# ELIMINATION OF POWER DIVERGENCES IN CONSISTENT MODEL FOR SPINLESS AND HIGH-SPIN PARTICLE INTERACTIONS 

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The currents for the interaction of the massive high-spin boson ( $J \geq 1$ ) with two spinless particles are derived. These currents obey the theorem on currents and fields as well as the theorem on current asymptotics. In one-loop approximation the contributions of high-spin boson to the self-energy operator for a spinless particle are calculated. It is shown that in one loop approximation the high-spin boson contributions for any spin $J$ and mass lead to finite self-energy operators of spinless-particle.

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## 1. INTRODUCTION

The improvement of the accuracy for the calculations of the hadron reaction amplitudes at low and intermediate energies demands taking into consideration of the high-spin particle contributions. Such investigations are performing more than forty years. In the amplitude calculations the Feynman rules, the propagators and the vertex functions related to the interactions current are used for high-spin particles like to these ones for the 0 and $1 / 2$-spin particles. We name the approaches used for these calculations as the common approaches. Unfortunately the common approaches have the defects such as: 1) the inconsistency of the equations; 2) the power divergences or energy increasing; 3) the ambiguities in the vertex functions; 4) the contradictions to the experimental data.

### 1.1. INCONSISTENSY OF EQUATIONS

Assume that the interactions of the high-spin particles are described by the non-homogeneous Klein Gordon or Dirac equations. For the integer spin $J=l$ we have

$$
\begin{equation*}
\left(\square+M^{2}\right) U(x)_{\mu_{1} \ldots \mu_{l}}=j(x)_{\mu_{1} \ldots \mu_{l}}, \tag{1}
\end{equation*}
$$

where $U(x)_{\mu_{1} \ldots \mu_{l}}$ and $j(x)_{\mu_{1} \ldots \mu_{l}}$ are the symmetrical field and current tensors, respectively, $M$ is the particle mass. It is known that the field tensors for the highspin boson (HSB) obey the auxiliary conditions:

$$
\begin{align*}
& \partial_{\mu_{k}} U(x)_{\mu_{1} \ldots \mu_{l}}=0  \tag{2}\\
& g_{\mu_{i} \mu_{k}} U(x)_{\mu_{1} \ldots \mu_{l}}=0 \tag{3}
\end{align*}
$$

where $i, k=1,2, \ldots, l$. In the common approaches the current tensors obey the symmetric condition only. Therefore the equation systems are inconsistent. To see this we consider the Fourier components of the field and the current tensors. Then the system of the partial differential equations (1) for the Fourier components is the system of the linear algebraic equations. This system is inconsistent as the conditions like to (2), (3) are only for left hand of (1).

### 1.2. POWER DIVERGENCES

The substitutions of the propagators and the vertex functions for the high-spin particles in the reaction amplitudes instead of the propagators and the vertex functions of the 0 and $\frac{1}{2}$-spin particles lead to the power divergences for the amplitudes corresponding to the loop diagrams and the energy increasing for the amplitudes corresponding to the tree diagrams. The common approaches have two sources of the power divergences.

### 1.2.1. PROPAGATORS

As is known (for example in [1-3]) the propagator of spin-1 particle include term $p_{\mu} p_{v} / M^{2}$, where $p$ is the particle momentum. For $J=l$ the propagator include term

$$
\begin{equation*}
\frac{p_{\mu_{1}} \ldots p_{\mu_{l}} \cdot p_{v_{1}} \ldots p_{v_{l}}}{M^{2 l}\left(p^{2}-M^{2}\right)} . \tag{4}
\end{equation*}
$$

Therefore the scale dimension of the particle propagator is equal to $2 l-2$. The HSB momentum can be the integration momentum for the loop-diagram amplitudes and this give the power divergences. For the treediagram amplitudes the HSB momentum is expressed through the external particle moments and this leads to the energy increasing at high energies.

