# T-ODD CORRELATIONS IN $\mathbf{B}^{ \pm} \rightarrow \mathbf{D}^{*} \mathbf{K}^{* \pm}$ DECAYS 

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We examine T violating triple product correlations in the processes $\mathrm{B}^{ \pm} \rightarrow \mathrm{D}^{*} \mathrm{~K}^{* \pm}, \mathrm{D}^{*} \rightarrow \mathrm{D} \pi^{0}, \mathrm{D} \gamma, \mathrm{D} \rightarrow \mathrm{f}$, where the neutral $\mathrm{D}\left(\mathrm{D}^{*}\right)$ meson is a superposition of $\mathrm{D}^{0}\left(\mathrm{D}^{* 0}\right)$ and $\overline{\mathrm{D}}^{0}\left(\overline{\mathrm{D}}^{* 0}\right)$. It is shown that large T violating asymmetries ( $\sim 18 \%$ for the weak phase $\gamma=62^{\circ}$ ) are possible within the Standard Model for hadronic final states 'f' such that $\mathrm{D}^{0} \rightarrow \mathrm{f}$ is doubly Cabibbo suppressed mode while $\overline{\mathrm{D}}^{0} \rightarrow \mathrm{f}$ is Cabibbo allowed mode.

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## 1. INTRODUCTION

Within the Standard Model (SM), CP violation arises due to the presence of a nonzero complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [1,2]. Although the CKM phase is, very likely, the dominant source of CP violation in lowenergy flavor-changing processes. Nevertheless, our knowledge on the origin of CP violation is still unclear because it is known that the same CP violating phase cannot explain the observed asymmetry of matter and antimatter [3]. That is, searching a new CP violating source is one of the most important issues in B factories.

The most straightforward indication of CP violation would be a rate asymmetry between two CP conjugate decays. For channels with only two (pseudo-)scalar mesons or a (pseudo-)scalar and a vector meson in the final state only of this kind the CP asymmetry can become apparent. However, for states with more complex spin content or larger number particles in the final states, asymmetries in the distributions of kinematics variables can also be used to search for CP violation. While traditionally most discussions of CP violation have centered on the partial rate asymmetries, there is another type of CP violating signal, which could potentially reveal the presence of physics beyond the SM. Thus, for example, the decay of the B meson into two vector mesons $B \rightarrow V_{1} V_{2}$ is possible by searching for a triple product correlation. The correlation takes the form $\vec{q} \cdot\left(\vec{\varepsilon}_{1}^{*} \times \vec{\varepsilon}_{2}^{*}\right)$, where $\vec{q}$ is the momentum of one of the final vector mesons in the rest frame of the $B$ meson, and $\vec{\varepsilon}_{1,2}$ are the polarizations of $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$. This quantity is sensitive to the T violation. The experimental searches for such correlations are in progress at the B factories $[4,5]$.

Some triple product correlations in decays of a B meson (charged or neutral) into two final-state vector mesons have been studied within the $S M$ in [6-9]. In these papers, it was found that these correlations with ground state vector mesons are almost all small. Detailed analysis of triple product correlations, based on generalized factorization and involving radiallyexcited mesons, has been performed in [10]. Moreover, it is set up that triple product asymmetries are maximal
when the strong-phase difference vanishes [6,10]. We will find that this result cannot always be correct.

In this paper, we study the possibility to observe T violating effects in the following processes:

$$
\begin{align*}
& B^{-} \rightarrow \widetilde{D}^{* 0}\left(\rightarrow \widetilde{D}^{0}(\rightarrow f) \pi^{0}\right) K^{*-}(\rightarrow K \pi) ;  \tag{1}\\
& B^{-} \rightarrow \widetilde{D}^{* 0}\left(\rightarrow \widetilde{D}^{0}(\rightarrow f) \gamma\right) K^{*-}(\rightarrow K \pi) ;  \tag{2}\\
& \left.B^{-} \rightarrow \widetilde{D}^{* 0}\left(\rightarrow \widetilde{D}^{0}(\rightarrow \bar{f})\right)^{0}\right) K^{*-}(\rightarrow K \pi) ;  \tag{3}\\
& B^{-} \rightarrow \widetilde{D}^{* 0}\left(\rightarrow \widetilde{D}^{0}(\rightarrow \bar{f}) \gamma\right) K^{*-}(\rightarrow K \pi) \tag{4}
\end{align*}
$$

