (the same) plane. The simplest such type process that probes VCS amplitude is the hard electroproduction $(ee'\gamma)$ reaction

$$e^-(k_1) + p(p_1) \to e^-(k_2) + \gamma(k) + p(p_2). \tag{1}$$

The whole amplitude of this process can be represented as a sum of they real Bethe–Heitler amplitude and VCS one, that has both the real and imaginary parts.

In present paper we want to pay attention that the one–loop correction to the lepton part of the Bethe–Heitler amplitude with radiation of a photon by the outgoing electron can generate the non–zero imaginary part, and, consequently, an additional contribution to the single-spin asymmetries, which has the status of radiative correction to main effect caused by imaginary part of VCS amplitude.

2. KINEMATICAL VARIABLES

To describe the physical observables in the process (1) usually used three dimensionless variables

$$x = -\frac{(k_1 - k_2)^2}{2p_1(k_1 - k_2)}, \quad y = \frac{2p_1(k_1 - k_2)}{V},$$
$$\rho = -\frac{(p_1 - p_2)^2}{V}, \quad V = 2p_1 k_1, \tag{2}$$

and azimuth angle Φ in the target proton rest frame that is simply the angle between leptonic and hadronic planes as shown in Fig. 1 for two different choices of Z-axis: opposite to direction $\mathbf{q}_1 = \mathbf{k}_1 - \mathbf{k}_2$ (Fig. 1,a) and along direction of $\mathbf{k}_1$ (Fig. 1,b).

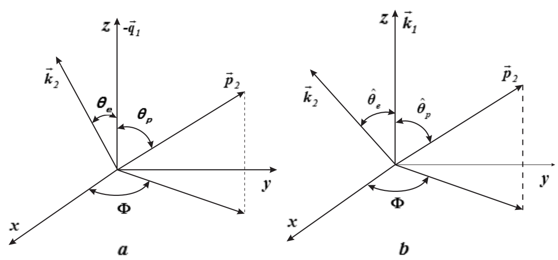

***Fig. 1**. Definition of angles in laboratory frame*

The energies and the 3–momentum modules of the particles do not depend on the choice of Z–axis and neglecting the electron mass read

$$\varepsilon_1 = \beta, \quad \varepsilon_2 = \beta(1-y), \quad q_{10} = \beta y,$$
$$E_2 = \beta(2\tau + \rho), \quad |\mathbf{p}_2| = \beta\sqrt{\rho(4\tau + \rho)}, \tag{3}$$
$$|\mathbf{q}_1| = \beta\sqrt{y^2 + 4xy\tau}, \quad \beta = \sqrt{\frac{V}{4\tau}}, \quad \tau = \frac{M^2}{V},$$

where $\varepsilon_1(\varepsilon_2)$ is the energy of ingoing (outgoing) electron, $E_2(\mathbf{p}_2)$ is the energy (3–momentum) of the recoil proton.

In contrast with the energies, the scattering angles depend on the choice of Z–axis direction. For the system K (Fig.1,a) one has

$$\cos\theta_e = -\frac{y(1-y-2x\tau)}{(1-y)\sqrt{y^2+4xy\tau}},$$

$$\cos\theta_p = -\frac{y\rho+2\tau(\rho+xy)}{\sqrt{\rho(4\tau+\rho)(y^2+4xy\tau)}}, \tag{4}$$

whereas in the case of system $\hat{K}$ (Fig.1,b):

$$\cos\hat{\theta}_e = \frac{1-y-2xy\tau}{1-y}, \quad \cos\hat{\theta}_p = \frac{\rho+2z\tau}{\sqrt{\rho(4\tau+\rho)}},$$

$$z = 1-\frac{2k_1p_2}{V}, \tag{5}$$

where $\theta_e(\theta_p)$ is the electron and proton scattering angles in system K, $\hat{\theta}_e(\hat{\theta}_p)$ is the same but in system $\hat{K}$ and we introduced for a convenience new dimensionless quantity z that has to be expressed through azimuth angle and invariant variables Eq. (2) in the final results.

