

THE METHOD OF UNITARY CLOTHING TRANSFORMATIONS IN QUANTUM FIELD THEORY: THE BOUND-STATE PROBLEM AND THE S -MATRIX

A.V. Shebeko

*National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine;
e-mail: shebeko@kipt.kharkov.ua*

Being aimed at reformulating quantum field theory (QFT) within the notion of the so-called clothed particles and interactions between them we consider the problem of finding the simplest eigenstates of the total field Hamiltonian H . Along this guideline H and other operators of great physical meaning, e.g., the Lorentz boost generators and the current density operators, which depend initially on the creation and destruction operators for the "bare" particles, are expressed through a new family of their "clothed" counterparts. We are stressing that this transition to the clothed-particle representation (CPR) has been fulfilled via certain unitary ("clothing") transformations (UT's) without changing the original Hamiltonian. It is shown how the S -matrix can be evaluated in the CPR.

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1. PREAMBLE

Finding the eigenstates of the total Hamiltonian H for interacting fields or its blockdiagonalization is a primary concern in quantum physics. The UT's in question do not blockdiagonalize H (except some simple models), but convert it into a form, which enables us to facilitate the initial, extremely complicated problem. We express H through new operators of particle creation and destruction and show that this can be regarded as a UT of H . The respective particles (these quasiparticles of our approach) may be called "clothed". They are identified with physical particles.

The Hamiltonian in the new form turns out to be dependent on the renormalized particle masses and not the initial "bare" ones [1-3]. In addition, it takes on a specific sparse structure in the Hilbert space (say, of the hadronic states in case of the interacting meson and nucleon fields). Forms of the same kind are derived for all the Poincare group generators. After constructing interactions [1-4] between the clothed particles we derive the approximate eigenvalue equations for the simplest bound and scattering states.

Keeping in mind the forthcoming applications of the method of clothing UT's in describing nuclear reactions (in particular, the meson production in nucleon-nucleon collisions), aspecial attention is paid to expressing the S -matrix in terms of the clothed-particle interactions responsible for physical processes in the system under consideration. It is proved that such a reduction becomes possible if the corresponding UT's in the Dirac (D) picture satisfy certain asymptotic conditions in the distant past and future [5].

As a whole, this talk is devoted to a simultaneous exposition of the key points of our approach with an emphasis on its practical aspects and perspectives.

2. UNDERLYING FORMALISM

Our departure point is a total Hamiltonian

$$\begin{aligned} H &\equiv H(a) = H_0(a) + H_I(a) \\ &= K_0(a_c) + K_I(a_c) \equiv K(a_c) \end{aligned} \quad (1)$$

expressed through "clothed" particle creation (destruction) operators $a_c^\dagger(a_c)$ such that

$$a_c(\mathbf{k})\Omega = 0, Ha_c^\dagger(\mathbf{k})\Omega = k_0 a_c^\dagger(\mathbf{k})\Omega, \forall k = (k_0, \mathbf{k}), \quad (2)$$

where Ω is the physical vacuum (the H lowest eigenstate). They obey the same algebra as "bare" operators $a^\dagger(a)$ do. Clothing itself is implemented via

$$a(\mathbf{k}) = W(a_c) a_c(\mathbf{k}) W^\dagger(a_c), \forall \mathbf{k}, \quad (3)$$

where unitary transformation

$$W(a_c) = W(a) = \exp R, \quad (4)$$

with $R^\dagger = -R$ removes from H some undesirable ("bad") terms that prevent the no-clothed-particle state Ω and the one-clothed-particle states to be H eigenvectors. In the context, $K_0(a_c) \neq H_0(a)$ but coincides with $H_0(a_c)$, viz.,

$$K_0(a_c) = H_0(a_c) = \int k_0 a_c^\dagger(\mathbf{k}) a_c(\mathbf{k}) d\mathbf{k} + \dots, \quad (5)$$

where $k_0 = \sqrt{\mathbf{k}^2 + \mu^2}$ with the physical mass μ . The operator

$$K_I(a_c) \equiv K(a_c) - H_0(a_c) = WH(a_c)W^\dagger - H_0(a_c) \quad (6)$$

consists of interactions between clothed particles, responsible for processes with physical particles. At the beginning of our clothing procedure we could consider such $R = R_1$ (see [1, 2] for details