### 1.2.2. CURRENTS

For the HSB interaction $J(p) \rightarrow O\left(k_{1}\right)+O\left(k_{2}\right)$ the currents in the common approach can be written in three forms:

$$
j_{\mu_{1} \ldots \mu_{l}}=\left\{\begin{array}{lc}
k_{1 \mu_{1}} \ldots k_{1 \mu_{l}}, & \text { or }  \tag{5}\\
k_{2 \mu_{1}} \ldots k_{2 \mu_{l}}, & \text { or } \\
\left(k_{1}-k_{2}\right)_{\mu_{1}} \ldots\left(k_{1}-k_{2}\right)_{\mu_{l}}
\end{array}\right.
$$

We see that the current tensors (5)-(7) include the products of the particle moments additional to the spinless particle current. These moments can be expressed through the integration moments or the external particle moments. This leads also to the power divergences or the energy increasing.

### 1.3. AMBIGUITIES

The using of the current tensors (5)-(7), including $k_{1 \mu_{i}}$ or $k_{2 \mu_{i}}$ or $\left(k_{1}-k_{2}\right)_{\mu_{i}}$ for $J \geq 1$ gives different expressions for the amplitudes and even the different powers of the divergences. The question is: what current is correct? We can say: all these currents are wrong.

### 1.4. CONTRADICTIONS TO EXPERREMENTAL DATA

In the common approaches the reaction amplitudes rise with the energy and the spin value by the power law. Moreover the quantity of the diverging loopdiagram amplitudes increases and the powers of the divergence in each such amplitude increase too. It contradicts to the experimental data, as in the reality the cross-sections decrease with the energy and the spin value $J$ at high energies.

The performed consideration shows that the common approaches must be modified. To eliminate the defects of the common approaches we continue the investigations, which had been begun in $[4,6]$.

## 2. PROPERTIES OF CURRENTS IN CONSISTENT APPROACH

### 2.1. THEOREM ON CURRENTS AND FIELDS

For the consistency of the non-homogeneous partial differential equations of Klein-Gordon the current and the field tensors must have the same properties, i.e. must be

$$
\begin{array}{ll}
g_{\mu_{i} \mu_{k}} j(x)_{\mu_{1} \ldots \mu_{l}}=0, & g_{\mu_{i} \mu_{k}} j(p)_{\mu_{1} \ldots \mu_{l}}=0 \\
\partial_{\mu_{k}} j(x)_{\mu_{1} \ldots \mu_{l}}=0 . & p_{\mu_{k}} j(p)_{\mu_{1} \ldots \mu_{l}}=0 .
\end{array}
$$

coordinate
representation
momentum
representation
It is easy to see that the conditions (8), (9) must exit as the operator of the Klein -Gordon equation is the scalar operator. Therefore in the left and right parts of the non-homogeneous equations must be the representations of the same dimension. This gives the same properties of the field tensors and the current tensors.

In consequence of the current conservation the contributions of the terms including $p_{\mu} / M$ in the propagator numerator to the product of the propagator and the currents disappear and the momentum dependences of the propagator part retained are the same for any $l$.

To construct the current tensor which obey (8), (9) we modify the projection operator [7]
$\Pi(p, a, b)=a_{\mu_{1} \ldots a_{\mu_{l}} \Pi(p)_{\mu_{1} \ldots \mu_{l}, v_{1} \ldots v_{l}} b_{v_{1}} \ldots b_{v_{l}}}$
$=\frac{l!\left(-\widetilde{a}^{2}\right)^{\frac{l}{2}}\left(-\widetilde{b}^{2}\right)^{\frac{l}{2}}}{(2 l-1)!!} P_{l}(z), \quad \widetilde{a}_{\mu}=a_{\mu}-\frac{p_{\mu}(a p)}{p^{2}}$,
$\widetilde{b}_{\mu}=b_{\mu}-\frac{p_{\mu}(b p)}{p^{2}}, \quad z=-\frac{(\widetilde{a} \widetilde{b})}{\sqrt{-\widetilde{a}^{2} \sqrt{-\widetilde{b}^{2}}}, ~}$
where $P_{l}(z)$ is the Legendre polynomial. At $\vec{p}=0, z=\cos \Theta, \widetilde{a}=(0, \vec{a}), \tilde{b}=(0, \vec{b})$. The projection
operator $\Pi(p)_{\mu_{1} \ldots \mu_{l}, v_{1} \ldots v_{l}}$ can be derived from $\Pi^{l}(p, a, b)$ by the differentiations with respect to the vectors $a_{\mu_{k}}$ and $b_{v_{k}}$. For example for $J=1$ we have [5]

$$
\begin{equation*}
\Pi(p)_{\mu \nu}=-g_{\mu \nu}+p_{\mu} p_{v} / p^{2} \tag{11}
\end{equation*}
$$

One can be shown that the scale dimension of the propagators for any spin $J$ equals -2 in our approach, whereas in the common approaches it equals $2 J-2$. Such a way, one source of the divergences disappears in consequence of the theorem on currents and fields.