and their CP conjugate partners at B factories. Here we use notation $\widetilde{D}^{0}\left(\widetilde{D}^{* 0}\right)$ to indicate a superposition of $D^{0}$ and $\bar{D}^{0}\left(D^{* 0}\right.$ and $\left.\bar{D}^{* 0}\right)$. The final state ' f ' is chosen to be Cabibbo-allowed mode of $\bar{D}^{0}$ and, therefore, a double Cabibbo-suppressed mode of $D^{0}$ (e.g., ' $\mathrm{f}^{\prime}=\mathrm{K}^{+} \pi^{-}, \mathrm{K}^{*+} \pi^{-}$, $\mathrm{K}^{+} \pi^{-} \pi^{0}$, etc.). Recall that the main decay modes of the $D^{* 0}$ are $D^{0} \pi^{0}(62 \%)$ and $D^{0} \gamma$ (38\%) [11]. We will show that combined analyses of these $\widetilde{D}^{* 0}$ decay modes are useful not only for increasing statistics, but also, due to the effective phase difference of $\pi$ between the $\widetilde{D}^{* 0}$ decays in $\widetilde{D}^{0} \pi^{0}$ and $\widetilde{D}^{0} \gamma$ [12], for clearing the new ways to search for T violating effects in decays of $B^{ \pm} \rightarrow D^{*} K^{* \pm}$.

## 2. TRIPLE PRODUCT ASYMMETRIES

A decay of a B meson, into two vector mesons $B \rightarrow V_{1} V_{2}$, is characterized by three amplitudes. In the transversity basis [8], the decay amplitudes correspond to linear polarization states of the vector mesons which are either longitudinal ( 0 ), or transverse to their directions of motion and parallel $(\|)$ or perpendicular $(\perp)$ to one another. The states 0 and $\|$ are P-even, while the state $\perp$ is P -odd.
$\mathrm{B}^{-}$meson can decay into a $D^{* 0} K^{*-}$ final state via a $b \rightarrow c \bar{u} s$ transition or into a $\bar{D}^{* 0} K^{*-}$ final state via a $b \rightarrow u \bar{c} s$ transition. In the SM, the decay amplitudes for each of the three possible helicity states may be written as

$$
\begin{aligned}
& A_{\lambda}\left(B^{-} \rightarrow D^{* 0} K^{*-}\right) \equiv A_{c \lambda}=\left|V_{c b} V_{u s}^{*}\right| a_{c \lambda} e^{i \delta_{c \lambda}} \\
& A_{\lambda}\left(B^{-} \rightarrow \bar{D}^{* 0} K^{*-}\right) \equiv A_{u \lambda}=\left|V_{u b} V_{c s}^{*}\right| a_{u \lambda} e^{i\left(\delta_{u \lambda}-\gamma\right)},
\end{aligned}
$$

where the helicity index $\lambda$ takes the values $\{0, \|, \perp\}$. In above, $a_{c \lambda}$ and $a_{u \lambda}$ are positive parameters, $\delta_{c \lambda}$ and $\delta_{u \lambda}$ are strong phases, $V_{i j}$ are CKM matrix elements, $\gamma=\arg \left(V_{u b}^{*}\right)$ [13]. These decay amplitudes interfere, when $D^{* 0}$ and $\bar{D}^{* 0}$ decay into the same final state, which can lead to possibility of observing T odd CP violation in these decays. Notice that these types of interferences are similar to those proposed for the extraction of the weak phase $\gamma[14,15]$.

Let us denote the helicity amplitudes for the cascade decays (1) - (:4) as $A_{\lambda}^{f \pi}, A_{\lambda}^{f \gamma}, A_{\lambda}^{\bar{f} \pi}$ and $A_{\lambda}^{\bar{f} \gamma}$, respectively, and its CP conjugate processes as $\bar{A}_{\lambda}^{\bar{f} \pi}$, $\bar{A}_{\lambda}^{\bar{f} \gamma}, \quad \bar{A}_{\lambda}^{f \pi}$ and $\bar{A}_{\lambda}^{f \gamma}$, respectively. In the SM , ignoring possible small effects due to $D^{0}-\bar{D}^{0}$ mixing [16] and taking into account the effective phase difference of $\pi$ between the $D^{*}$ decays in $D \pi^{0}$ and $D \gamma$ [12], the helicity amplitudes for these decays can be written as:

$$
\begin{align*}
& A_{\lambda}^{f \pi(\gamma)}=\left(r_{D f} \pm z_{\lambda}^{-}\right) A_{c \lambda}  \tag{5}\\
& A_{\lambda}^{\bar{f} \pi(\gamma)}=\left(1 \pm r_{D f} z_{\lambda}^{-} e^{-2 i \delta_{D f}}\right) A_{c \lambda}  \tag{6}\\
& \bar{A}_{\lambda}^{\bar{f} \pi(\gamma)}= \pm \sigma_{\lambda}\left(r_{D f} \pm z_{\lambda}^{+}\right) A_{c \lambda}  \tag{7}\\
& \bar{A}_{\lambda}^{f \pi(\gamma)}=\sigma_{\lambda}\left(1 \pm r_{D f} z_{\lambda}^{+} e^{-2 i \delta_{D f}}\right) A_{c \lambda} \tag{8}
\end{align*}
$$

where $\sigma_{0}=\sigma_{\|}=1, \quad \sigma_{\perp}=-1$,

$$
\begin{aligned}
& \delta_{D f} \equiv \arg \left(\frac{A\left(\bar{D}^{0} \rightarrow f\right)}{A\left(D^{0} \rightarrow f\right)}\right), r_{D f} \equiv \sqrt{\frac{\operatorname{Br}\left(D^{0} \rightarrow f\right)}{\operatorname{Br}\left(\bar{D}^{0} \rightarrow f\right)}}, \\
& z_{\lambda}^{ \pm} \equiv r_{B \lambda} e^{i\left(\delta_{\lambda} \pm \gamma\right)}, r_{B \lambda} \equiv\left|\frac{A_{u \lambda}}{A_{c \lambda}}\right|,
\end{aligned}
$$

where $\delta_{B \lambda} \equiv \delta_{u \lambda}-\delta_{c \lambda}, \delta_{\lambda} \equiv \delta_{B \lambda}+\delta_{D f}$.
In the above, $\pm=+$ for the decays (1) and (3) and its CP conjugate and $\pm=-$ for the decays (2) and (4) and its CP conjugate.

The differential decays rates of a B meson into two vector particles

$$
\mathrm{B} \rightarrow \mathrm{~V}_{1} \mathrm{~V}_{2}
$$

with subsequent decays of $V_{1} V_{2}$ have been studied in [7,8,17].

It is convenient to define ratios between the differential decay rate (1) and the decay rate (3) (the differential decay rate (2) and the decay rate (4)) as well as ratios for its CP conjugate decays as:

$$
\begin{align*}
& \frac{d^{3} R_{f, D \pi(\gamma)}}{d \cos \theta_{1} d \cos \theta_{2} d \Phi} \equiv \\
& \Gamma_{\bar{f}, D \pi(\gamma)}^{-1} \frac{d^{3} \Gamma_{f, D \pi(\gamma)}}{d \cos \theta_{1} d \cos \theta_{2} d \Phi}  \tag{9}\\
& \frac{d^{3} \bar{R}_{\bar{f}, D \pi(\gamma)}}{d \cos \theta_{1} d \cos \theta_{2} d \Phi} \equiv \\
& \bar{\Gamma}_{f, D \pi(\gamma)}^{-1} \frac{d^{3} \bar{\Gamma}_{\bar{f}, D \pi(\gamma)}}{d \cos \theta_{1} d \cos \theta_{2} d \Phi} \tag{10}
\end{align*}
$$

The advantage of using of the ratios is that most theoretical and experimental uncertainties cancel. Equations (9) and (10) assume no CP violation in the normalization modes (3) and (4). In fact, we can make the observations, which are based on Eqs. ${ }^{\circ}(6)$, (7), that CP violation in these decays are expected to be small since $r_{D f} r_{B \lambda} \approx 0.01$. Certainly, $r_{D f}$ and $r_{B \lambda}$ are key quantities which values have a significant impact on the ability to measure CP violation in the decays (1)-(4) at the B-factories. Detailed analysis of possible values of these parameters has been performed in [18]. Here we only note that for numerical results we will use the following inputs: the final state ' f ' $=\mathrm{K}^{+} \pi^{-}$, for which $r_{D\left(K^{+} \pi^{-}\right)}, r_{D\left(K^{+} \pi^{-}\right)}=0.06 \pm 0.002$ [19], and the values of $r_{B \lambda}$ could be an order of 0.2 [20] for all helicities.