$$\{H_I\}_{bad} = [H_0, R_1] \quad (7)$$

to eliminate the "bad" terms of the g^1 -order, if any, from $H_I = V + M_{ren}$, where V denotes primary interactions between bare particles, $M_{ren} = O(g^2)$ are necessary mass counterterms. After eliminating these bad terms via $W_1 = \exp R_1$,

$$\begin{aligned} K(a_c) &= K_0(a_c) + M_{ren}(a_c) + \frac{1}{2}[R_1, V] \\ &+ [R_1, M_{ren}] + \frac{1}{3}[R_1, [R_1, V]] + \dots \end{aligned} \quad (8)$$

Four-operator (g^2 -order) interactions between clothed particles stem from $\frac{1}{2}[R, V]$. Two-operator contributions to it can be compensated by species $M_{ren}^{(2)}(a_c)$ carrying definition of the latter (see, e.g., another contribution to the Conference by Korda et al.).

3. GENERATORS FOR SPACE TRANSLATIONS AND OTHER SYMMETRIES

We have expressed time translation generator H through the set of a_c . How do the total linear and angular momenta and the Lorentz boost generators depend on the clothed particle operators? The answer will allow us to formulate the transformation properties of the clothed no-particle and one-particle states with respect to the Poincaré group. It is convenient, but not necessary, to use

$$R = -i \lim_{\varepsilon \rightarrow 0^+} \int_0^\infty V(t) e^{-\varepsilon t} dt, \quad (9)$$

where $V(t) = \exp(iK_0 t) V \exp(-iK_0 t)$ the interaction operator in the D picture.

Further, since $[\mathbf{P}, V] = 0$, then $[R, \mathbf{P}] = 0$ and $\mathbf{P}(a) = \mathbf{P}(W(a_c) a_c W^\dagger(a_c)) = W(a_c) \mathbf{P}(a_c) W^\dagger(a_c) \equiv \mathbf{P}(a_c)$, i.e., the total linear momentum \mathbf{P} is the same function of clothed operators as of bare ones. Analogous statements are valid for the total angular momentum and the baryon (fermion) number operator $B = \int \psi^\dagger(\mathbf{x}) \psi(\mathbf{x}) d\mathbf{x}$. This means that the clothed states Ω , $a_c^\dagger \Omega$, $b_c^\dagger \Omega$, and $d_c^\dagger \Omega$ have the following properties: a) they are eigenvectors of \mathbf{P} ; b) they are transformed under space rotations in the same manner as relevant bare states do; c) they possess definite B values. As supplementary requirements: clothed operators and clothed one-particle states must have the same transformation properties with respect to space inversion, time reversal and charge conjugation as their bare counterparts.

Henceforth, to be more definite let me refer to the model, where the neutral pion field ϕ interacts with the nucleon field ψ via the Yukawa coupling, $V = ig \bar{\psi} \gamma_5 \psi \phi$, implying that the set of clothed operators a_c is composed of the meson (pion) operators a_c with the same notation and the fermion operators $b_c(\mathbf{p}, r)$ (nucleons) and $d_c(\mathbf{p}, r)$ (antinucleons) with the polarization index r (afterwards, it will be omitted for short).

4. LORENTZ BOOSTS

In the ‘‘instant’’ form of the relativistic QFT (after Dirac) the generators $\mathbf{N} = (N^1, N^2, N^3)$ of the Lorentz boost $\Lambda = \exp(i\beta \mathbf{N})$ (here $\beta = \beta \mathbf{n}$, $\mathbf{n} = \mathbf{v}/v$ and $\tanh \beta = v$, where v is the velocity of a reference frame) contains interaction terms, e.g., for the Yukawa model,

$$\mathbf{N} = \mathbf{N}_F - \int \mathbf{x} V(\mathbf{x}) d\mathbf{x} + \mathbf{N}_{ren}, \quad (10)$$

where \mathbf{N}_F is the free part of \mathbf{N} : $\mathbf{N}_F = \mathbf{N}_{ferm} + \mathbf{N}_{mes}$ with