Consider the physical currents $j(x)_{\mu_{1} \ldots \mu_{l}}$, (which obey (8), (9) and have $2 l+1$ components) and the common currents $\quad \eta(x)_{v_{1} . . v_{l}} \quad$ (which have $(l+1)(l+2)(l+3) / 6$ components). The physical currents may be derived as

$$
\begin{equation*}
j(x)_{\mu_{1} \ldots \mu_{l}}=\square^{l} \Pi(x)_{\mu_{1} \ldots \mu_{l}, v_{1} \ldots v_{l}} \eta(x)_{v_{1} \ldots v_{l}} . \tag{12}
\end{equation*}
$$

We consider similar the physical fields $U(x)_{\mu_{1} . \mu_{l}}$ and the common fields $\varphi(x)_{v_{1} \ldots v_{l}}$. Then the Klein-Gordon equations may be written as

$$
\begin{align*}
& \square^{l}\left(\square+M^{2}\right) \Pi(x)_{\mu_{1} \ldots \mu_{l}, v_{1} \ldots v_{l}} \varphi(x)_{v_{1} \ldots v_{l}} \\
& =\square^{l} \Pi(x)_{\mu_{1} \ldots, \mu_{l}, v_{1} \ldots v_{l}} \eta(x)_{v_{1} \ldots v_{l}} . \tag{13}
\end{align*}
$$

### 2.2. THEOREM ON CURRENT ASYMPTOTICS

For the existence of the physical currents $j(x)_{\mu_{1} \ldots \mu_{l}}$ their Fourier components must decrease at $\left|p_{\nu}\right| \rightarrow \infty$ such that the double integrals of the modulus for the product of these current and the high-spin particle momentum with respect to these momentum components must converge.

Indeed, the equations have the order $2 l+2$ and include the derivatives of the $\eta(x)_{v_{1} . v_{l}}$ up to order $2 l$. In [5] it is shown that $2 l+1$ equations will be for $2 l+1$ physical states if the common currents $\eta(x)_{v_{1} . . v_{l}}$ have the derivatives up to order $2 l+1$. Then the physical currents must have the first derivatives (to obey (9)).

The HSB moves along some direction in the 3space. Let HSB moves along the $z$-axis, i.e. $p=\left(p_{0}, 0,0, p_{3}\right)$. According to the Weierstrass test for uniform convergence the existence of the derivatives for the current in the coordinate representation are provided by the convergence of the integrals for the Fourier components of the physical currents

$$
\begin{equation*}
\int_{-\infty}^{+\infty} d p_{0} \int_{-\infty}^{+\infty} d p_{3}\left|j(p)_{\mu_{1} \ldots \mu_{l}} p_{v}\right|, \tag{14}
\end{equation*}
$$

and for the common currents

$$
\begin{equation*}
\int_{-\infty}^{\infty} d p_{0} \int_{-\infty}^{\infty} d p_{3}\left|\eta(p)_{v_{1} \ldots v_{l}} p_{0}^{m_{0}} p_{3}^{m_{3}}\right|, \tag{15}
\end{equation*}
$$

where $m_{0}+m_{3}=m ; \quad m=0,1, \ldots, 2 l+1$. The integrals (15) must converge for all these $m_{0}$ and $m_{3}$. Therefore the currents (15) must include the functions (formfactors) which provide the convergence of the integrals (14) and (15).