In what follows we will present the analytical formulae only for K-system. Usually the photon is not recorded experimentally and therefore we have to exclude the photon 4–momentum from the phase space of final particles by means of the overall $\delta^{(4)}$–function. Thus, we have to define

$$dF = \frac{d^3k_2}{\varepsilon_2}\frac{d^3p_2}{E_2}\delta(k^2). \tag{6}$$

Elimination of $\delta(k^2)$ is trivial in system K

$$\delta(k^2)d\cos\theta_p = \frac{1}{2|\mathbf{q}_1||\mathbf{p}_2|},$$

that leads to

$$dF = \frac{\pi V}{2}\frac{y}{\sqrt{y^2+4xy\tau}}dxdyd\rho d\Phi. \tag{7}$$

The invariant variable z can be expressed through x, y, ρ and $\cos\Phi$, namely (see also Ref. [1])

$$z = \frac{1}{y+4x\tau}$$
$$\times\left[2K\cos\Phi+2x\tau(xy+\rho)+xy+\rho(1+xy-2x)\right]; \tag{8}$$
$$K^2 = \frac{x(1-y-xy\tau)(y-xy+\tau)(\rho_+-\rho)(\rho-\rho_-)}{y}.$$

Quantities $\rho_\pm$ in the last expression have a sense of the minimum and maximum value of ρ at fixed x and y

$$\rho_\pm = \frac{y}{2[y(1-x)+\tau]}\left[(1-x)\left(y\pm\sqrt{y^2+4xy\tau}\right)+2x\tau\right]. \tag{9}$$

3. THE SINGLE–SPIN TARGET ASYMMETRY IN ELECTROPRODUCTION

In this paper we will concentrate on the single–spin target asymmetries. They can be written in terms of contraction of leptonic and hadronic tensors. For the hard electroproduction process (1) we have [2]

$$A_t = -\frac{\alpha}{4\pi}\frac{H_{[\mu\nu]}B^{(1)}_{[\mu\nu]}}{H_{(\mu\nu)}B_{(\mu\nu)}}, \tag{10}$$

where $H_{(\mu\nu)}$ and $H_{[\mu\nu]}$ are the symmetrical and antisymmetrical parts of hadronic tensor. They can be expressed through the proton electromagnetic form factors (see Ref. [2]). Tensor $B_{(\mu\nu)}$ is the leptonic tensor in the Born approximation and $B^{(1)}_{[\mu\nu]}$ is antisymmetrical imaginary part of leptonic tensor that generated by the loop radiative correction. Here we bear in mind that the lepton beam is unpolarized and the target proton has an arbitrary polarization. We use the result of Ref. [3] and write one–loop corrected unpolarized leptonic tensor in the form

$$iB^{(1)}_{[\mu\nu]}+B^{(1)}_{(\mu\nu)} = (T_g+T_g^*)\tilde{g}_{\mu\nu}+(T_{11}+T_{11}^*)\tilde{k}_{1\mu}\tilde{k}_{1\nu}$$
$$+(T_{22}+T_{22}^*)\tilde{k}_{2\mu}\tilde{k}_{2\nu}+(T_{12}+T_{21}^*)\tilde{k}_{1\mu}\tilde{k}_{2\nu}$$
$$+(T_{21}+T_{12}^*)\tilde{k}_{2\mu}\tilde{k}_{1\nu}.$$

It is easy to divide the right-hand side of above equation by its symmetrical and antisymmetrical pieces and we arrive at

$$B^{(1)}_{[\mu\nu]} = \Im(T_{12}-T_{21})[\tilde{k}_1\tilde{k}_2]_{\mu\nu}. \tag{11}$$

Quantities T_{12} and T_{21} are found in Ref. [3], and the extraction of the respective imaginary part leads to result

$$B^{(1)}_{[\mu\nu]} = 2\pi\left(\tilde{k}_{1\mu}\tilde{k}_{2\nu}-\tilde{k}_{1\nu}\tilde{k}_{2\mu}\right)T;$$
$$T = \frac{q^2}{st}\left[\frac{2st}{c^2}+4u\left(\frac{1}{t}\ln\frac{u+t}{u}-\frac{1}{c}\right)\right], \tag{12}$$

where we used the same variables as in Ref. [3]

$u = -2k_1k_2,\ s = 2kk_2,\ t = -2kk_1,\ q^2 = s+t+u,\ c = u+t.$

Note that quantity T does not have singularity at $t\to 0$ and goes to zero when $q^2\to 0$.