The ratio between the differential decay rate (1) and the decay rate (3) in the linear polarization basis can be expressed as follows:
$\frac{d^{3} R_{f, D \pi}}{d \cos \theta_{1} d \cos \theta_{2} d \Phi}=\frac{9}{16 \pi}\left(2 R_{0}^{\pi} \cos ^{2} \theta_{1} \cos ^{2} \theta_{2}\right.$
$\left.R^{\pi} \cos ^{2} \Phi+R^{\pi} \sin ^{2} \Phi-\xi \pi \sin 2 \Phi\right) \sin ^{2} \theta_{1}$
$+\left(R_{\|}^{\pi} \cos ^{2} \Phi+R_{\perp}^{\pi} \sin ^{2} \Phi-\xi_{\|}^{\pi} \sin 2 \Phi\right) \sin ^{2} \theta_{1}$
$\left.\times \sin ^{2} \theta_{2}+\frac{\zeta^{\pi} \cos \Phi-\xi_{0}^{\pi} \sin \Phi}{\sqrt{2}} \sin 2 \theta_{1} \sin 2 \theta_{2}\right)$
and the ratio between the differential decay rate (2) and the decay rate (4) as

$$
\begin{align*}
& \frac{d^{3} R_{f, D \gamma}}{d \cos \theta_{1} d \cos \theta_{2} d \Phi}=\frac{9}{32 \pi}\left(2 R_{0}^{\gamma} \sin ^{2} \theta_{1} \cos ^{2} \theta_{2}\right. \\
& -\left(R_{\|}^{\gamma} \cos ^{2} \Phi+R_{\perp}^{\gamma} \sin ^{2} \Phi-\xi_{\|}^{\gamma} \sin 2 \Phi\right) \sin ^{2} \theta_{1} \\
& \times \sin ^{2} \theta_{2}+\left(R_{\|}^{\gamma}+R_{\perp}^{\gamma}\right) \sin ^{2} \theta_{2} \\
& \left.-\frac{\zeta^{\gamma} \cos \Phi-\xi_{0}^{\gamma} \sin \Phi}{\sqrt{2}} \sin 2 \theta_{1} \sin 2 \theta_{2}\right), \tag{12}
\end{align*}
$$

where $\theta_{1}$ is the angle between the direction of the $D$ momentum from the $\mathrm{D}^{*} \rightarrow \mathrm{D} \pi$ or $\mathrm{D}^{*} \rightarrow \mathrm{D} \gamma$ and the direction opposite the B momentum in the $\mathrm{D}^{*}$ rest frame; $\theta_{2}$ is the angle between the direction of the K momentum from the $\mathrm{K}^{*} \rightarrow \mathrm{~K} \pi$ and the direction opposite the B momentum in the $\mathrm{K}^{*}$ rest frame; $\Phi$ is the angle between the decay planes of $\mathrm{D}^{*}$ and $\mathrm{K}^{*}$ in the B rest frame. The observables $R_{\lambda}^{\pi(\gamma)}, \xi_{0, \|}^{\pi(\gamma)}$ and $\zeta^{\pi(\gamma)}$ can be written as

$$
\begin{align*}
& R_{\lambda}^{\pi(\gamma)}=R_{c \lambda}\left(\left(r_{D f} \pm x_{\lambda}^{-}\right)^{2}+\left(y_{\lambda}^{-}\right)^{2}\right) ;  \tag{13}\\
& \xi_{i}^{\pi(\gamma)} \equiv \frac{\mathfrak{T}\left(A_{\perp}^{f \pi(\gamma)}\left(A_{i}^{f \pi(\gamma)}\right)^{*}\right)}{\sum_{\lambda=0, \|, \perp}\left|A_{\lambda}^{\overline{f \pi}(\gamma)}\right|^{2}} ; \\
& \zeta^{\pi(\gamma)} \equiv \frac{\mathfrak{R}\left(A_{\|}^{f \pi(\gamma)}\left(A_{0}^{f \pi(\gamma)}\right)^{*}\right)}{\sum_{\lambda=0, \|, \perp}\left|A_{\lambda}^{\bar{f}(\gamma)}\right|^{2}} \tag{14}
\end{align*}
$$