## 3. CONSISTENT MODEL <br> FOR INTERACTION OF MASSIVE HIGH-SPIN BOSON WITH TWO SPINLESS PARTICLES

In our approach the physical currents for $J(p) \leftrightarrow O\left(k_{1}\right)+O\left(k_{2}\right)$ - transition are given by

$$
\begin{align*}
& j(p, q)_{\mu_{1} \ldots \mu_{l}}=g f(p, q) p^{2 l} \Pi(p)_{\mu_{1} \ldots \mu_{l}, v_{1} \ldots v_{l}}  \tag{16}\\
& \times q_{v_{1} \ldots v_{v_{l}} \cdot \varphi} .
\end{align*}
$$

where $q=k_{1}-k_{2}, \quad p=k_{1}+k_{2}, g$ is the coupling constant. Then the integral (15) written as

$$
\begin{equation*}
\int_{-\infty}^{+\infty} d p_{0} \int_{-\infty}^{+\infty} d p_{3}\left|f(p, q) p_{0}^{m_{0}} p_{3}^{m_{3}}\right| \tag{17}
\end{equation*}
$$

must converge. The function $f(p, q)$ is such that:

1) $f(p, q)$ exists for any values of the moment $p$ and $q ; 2$ ) we choose $f(p, q)>0$, as the integrands in (15), (17) include the modulus of the $f(p, q)$; 3) $f(p, q)$ is the relativistic scalar and depends on the relativistic invariants $p^{2}=p_{0}^{2}-p_{3}^{2},(p, q)=p_{0} q_{0}-p_{3} q_{3}, q^{2}$;
2) $f(p, q)$ can be the rational fraction, as for such function the integrals (14), (15), (17) for low $m$ can converge but for large $m$ can diverge. The theorem on current asymptotics allows the discontinuities in the derivatives of the common currents for the order more than $2 l+1$.

It can be shown that the integrals (17) diverge for the functions $f(p, q)=f\left(p^{2}, q^{2}\right)$ and $f\left((p, q), q^{2}\right)$ at some $m_{0}$ and $m_{3}$. Now we consider function

$$
\begin{align*}
& f(p, q)=\left[(p q)^{2 n_{1}}+a^{4 n_{1}}\right]^{-1} \\
& \times\left[\left(2(p q)^{2} / q^{2}-p^{2}\right)^{2 n_{2}}+b^{4 n_{2}}\right]^{-1} \tag{18}
\end{align*}
$$

where $a$ and $b$ are positive constants and $n_{1}, n_{2}$ are the positive integer number. This function gives the convergent integrals (14), (15), (17) at $n_{1} \geq 2 l+3, n_{2} \geq \frac{l}{2}+2$ for all the vectors $q \neq\left(0, q_{1}, q_{2}, 0\right)$. These integrals diverge logarithmically at $q=\left(0, q_{1}, q_{2}, 0\right)$. We can make the convergent integrals (14), (15), (17) for all the vectors $q$ if we shall consider the function (18) multiplied by $(p, q)^{m_{1}}$, where
$m_{1}$ is the positive integer number. But at new function $f(p, q)$ the well-known decays $\rho \rightarrow 2 \pi, f^{0}(1520) \rightarrow 2 \pi, \varphi \rightarrow K \bar{K}$ will vanish.

The logarithmic divergence of the integrals (14), (15), (17) for $q=\left(0, q_{1}, q_{2}, 0\right)$ may be compared with the quadratic divergence of similar integral for the superrenormalized $\lambda \varphi^{3}$ - theory.

Using the function (18) we shown that the physical currents at $\left|p_{v}\right| \rightarrow \infty$ behave as

$$
\begin{equation*}
\left|j(p, q)_{\mu_{1} \ldots \mu_{l}}\right| \leq\left|p_{v}\right|^{-4 l-16} \tag{19}
\end{equation*}
$$

Therefore we expect that in our approach the convergence for the HSB interactions will be better than in the $\lambda \varphi^{3}$-theory.

Note that in our approach the currents for the HSB and two spinless particle interactions have no the ambiguities presented in Ch.1.3. Indeed the products of the projection operator derived from the contracted projection operator $\Pi(p, a, b)$ (10) and momentum components $p_{\mu_{k}}$ or $p_{v_{k}}$ vanish, as follow from (2).
Then we have

$$
\begin{align*}
& 2 \Pi(p)_{\mu_{1} \ldots \mu_{l}, v_{1} \ldots v_{l}} k_{1 v_{i}}=-2 \Pi(p)_{\mu_{1} \ldots \mu_{l}, v_{1} \ldots v_{l}} k_{2 v_{i}}  \tag{20}\\
& =\Pi(p)_{\mu_{1} \ldots \mu_{l}, v_{1} \ldots v_{l}} q_{v_{i}}
\end{align*}
$$

Therefore in our approach the physical currents for three forms of the common currents (5)-(7) have the same momentum dependence.