The denominator in Eq. (10) in terms of invariant variables reads

$$H_{(\mu\nu)}B_{(\mu\nu)}$$
$$= -\frac{V}{(z-\rho)(z-xy)}\left[\chi_1(F_1+F_2)^2+2\chi_2(F_1^2+\frac{\rho}{4\tau}F_2^2)\right]; \tag{13}$$
$$\chi_1 = -\rho\left[2z(z-\rho-xy)+(\rho+xy)^2\right];$$
$$\chi_2 = \left(z-\rho-xy\right)\left[2z\tau+\rho(1-y)\right]$$
$$+\tau\left(\rho+xy\right)^2-\rho\left[1-z+(1-y)^2\right],$$

where we used also $s = V(z-\rho), t = -V(z-xy),$ and $F_1(F_2)$ is the Dirac (Pauli) proton form factor.

The numerator in Eq. (10) is expressed via the target–proton polarization 4-vector S

$$H_{[\mu\nu]}B^{(1)}_{[\mu\nu]} = -\frac{4\pi M\rho(F_1+F_2)}{V(z-\rho)(z-xy)}$$
$$\times\left(-1+\frac{\rho-xy}{z}+\frac{xy\rho}{z^2}+\frac{2xy}{z-xy}\ln\frac{z}{xy}\right)G_s; \tag{14}$$
$$G_s = 2(k_1k_2qS)\left(F_1-\frac{\rho}{4\tau}F_2\right)+\frac{(k_1k_2qp_1)(p_2S)}{V\tau}F_2,$$

where $(abcd) = \varepsilon_{\alpha\beta\gamma\delta} a^\alpha b^\beta c^\gamma d^\delta$.

In general the one-loop correction to the leptonic part of interaction generates the three types of target single-spin asymmetries when the target proton has three different directions of its polarization 3-vector in laboratory system.

If the longitudinal (L) target proton polarization in laboratory system is chosen along direction of $\mathbf{k}_1$, the transverse (T) polarization belongs to plane $(\mathbf{k}_1, \mathbf{k}_2)$ and the normal (N) one – along direction $\mathbf{k}_1 \times \mathbf{k}_2$, the respective polarization 4-vectors $S_1^{(L,T,N)}$ can be expressed through the particles 4-momenta as Ref. [4].

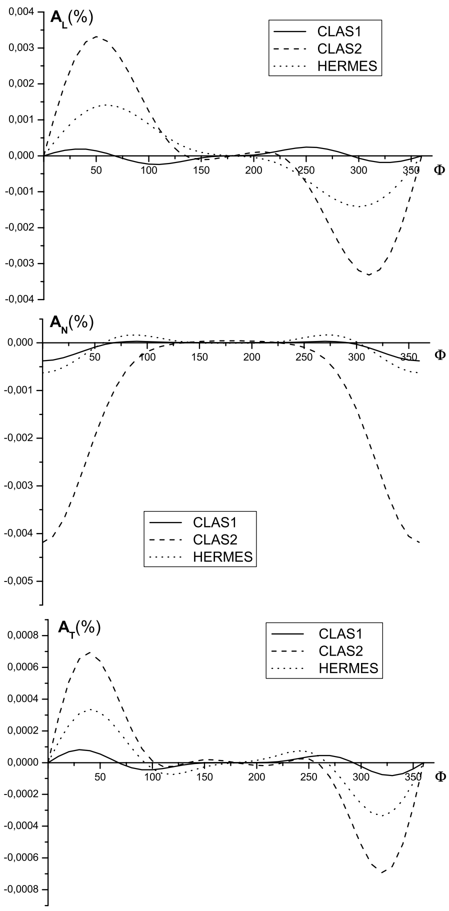

***Fig. 2**. The target single-spin asymmetries that are suitable for choice (16) of the target-proton polarizations as a function of angle* Φ

$$S_{1\mu}^L = \frac{2\tau k_{1\mu} - p_{1\mu}}{\sqrt{V\tau}}, \quad S_{1\mu}^N = -\frac{2(\mu k_1 k_2 p_1)}{\sqrt{V^3 xy(1-y-xy\tau)}}, \qquad (15)$$

$$S_{1\mu}^T = \frac{k_{2\mu} - (1-y-2xy\tau)k_{1\mu} - xyp_{1\mu}}{\sqrt{Vxy(1-y-xy\tau)}},$$