where $\mathrm{i}=\{0, \|\}, \mathrm{x}^{ \pm}{ }_{\lambda}$ and $\mathrm{y}^{ \pm}{ }_{\lambda}$ are defined as the real and imaginary parts of the complex parameters $\mathrm{z}^{ \pm} \lambda$, respectively. In the above, $R_{c \lambda} \equiv \frac{a_{c \lambda}^{2}}{\sum_{\lambda=0, \|, \perp}^{2} a_{c \lambda}^{2}}$, where $R_{c 0}, R_{c \|}$ and $R_{c \perp}$ denotes longitudinal, transverse parallel and transverse perpendicular polarization fraction in the decays (3) and (4). Recall, that the value of $\quad R_{c} 0$ has been measured to be $R_{c 0}=0.86 \pm 0.06 \pm 0.03$ [21]. The ratios for the case of $\mathrm{B}^{+}$mesons are identical to those ones of Eqs. (11) or (12), where $\Phi, R_{\lambda}^{\pi(\gamma)}, \xi_{i}^{\pi(\gamma)}$, and $\zeta^{\pi(\gamma)}$ were replaced by $-\Phi, \quad \bar{R}_{\lambda}^{\pi(\gamma)}, \quad \bar{\xi}_{i}^{\pi(\gamma)}$, and $\bar{\zeta}^{\pi(\gamma)}$, respectively. Moreover, the observables $\bar{R}_{\lambda}{ }^{\pi(\gamma)}$ are given as

$$
\begin{equation*}
\bar{R}_{\lambda}^{\pi(\gamma)}=R_{c \lambda}\left(\left(r_{D f} \pm x_{\lambda}^{+}\right)^{2}+\left(y_{\lambda}^{+}\right)^{2}\right), \tag{15}
\end{equation*}
$$

and the expressions for the observables $\bar{\xi}_{i}^{\pi(\gamma)}$ and $\zeta^{\pi(\gamma)}$ are similar to those ones given in Eq. (14), with the replacements $A_{\lambda}^{f f(\gamma)} \rightarrow \bar{A}_{\lambda}^{\bar{f} \pi(\gamma)}$
and $A_{\lambda}^{\bar{f} \pi(\gamma)} \rightarrow \bar{A}_{\lambda}^{f \pi(\gamma)}$.

By performing angular analysis of the decays (1)-(4) and their corresponding CP conjugate processes, one can measure the observables $R_{\lambda}{ }^{\pi(\gamma)}, \bar{R}_{\lambda}{ }^{\pi(\gamma)}, \xi_{i}{ }^{\pi(\gamma)}$, $\bar{\xi}_{i}^{\pi(\gamma)}, \zeta^{\pi(\gamma)}, \bar{\zeta}^{\pi(\gamma)}$ and $R_{c \lambda}$ and determine the magnitudes $\delta_{c \lambda}$. Thus, we can measure CP violating asymmetries $R_{\lambda}^{\pi(\gamma)}-\bar{R}_{\lambda}^{\pi(\gamma)}$ and $\zeta^{\pi(\gamma)}-\bar{\zeta}^{\pi(\gamma)}$ as well as T-violating triple product correlations $\xi_{i}{ }^{\pi(\gamma)}+\bar{\xi}_{i}{ }^{\pi(\gamma)}$. Estimates of CP violating asymmetries $R_{\lambda}^{\pi(\gamma)}-\bar{R}_{\lambda}{ }^{\pi(\gamma)}$ can be found in [18].

Using Eqs. (5), (6) and (14), the expression for the observable $\xi_{i}^{\pi(\gamma)}$ can be written as

$$
\begin{align*}
& \xi_{i}^{\pi(\gamma)}=-\sqrt{R_{c \perp} R_{c i}}\left(r_{D f}^{2} \sin \left(\delta_{c i}-\delta_{c \perp}\right)\right. \\
& \pm r_{D f} r_{B \perp} \sin \left(\chi_{i}+\gamma\right) \pm r_{D f} r_{B i} \sin \left(\varphi_{i}-\gamma\right) \\
& \left.+r_{B \perp} r_{B i} \sin \left(\delta_{u i}-\delta_{u \perp}\right)\right), \tag{16}
\end{align*}
$$

where $\varphi_{i} \equiv \delta_{u i}-\delta_{c \perp}+\delta_{D f}$ and $\chi_{i} \equiv \delta_{c i}-\delta_{u \perp}-\delta_{D f}$, the two signs on the right-hand-side correspond to the
decays (1) and (2). We can obtain $\bar{\xi}_{i}^{\pi(\gamma)}$ by changing $\gamma$ to $-\gamma$ and multiplying the right-hand-side of Eq. (16) by the factor -1 . From Eq. (16), one sees that if the ratio $r_{B \lambda}$ is the same for all helicities, i.e., $r_{B 0}=r_{B \|}=r_{B \perp}$, $\xi_{i}^{\pi(\gamma)}$ is zero in absence of any final-state interactions.