## 4. PRODUCTS OF CURRENTS AND PROPAGATORS

In consequence of the conditions (2), (3) the physical fields $U(x)_{\mu_{1} . . \mu_{l}}$ are the implicit functions of the discrete variable (the spin projection $J_{z}$ ). Besides the physical fields $U(x)_{\mu_{1} \ldots \mu_{l}}$ and the currents $j(x)_{\mu_{1} . \mu_{l}}$ we consider other physical fields $U\left(x, J_{z}\right)$ and the currents $j\left(x, J_{z}\right)$, by analogy with the helicity formalism. The components of the tensor $U(p)_{\mu_{1} . \mu_{l}}$ can be expressed through the $U\left(p, J_{z}\right)$ fields (the helicity states) by means of the Clebsh-Gordan coefficients.

The Lagrangian can be written in the terms of the fields $U\left(x, J_{z}\right)$ by the sum of the states with the definite $J_{z}$ :

$$
\begin{align*}
& L(x)=\sum_{J_{z}=-l}^{l}\left[\partial_{\rho} U^{+}\left(x, J_{z}\right) \partial_{\rho} U\left(x, J_{z}\right)\right. \\
& -M^{2} U^{+}\left(x, J_{z}\right) U\left(x, J_{z}\right)  \tag{21}\\
& \left.+U\left(x, J_{z}\right) j^{+}\left(x, J_{z}\right)+U^{+}\left(x, J_{z}\right) j\left(x, J_{z}\right)\right]
\end{align*}
$$

The Lagrangian can be expressed through the fields $U(x)_{\mu_{1} \ldots \mu_{l}}$ as
$L(x)=\left(\partial_{\rho} U^{+}(x)_{\mu_{1} \ldots \mu_{l}}\right)\left(\partial_{\rho} U(x)_{\mu_{1} . . \mu_{l}}\right)$
$-M^{2} U^{+}(x)_{\mu_{1 . .} \mu_{l}} U(x)_{\mu_{1} . \mu_{l}}+U^{+}(x)_{\mu_{1} . . \mu_{l}} j(x)_{\mu_{1} . . \mu_{l}}$
$+U(x)_{\mu_{1} . \mu_{l}} j^{+}(x)_{\mu_{1} . . \mu_{l}}+$
$+\sum_{k=1}^{l}\left[\lambda_{\mu_{1} \ldots \mu_{l}}^{(k)} \partial_{\mu_{k}} U(x)_{\mu_{1} . . \mu_{l}}+\lambda_{\mu_{1} \ldots \mu_{l}}^{*(k)} \partial_{\mu_{k}} U^{+}(x)_{\mu_{1} . . \mu_{l}}\right]$
$+\sum_{k=1}^{l}\left[\rho_{\mu_{1} \ldots \mu_{l}}^{(i, k)} g_{\mu_{i} \mu_{k}} U(x)_{\mu_{1} . . \mu_{l}}\right.$
$\left.+\stackrel{k=1}{\stackrel{*}{\mu_{\mu_{1} \ldots \mu_{l}}}(i, k)} g_{\mu_{i} \mu_{k}} U^{+}(x)_{\mu_{1} . . \mu_{l}}\right]$.
The Lagrange factors $\lambda_{\mu_{1} . . \mu_{l}}^{(k)}$ and $\rho_{\mu_{1} \ldots \mu_{l}}^{(i, k)}$ are the tensors. The tensor $\lambda_{\mu_{1} . . \mu_{l}}^{(k)}$ has the rank $l-1$ and has no index $\mu_{k}$. The tensor $\rho_{\mu_{1} \ldots \mu_{l}}^{(i, k)}$ has the rank $l-2$ and has no the indices $\mu_{k}, \mu_{i}$. The Euler-Lagrange equations for the Lagrangian (22) give the equations (1) and the conditions (2), (3). Similarly using the Lagrangian (21) we can derive the non-homogeneous Klein-Gordon equations for the fields $U\left(x, J_{z}\right)$. By analogy with the spinless particle we can write the expansions of the physical field operators through the creation and the annihilation operators. Using these expansions and the commutators for the creation and the annihilation operators (similar to ones for spinless fields) we can derive the commutators and $T$-products of the free HSB operators. In [4] it is shown that the products of the HSB propagator numerators and the physical currents do not include the HSB-momentum components.