where $(S_1^I S_1^J) = -\delta_{IJ}$ and $(S_1^I p_1) = 0$; $I, J = L, T, N$. For this choice of the target polarization we have

$$G_{s1}^L = -\frac{(k_1 k_2 q p_1)}{\sqrt{V\tau}}\left(2F_1 + zF_2\right), \qquad (16)$$

$$G_{s1}^T = \frac{(k_1 k_2 q p_1)}{\sqrt{Vxy(1-y-xy\tau)}} \times \left[-2xyF_1 + \frac{F_2}{2\tau}(\rho + xy - yz(1+2x\tau))\right]; \qquad (17)$$

$$G_{s1}^N = -\frac{1}{2}\sqrt{\frac{V^3 xy}{(1-y-xy\tau)}} \times \left\{\left(F_1 - \frac{\rho}{4\tau}F_2\right) \times \left[z(2-y) - \rho - xy(1-\rho)\right] - \frac{4F_2(k_1 k_2 q p_1)^2}{V^4 xy\tau}\right\}, \qquad (18)$$

where the proton form factors depend on $q^2 = -Q^2 = -\rho V$. The target single-spin asymmetries corresponding to above choice of polarizations are shown on Fig. 2.

In principle, one can choose other directions to define polarizations of the target proton. The case when the longitudinal direction is along the 3-momentum of the recoil proton and the transverse one – in the plane $(\mathbf{k}_1, \mathrm{P}_2)$ were considered in [2].

4. SINGLE-SPIN TARGET ASYMMETRIES IN PAIR PRODUCTION

The amplitudes of the BH-processes (1) and the electron-positron pair production

$$\gamma(k) + p(p_1) \to e^+(k_1) + e^-(k_2) + p(p_2) \qquad (19)$$

are connected each others by well known substitution law [5].

By means of this substitution law one can calculate both, the symmetrical and antisymmetrical parts of leptonic tensor $L_{\mu\nu}^\gamma$ in process (19), using the known results for leptonic tensor in process Eq. (1), namely

$$L_{\mu\nu}^\gamma = -L_{\mu\nu}(k_1 \to -k_1, k \to -k, k_2 \to k_2). \qquad (20)$$

In one-loop approximation we have

$$iB_{[\mu\nu]}^{(1)\gamma} + B_{(\mu\nu)}^{(1)\gamma} = (T_g^\gamma + T_g^{\gamma *})\tilde{g}_{\mu\nu} + (T_{11}^\gamma + T_{11}^{\gamma *})\tilde{k}_{1\mu}\tilde{k}_{1\nu} + (T_{22}^\gamma + T_{22}^{\gamma *})\tilde{k}_{2\mu}\tilde{k}_{2\nu} + (T_{12}^\gamma + T_{21}^{\gamma *})\tilde{k}_{1\mu}\tilde{k}_{2\nu} + (T_{21}^\gamma + T_{12}^{\gamma *})\tilde{k}_{2\mu}\tilde{k}_{1\nu};$$

$$B_{[\mu\nu]}^{(1)\gamma} = \Im(T_{12}^\gamma - T_{21}^\gamma)[k_1 k_2]_{\mu\nu},$$

and the interesting for us term T_{12}^γ has the following form

$$T_{12}^\gamma = \frac{2}{ut}\left[\frac{aq^2(s-u)G^\gamma}{u^2} + \frac{q^2(sq^2-ut)\widetilde{G^\gamma}}{t^2} - 2q^2\left(\frac{sq^2}{ut} + \frac{2s-u+t}{c}\right)L_{qs} + 8s + 3t - u + \frac{2us}{a} - \frac{4(s^2-au)(q^2 L_{qs} - c)}{c^2} + \frac{q^2(2a+t)(ut-sq^2)}{a^2 t}L_{qu} - \frac{q^2 a(2s-u)}{bu}L_{qt}\right]; \qquad (21)$$

$$G^\gamma = L_{qs}(L_q + L_s - 2L_t) - \frac{\pi^2}{3} - 2Li_2\left(1 - \frac{q^2}{s}\right) + 2Li_2\left(1 - \frac{t}{q^2}\right),$$

$$\widetilde{G^{\gamma}} = G^{\gamma}(t \to u), \qquad Li_2(x) = -\int_0^x \frac{dy}{y} ln(1-y) \quad ,$$

where $s = (k_1 + k_2)^2, \quad t = -2kk_1, \quad u = -2kk_2,$ and also $a = s + t, \quad b = s + u, \quad c = u + t.$

The quantities L_{ik} in Eq. (21) are defined in the following way

$$L_{ik} = L_i - L_k, \quad i,k = s,t,u,q^2, \; L_i = \ln\frac{-i}{m^2}.$$

Quantity T_{21}^{γ} can be obtained from T_{12}^{γ} by change $t \leftrightarrow u$.