Let us define the T-odd quantities $R_{i T}^{\pi}$ and $R_{i T}^{\gamma}$ for the decays (1) and (2) as:

$$
\begin{align*}
& R_{i T}^{\pi(\gamma)} \equiv \mp \frac{\xi_{i}^{\pi(\gamma)}-\bar{\xi}_{i}^{\pi(\gamma)}}{2}= \pm \sqrt{R_{c \perp} R_{c i}}\left(r_{D f}^{2}\right. \\
& \times \sin \left(\delta_{c i}-\delta_{c \perp}\right)+r_{B \perp} r_{B i} \sin \left(\delta_{u i}-\delta_{u \perp}\right) \\
& \left. \pm r_{D f}\left(r_{B \perp} \sin \chi_{i}+r_{B i} \sin \varphi_{i}\right) \cos \gamma\right) . \tag{17}
\end{align*}
$$

From Eq. (17), one sees that even if the weak phase $\gamma$ vanishes, $R_{i T}^{\pi(\gamma)}$ is nonzero in the presence final-state interactions. In addition to these of quantities we define the T-violating quantities $A_{i T}^{\pi}$ and $A_{i T}^{\gamma}$ :

$$
\begin{align*}
& A_{i T}^{\pi(\gamma)} \equiv \mp \frac{\xi_{i}^{\pi(\gamma)}+\xi_{i}^{\pi(\gamma)}}{2}=r_{D f} \sqrt{R_{c \perp} R_{c i}} \\
& \times\left(r_{B \perp} \cos \chi_{i}-r_{B i} \cos \varphi_{i}\right) \sin \gamma . \tag{18}
\end{align*}
$$

This is a true T -violating signal in that it is nonzero only if $\gamma \neq 0$ (i.e., if CP violation is present). The two signs on the right-hand-side of Eqs. (17), (18) correspond to the decays (1) and (2).

Note that if the ratio $r_{B \lambda}$ is the same for all helicities, i.e., $r_{B 0}=r_{B \|}=r_{B \perp}$, then it is easy to show that the quantities $A_{i T}^{\pi(\gamma)}$ are given by

$$
\begin{align*}
& A_{i T}^{\pi(\gamma)}=2 r_{D f} r_{B \perp} \sqrt{R_{c \perp} R_{c i}} \\
& \times \sin \frac{\varphi_{i}+\chi_{i}}{2} \sin \frac{\delta_{i}+\delta_{\perp}}{2} \sin \gamma . \tag{19}
\end{align*}
$$

From Eq. (19), one sees that $A_{i T}^{\pi(\gamma)}$ is zero if the strong phases differences vanish as well as if they satisfy very special conditions, for instance, $\delta_{c i}+\delta_{u i}=\delta_{c \perp}+\delta_{u \perp}$. Therefore, the statement that triple product asymmetries are maximal when the strong-phase difference vanishes $[6,10]$ is not accurate. On the other hand, if the value of $r_{B \lambda}$ will different for the different helicity states then the quantities $A_{i T}^{\pi(\gamma)}$ for $\varphi_{i}=\chi_{i}=0$ can be written as

$$
\begin{equation*}
A_{i T}^{\pi(\gamma)}=r_{D f}\left(r_{B \perp}-r_{B i}\right) \sqrt{R_{c \perp} R_{c i}} \sin \gamma . \tag{20}
\end{equation*}
$$

Thus, a true T -violating signal in the decays (1) and (2) depend not only on the weak phase $\gamma$ and unknown strong phases from the B and D decays, but it also depends on whether the magnitude of the ratios of the amplitudes $B^{-} \rightarrow \bar{D}^{* 0} K^{*-}$ decay to $B^{-} \rightarrow D^{* 0} K^{*-}$ decay depends on the polarization states the vector mesons.