Now we show that these products expressed through the common currents. Using (12) and property $\Pi^{2}(p)=(-1)^{l} \Pi(p)$ we have

$$
\begin{align*}
& j_{\mu_{1} \ldots \mu_{l}}^{(2)} \frac{\Pi(p)_{\mu_{1} \ldots \mu_{l}, v_{1} \ldots v_{l}}}{p^{2}-M^{2}+i \varepsilon} j_{v_{1} \ldots v_{l}}^{(1)} \equiv j j^{(2)} \frac{\Pi(p)}{p^{2}-M^{2}+i \varepsilon} j^{(1)} \\
& \quad=\left(p^{2}\right)^{2 l} \eta^{(2)} \frac{\Pi^{3}(p)}{p^{2}-M^{2}+i \varepsilon} \eta^{(1)} \\
& \quad=\left(p^{2}\right)^{2 l} \eta^{(2)} \frac{\Pi(p)}{p^{2}-M^{2}+i \varepsilon} \eta^{(1)} . \tag{23}
\end{align*}
$$

These products are similar to ones in the common approaches. But in (23) the projection operator is dimensionless. For the currents (16) of the $J(p) \leftrightarrow O\left(k_{1}\right)+O\left(k_{2}\right) \quad\left(O\left(k_{1}\right) \leftrightarrow J(p)+O\left(k_{2}\right)\right) \quad-$ transitions the product (23) can be expressed through the contracted projection operator (10):

$$
\begin{align*}
& j^{(2)} \frac{\Pi\left(p, q^{\prime}, q\right)}{p^{2}-M^{2}+i \varepsilon} j(1) \\
& =g^{2} f^{2}(p, q)\left(p^{2}\right)^{2 l} \frac{\Pi\left(p, q^{\prime}, q\right)}{p^{2}-M^{2}+i \varepsilon} . \tag{24}
\end{align*}
$$

The $S$-matrix for the HSB interactions can be derived similarly to the $S$-matrix for the 0 -and-1/2particle interactions [1-3] (by means of the $T$-product of the exponent for the interaction Lagrangian).

## 5. TEST OF CONVERGENCE IN ONE-LOOP APPROXIMATION

Consider the contribution of HSB to the self-energy operator of the spinless particle in the one-loop approximation. In this approximation the interaction currents (16) include the vector $q=q^{\prime}=2 k-p$. Then the product (24), as $z=1$ and $P_{l}(1)=1$ in (12), is given by

$$
\begin{align*}
& j^{(2)} \frac{\Pi(p)}{p^{2}-M^{2}+i \varepsilon} j^{(1)}=  \tag{25}\\
& =\frac{(l!)^{2}}{(2 l-1)!!} g^{2} f^{2}(p, q)\left(p^{2}\right) l\left[(p q)^{2}-p^{2} q^{2}\right]^{l}
\end{align*}
$$

Now we derive for the HSB contribution to the selfenergy operator using (25):

$$
\begin{align*}
& \sum\left(k^{2}\right)=\frac{(l!)^{2}}{(2 l-1)!!} g^{2} \int \frac{d^{4} p}{(2 \pi)^{4}} f^{2}(p, q)\left(p^{2}\right)^{l} \\
& {\left[(p q)^{2}-p^{2} q^{2}\right] l}  \tag{26}\\
& {\left[p^{2}-M^{2}+i \varepsilon\right] \cdot\left[(k-p)^{2}-\mu^{2}+i \varepsilon\right]}
\end{align*}
$$

where $\mu$ is the mass of the virtual spinless particle. To simplify the calculations we use the dispersion relations induced by the Lehman representation:

$$
\begin{equation*}
\operatorname{Re} \sum(s)=\frac{P}{\pi} \int_{(M+\mu)^{2}}^{\infty} \frac{\operatorname{Im} \sum(t)}{t-s} d t, \quad s=k^{2} \tag{27}
\end{equation*}
$$