The extraction of imaginary part leads to

$$\Im(T_{12}^{\gamma} - T_{21}^{\gamma}) = = \frac{8\pi q^2}{ut}\left[\frac{s}{t}\ln\frac{u+t}{t} - \frac{s}{u}\ln\frac{u+t}{u} + \frac{q^2(u-t)}{(u+t)^2}\right]. \qquad (22)$$

It is convenient also to introduce the appropriate for the process (19) dimensionless variables

$$x = -\frac{(k-k_2)^2}{2p_1(k-k_2)}, y = \frac{2p_1(k-k_2)}{V}, \rho = -\frac{(p_1-p_2)^2}{V}, \qquad (23)$$

$$z = \left(1 - \frac{2kp_2}{V}\right), \quad V = 2p_1k.$$

In terms of these variables we have

$$H_{[\mu\nu]}B_{[\mu\nu]}^{(1)\gamma} = \frac{8\pi\rho M(F_1+F_2)}{Vxy(z-xy)}$$

$$\times\left[\frac{z-\rho}{xy}\ln\frac{z}{xy} - \frac{z-\rho}{z-xy}\ln\frac{z}{z-xy} - \frac{\rho(z-2xy)}{z^2}\right]G_s^{\gamma};$$

$$H_{(\mu\nu)}B_{(\mu\nu)}^{\gamma} = -\frac{V}{xy(z-xy)}\times$$

$$\left[\chi_1^{\gamma}(F_1+F_2)^2 + 2\chi_2^{\gamma}(F_1^2 + \frac{\rho}{4\tau}F_2^2)\right],$$

where

$$\chi_1 = -\rho\left[(z-\rho-xy)^2 + (\rho-xy)^2\right];$$

$$\chi_2 = \tau\left[(z-\rho-xy)^2 + (\rho-xy)^2\right]$$

$$-\rho\left[1 + (1-y)^2 + \rho - xy - (1-z)(z-\rho-xy)\right].$$

The function G_s^{γ} , that depends on the target polarizations, has the same form in terms of variables of Eq. (23) as function G_s in Eq. (14) in terms of variables Eq. (2).

The single–spin target asymmetry in the photoproduction process (19) is

$$A_t^{\gamma} = -\frac{\alpha}{4\pi}\frac{H_{[\mu\nu]}B_{[\mu\nu]}^{(1)\gamma}}{H_{(\mu\nu)}B_{(\mu\nu)}^{\gamma}}. \qquad (24)$$

In our numerical evaluations we use the parameterization Eq. (15) of the target proton polarizations in which k_1 changed by k . The results are shown on Fig. 3 for CLAS1 experimental condition.

5. CONCLUSIONS

In present paper we studied the single-spin parity conserving target asymmetries in the Bethe-Heitler processes of hard electroproduction (1) and electron-positron pair photoproduction (19). Effect arises due to appearance of non zero imaginary part of the amplitudes on the level of radiative corrections. During the calculations we used the substitution law to obtain the one-loop corrected leptonic tensor in process (19) using corresponding and known tensor for the process (1). The numerical estimations in conditions of current experiments CLAS (JLab) and HERMES (DESY) indicate very small values of any kind asymmetry in process (1).