The expectation for the size of the triple products asymmetry in the decays (1) and (2) will, in general,
depend on the values $r_{B \lambda}$ and $\gamma$, the strong phases of the $B$ decay and the strong phase of the $D$ decays, the polarization fractions of the vector mesons in the decays (3) and (4). The strong phases $\delta_{c \lambda}, \delta_{u \lambda}$ and $\delta_{D f}$ that the result from hadronic final state interactions cannot be reliably calculated with any known method and must be determined experimentally. Therefore, for our illustrative calculations we will assume that these phases are chosen completely arbitrary. From Eq. (19) with $R_{c 0}=0.86$ [21] we get:

$$
\left|A_{0 T}^{\pi(\gamma)}\right| \leq 0.69 r_{D f} r_{B \perp}|\sin \gamma|
$$

i.e., for mode ' f ' $=\mathrm{K}^{+} \pi^{-}$and for $r_{B \perp}=0.18$ and $\gamma=62^{\circ}$ [22] it means that $\left|A_{0 T}^{\pi(\gamma)}\right| \leq 0.007$. However, this small value does not indicates that T -violating effects in the decays (1) and (2) are small since the quantities $A_{i T}^{\pi(\gamma)}$ are proportional to $R^{\pi(\gamma)}$,

$$
\begin{aligned}
& R^{\pi(\gamma)} \equiv \frac{R_{f, D \pi(\gamma)}+\bar{R}_{\bar{f}, D \pi(\gamma)}}{2}=r_{D f}^{2} \\
& +\sum_{\lambda=0, \|, \perp} R_{c \lambda} r_{B \lambda}\left(r_{B \lambda} \pm 2 r_{D f} \cos \delta_{\lambda} \cos \gamma\right)
\end{aligned}
$$

which itself is small magnitude.
Therefore, the size of the T-violating effects in the decays (1) and (2) will be better characterized by another parameters $\kappa_{i T}^{\pi(\gamma)}$ or $\kappa_{i T}$, which we define as: $A_{i T}^{\pi(\gamma)}=\kappa_{i T}^{\pi(\gamma)} R^{\pi(\gamma)}$ and $A_{i T}^{\pi(\gamma)}=\kappa_{i T} \frac{R^{\pi}+R^{\gamma}}{2}$. Now, from Eq. (19) and the observed size for $R_{c 0}$ [21] it follows that $\left|\kappa_{0 T}\right| \leq 0.69|\sin \gamma| \frac{r_{D f} r_{B \perp}}{r_{D f}^{2}+r_{B \perp}^{2}}$, i.e., for mode ' f ' $=\mathrm{K}^{+} \pi^{-}$and for $r_{B \perp}=0.18$ and $\gamma=62^{\circ}$ [22] it means that $\left|\kappa_{0 T}\right| \leq 0.18$ for either of the two modes $D^{* 0}$ decays.

Thus, combined analysis of two $D^{* 0}$ decay modes, $D^{0} \pi^{0}$ and $D^{0} \gamma$, leads to the conclusion that the Tviolating asymmetries are expected to be large for $B^{ \pm} \rightarrow D^{*} K^{* \pm}$.

Note as well that combined investigation of $D^{* 0}$ decays in $D^{* 0} \rightarrow D^{0} \pi^{0}$ and $D^{* 0} \rightarrow D^{0} \gamma$ can allow the extraction of the T -odd effects in the decays $B^{ \pm} \rightarrow D^{*} K^{* \pm}$. In fact, from Eq. (17) it follows that

$$
\begin{align*}
& \Delta R_{i T} \equiv R_{i T}^{\pi}-R_{i T}^{\gamma}=2 \sqrt{R_{c \perp} R_{c i}}\left(r_{D f}^{2}\right. \\
& \left.\times \sin \left(\delta_{c i}-\delta_{c \perp}\right)+r_{B \perp} r_{B i} \sin \left(\delta_{u i}-\delta_{u \perp}\right)\right) ;  \tag{21}\\
& R_{i T}^{\pi}+R_{i T}^{\gamma}=2 r_{D f} \sqrt{R_{c \perp} R_{c i}}\left(r_{B i} \sin \varphi_{i}\right. \\
& \left.+r_{B \perp} \sin \chi_{i}\right) \cos \gamma . \tag{22}
\end{align*}
$$

Thus, the quantities $\Delta R_{i T}$ include contributions $b \rightarrow c \bar{u} s$ and $b \rightarrow u \bar{c} s$ transitions in the T-odd
effects while the quantities (22) include contributions only from the interference of $b \rightarrow c \bar{u} s$ and $b \rightarrow u \bar{c} s$ transitions in these effects.