We calculate $\operatorname{Im} \sum(t)$ in (26) by means of changes like to $\left(p^{2}-M^{2}+i \varepsilon\right)^{-1} \rightarrow-2 \pi i \delta\left(p^{2}-M^{2}\right)$. Using the function $f(p, q)$ (18) we derive the exact expression for $\operatorname{Re} \sum(s)$ :

$$
\begin{align*}
& \operatorname{Re} \sum(s)=-\frac{1}{4 \pi} \frac{(l!)^{2}}{(2 l-1)!!} g^{2} M^{2 l} . \\
& \times P \int_{(M+\mu)^{2}}^{\infty} d t \sqrt{\left(\frac{t+M^{2}-\mu^{2}}{2 t}\right)^{2}-\frac{M^{2}}{t}} \cdot \frac{1}{t-s}  \tag{28}\\
& \times\left[\left(t-\mu^{2}\right)^{2}-M^{2}\left(2 t+2 \mu^{2}-M^{2}\right)\right]^{l} \\
& \times\left[\left(t-\mu^{2}\right)^{2 n_{1}}+a^{4 n_{1}}\right]^{-2} \\
& \times\left[\left(2 \frac{\left(t-\mu^{2}\right)^{2}}{\left.\left.\left(2 t+2 \mu^{2}-M^{2}\right)^{-M^{2}}\right)^{2 n_{2}}+b^{4 n_{2}}\right]^{-2}} .\right.\right.
\end{align*}
$$

The integral (28) converges as $n_{1} \geq 2 l+3, n_{2} \geq \frac{l}{2}+2$.
It can be shown that $\operatorname{Re} \sum(s)$ decreases at least as $1 / s$ at $s \rightarrow \infty$. Such a way, the convergence for HSB contribution is better than in the $\lambda \varphi^{3}$-theory for spinless particles, as is known that $\sum(s)$ diverges logarithmically for the contribution of two spinless virtual particles.

## 6. CONCLUSIONS

1. The physical currents of the HSB interactions must be conserved (theorem on currents and fields).
2. The physical currents of HSB interactions must include the functions, which provide the convergence of the integrals for the module of these currents multiplied by the HSB momentum with respect these momentum components (theorem on current asymptotics).
3. The investigations of the HSB contribution (at any spin $J \geq 1$ and any mass $M$ ) to the self-energy operator for the spinless particle show that in our consistent model this self-energy operator is finite and decreases with $s$ at least as $1 / s$.
4. We expect that the currents of HSB interactions, which obey to the theorem on currents and fields as well as the theorem on current asymptotics lead to better convergence than the $\lambda \varphi^{3}$-theory.

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# УСТРАНЕНИЕ СТЕПЕННЫХ РАСХОДИМОСТЕЙ В НЕПРОТИВОРЕЧИВОЙ МОДЕЛИ ВЗАИМОДЕЙСТВИЙ ВЫСОКОСПИНОВОЙ ЧАСТИЦЫ С БЕССПИНОВЫМИ <br> Ю.В. Кулиш, Е.В. Рыбачук 

Получены токи взаимодействия массивного высокоспинового бозона ( $J \geq 1$ ) с двумя бесспиновыми частицами. Эти токи удовлетворяют теореме о полях и токах, а также теореме об асимптотике токов. В однопетлевом приближении показано, что вклады высокоспинового бозона при любых массе и спине дают конечный оператор собственной энергии бесспиновой частицы.

# УСУНЕННЯ СТЕПЕНЕВИХ РОЗБГЖНОСТЕЙ У НЕСУПЕРЕЧЛИВІЙ МОДЕЛІ ВЗАЄМОДІЙ виСОкоСПІНОвОЇ ЧАСТИНКИ з БЕЗСПІНОВИМИ 

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Одержано струми взаємодій масивних високоспінових бозонів $(J \geq 1)$ з двома безспіновими частинками. Ці струми задовольняють теоремі про поля та струми, а також теоремі про асимптотику струмів. В однопетльовому наближенні показано, що внески високоспінових бозонів при довільних спіні $J$ і масі дають скінчений оператор власної енергії безспінової частинки.