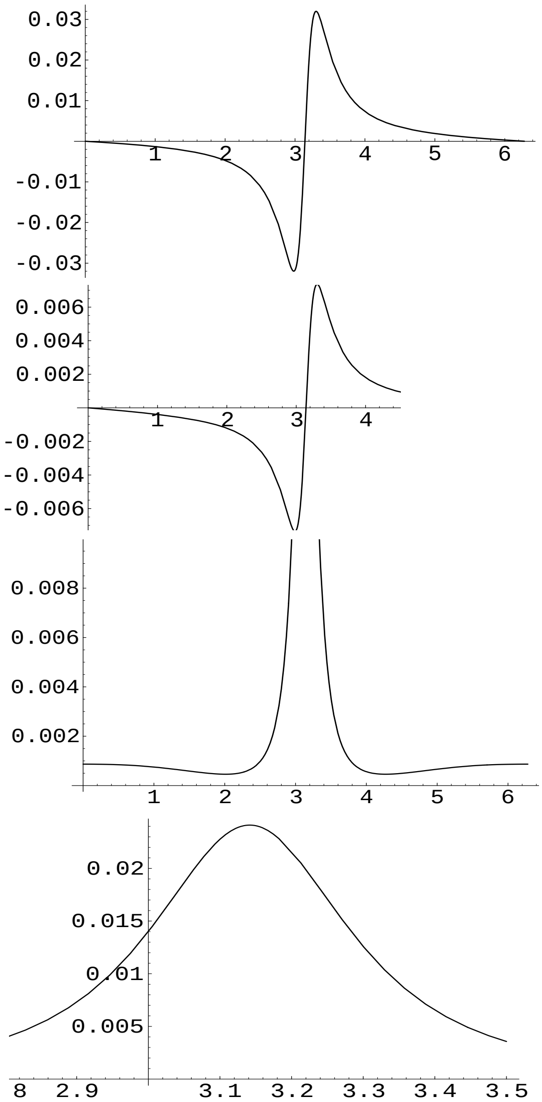

Fig. 3. *The Φ -dependence of target asymmetries in photoproduction for clas1 conditions. From top: $A^{\gamma L}$, $A^{\gamma T}$, $A^{\gamma N}$; angle Φ is given in radians*

In fact, in this reaction there is additional suppression due to used kinematical restrictions: small values of invariants t and q^2.This suppression leads to asymmetries which do not exceed 10^{-4}. At the same kinematics the asymmetries in process (19) can reach for about two order more values. Such situation, in principle, gives the possibility to use process of pair photoproduction to determine the polarization states of the proton and even for independent measurement of the proton electromagnetic form factors.

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ОДНОСПИНОВЫЕ АСИММЕТРИИ В ЭЛЕКТРОН-ПРОТОННОМ И ФОТОН-ПРОТОННОМ РАССЕИВАНИИ В ПРОЦЕССАХ БЕТЕ-ГАЙТЛЕРА, ИНДУЦИРОВАННЫЕ ПЕТЛЕВЫМИ ПОПРАВКАМИ

А.В. Афанасьев, М.И. Кончатный, Н.П. Меренков

Исследованы односпиновые асимметрии мишени в процессах "жесткого" электророждения $e^- + p \to e^- + \gamma + p$ и в фоторождении e^+e^--пар $\gamma + p \to e^+ + e^- + p$, индуцированные петлевыми радиационными поправками в лептонной части взаимодействия. Физической причиной, обуславливающей такого вида асимметрии, является ненулевая мнимая часть Бете-Гайтлеровской амплитуды, которая появляется на уровне радиационной поправки. Односпиновые асимметрии мишени в случае неполяризованного электронного (или фотонного) пучка и произвольной поляризации протона-мишени вычислены в кинематических условиях экспериментов по электророждению CLAS (Jefferson Lab, USA) и HERMES (DESY).

ОДНОСПІНОВІ АСИМЕТРІЇ В ЕЛЕКТРОН-ПРОТОННОМУ І ФОТОН-ПРОТОННОМУ РОЗСІЮВАННІ В ПРОЦЕСАХ БЕТЕ-ГАЙТЛЕРА, ІНДУКОВАНІ ПЕТЛЕВИМИ ПОПРАВКАМИ

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Досліджені односпінові асиметрії мішені в процесах "жорсткого" електронародження $e^- + p \to e^- + \gamma + p$ і в фотонародженні e^+e^--пар $\gamma + p \to e^+ + e^- + p$, індуковані петлевими радіаційними поправками в лептонній частині взаємодії. Фізичною причиною, що обумовлює такого виду асиметрії, є ненульова уявна частина Бете-Гайтлерівської амплітуди, яка з'являється на рівні радіаційної поправки. Односпінові асиметрії мішені у випадку неполяризованого електронного (чи фотонного) пучків та довільної поляризації протона-мішені обчислені в кінематичних умовах експериментів по електронародженню CLAS (Jefferson Lab, USA) і HERMES (DESY).