## 3. CONCLUSION

We have considered the T-violation effects in $B^{ \pm} \rightarrow D^{*} K^{* \pm}$ decays, where the neutral $D^{*}$ meson is a superposition of $D^{* 0}$ and $\bar{D}^{* 0}$. To study tripleproduct correlations in these decays, we suggest to use two $D^{*}$ decay modes, namely: $D^{0} / \bar{D}^{0} \pi^{0}$ and $D^{0} / \bar{D}^{0} \gamma$, with $D^{0} / \bar{D}^{0}$ meson further decaying to a final state ' f ' that is common to both $D^{0}$ and $\bar{D}^{0}$. The final state ' f ' is chosen to be a Cabibbo-allowed mode of $\bar{D}^{0}$ and, therefore, a double Cabibbo-suppressed mode of $D^{0}$ (e.g., ' $\mathrm{f}^{\prime}=\mathrm{K}^{+} \pi^{-}, \mathrm{K}^{*+} \pi^{-}, \mathrm{K}^{+} \pi^{-} \pi^{0}$, etc.). We showed that combined analyses of these $D^{*}$ decay modes are useful not only for increasing statistics, but also, due to the effective phase difference of $\pi$ between the $D^{*}$ decays in $D \pi^{0}$ and $D \gamma$, it allows to separate contributions of $b \rightarrow c \bar{u} s$ and $b \rightarrow u \bar{c} s$ transitions into the T-odd triple-product correlations. We found that a true T -violating effect depends not only on the weak phase $\gamma$ and on unknown strong phases, from the $B$ and D decays, but it also depends on whether the magnitude of the ratios of the amplitudes $B^{-} \rightarrow \bar{D}^{* 0} K^{*-}$ decay to $B^{-} \rightarrow D^{* 0} K^{*-}$ decay depends on polarization states the vector mesons. We illustrated that the large T-violating effect ( $\sim 18 \%$ for $\gamma=62^{\circ}$ ) is possible within the SM.

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## Т-НЕЧЕТНЫЕ КОРРЕЛЯЦИИ В РАСПАДАХ В ${ }^{ \pm} \rightarrow \mathbf{D}^{*} \mathbf{K}^{* \pm}$

## В.А. Ковальчук

Исследовали тройные корреляции, которые нарушают T-симметрию, в процессах $\mathrm{B}^{ \pm} \rightarrow \mathrm{D}^{*} \mathrm{~K}^{* \pm}, \mathrm{D}^{*} \rightarrow \mathrm{D} \pi^{0}$, $\mathrm{D} \gamma, \mathrm{D} \rightarrow \mathrm{f}$, где нейтральный $\mathrm{D}\left(\mathrm{D}^{*}\right)$ мезон есть суперпозиция $\mathrm{D}^{0}\left(\mathrm{D}^{* 0}\right)$ и $\overline{\mathrm{D}}^{0}\left(\overline{\mathrm{D}}^{*}\right)$. Показано, что в рамках стандартной модели возможны большие Т-нарушающие асимметрии ( $\sim 18 \%$ для слабой фазы $\gamma=62^{\circ}$ ) для конечных адронных состояний ‘f’ таких, что $\mathrm{D}^{0} \rightarrow \mathrm{f}$ - дважды Кабиббо-подавленная мода, тогда как $\overline{\mathrm{D}}^{0} \rightarrow \mathrm{f}-$ Кабиббо-разрешенная мода.

## Т-НЕПАРНІ КОРЕЛЯЦІЇ У РОЗПАДАХ $\mathbf{B}^{ \pm} \rightarrow \mathbf{D}^{*}$ K $^{* \pm}$

## В. А. Ковальчук

Досліджували потрійні кореляції, які порушують Т-симетрію, у процесах $\mathrm{B}^{ \pm} \rightarrow \mathrm{D}^{*} \mathrm{~K}^{* \pm}, \mathrm{D}^{*} \rightarrow \mathrm{D} \pi^{0}, \mathrm{D} \gamma, \mathrm{D} \rightarrow \mathrm{f}$, де нейтральний $D\left(D^{*}\right)$ мезон є суперпозиція $D^{0}\left(D^{* 0}\right)$ та $\bar{D}^{0}\left(\bar{D}^{* 0}\right)$. Показано, що у рамках стандартної моделі можливі великі Т-порушуючі асиметрії ( $\sim 18 \%$ для слабкої фази $\gamma=62^{\circ}$ ) для кінцевих адронних станів `f’ таких, що $\mathrm{D}^{0} \rightarrow \mathrm{f}$ - подвійно Кабіббо пригнічена мода, тоді як $\overline{\mathrm{D}}^{0} \rightarrow \mathrm{f}-$ Кабіббо-дозволена мода.