$$\begin{aligned} \mathbf{N}_{ferm} &= - \int \mathbf{x} \bar{\psi}(\mathbf{x}) [-i\gamma \nabla + m] \psi(\mathbf{x}) d\mathbf{x} + \frac{i}{2} \int \bar{\psi}(\mathbf{x}) \gamma \psi(\mathbf{x}) d\mathbf{x}, \\ \mathbf{N}_{mes} &= - \frac{1}{2} \int \mathbf{x} [\pi^2(\mathbf{x}) + \nabla \phi(\mathbf{x})^2 + \mu^2 \phi^2(\mathbf{x})] d\mathbf{x}. \end{aligned} \quad (11)$$

We have separated contribution \mathbf{N}_{ren} to \mathbf{N} that comes from the meson and fermion mass counterterms.

It is reasonable to anticipate that the physical vacuum Ω and clothed one-particle states (e.g., $a_c^\dagger(k)\Omega$) should be, respectively, the no-clothed-particle state and one-clothed-particle states from the point of view of a moving observer. More exactly, they should meet the requirements: $\Lambda \Omega = \Omega$ and $\Lambda a_c^\dagger(k)\Omega = a_c^\dagger(Lk)\Omega$, where L is the Lorentz transformation. How can one provide them? Again, in the CPR,

$$\begin{aligned} \mathbf{N} &\equiv \mathbf{N}(a) = W \mathbf{N}(a_c) W^\dagger \\ &\equiv \mathbf{B}(a_c) = \mathbf{N}_F + \mathbf{N}_I + [R, \mathbf{N}_F] + [R, \mathbf{N}_I] + \dots \end{aligned} \quad (12)$$

and we are looking for R such that

$$[\mathbf{N}_F, R] = \mathbf{N}_I = - \int \mathbf{x} V(\mathbf{x}) d\mathbf{x} \quad (13)$$

to eliminate the bad terms linear in \mathbf{N}_I . Now, by using the property of interaction density $V(x) = V(\mathbf{x}, t) = e^{iK_0 t} V(\mathbf{x}) e^{-iK_0 t}$ to be a scalar, i.e., $e^{i\beta \mathbf{N}_F} V(x) e^{-i\beta \mathbf{N}_F} = V(Lx)$, one can show that Eq. (13) will hold if $R = R_I$. Thus, the UT $W_I = \exp R_I$ removes bad terms (for instance, all three-operator (‘‘three-legs’’) terms of the g^1 -order in the field models with Yukawa-type couplings) simultaneously from the total Hamiltonian $K(a_c)$ and the boost generators $\mathbf{B}(a_c)$. One should emphasize that this result is valid for any Lorentz-scalar function $V(\mathbf{x}, t)$. The explicit expressions for \mathbf{N}_F are not required as well.

After this operation,

$$\begin{aligned} \mathbf{B}(\alpha) &= \mathbf{N}_F(\alpha) + \mathbf{N}_{ren}(\alpha) + \frac{1}{2} [R, \mathbf{N}_I] \\ &+ [R, \mathbf{N}_{ren}] + \frac{1}{3} [R, [R, \mathbf{N}_I]] + \dots \end{aligned} \quad (14)$$

that repeats the structure of the corresponding contributions to $K(\alpha)$. It is important from the physical point of view if we want to have $\Lambda \Omega = \Omega$, $\Lambda a_c^\dagger(k)\Omega = a_c^\dagger(Lk)\Omega$. The latter does not mean $\Lambda a_c^\dagger(k)\Lambda^{-1} = a_c^\dagger(Lk)$.

5. RELATIVISTIC INTERACTIONS IN MESON-NUCLEON SYSTEMS IN CPR

Within the method of UT’s a huge amount of virtual processes induced by a meson absorption/emission, a $N\bar{N}$ -pair annihilation/production and other cloud effects can be accumulated in the creation (destruction) operators for the **clothed** (physical) mesons and nucleons. Such a bootstrap reflects the most significant distinction between the concepts of clothed and bare particles.

Performing the normal ordering of the clothed-particle operators involved, we get a simple recipe to select the $2 \longleftrightarrow 2$ and $2 \longleftrightarrow 3$ interaction operators of the g^2 - and g^3 -orders between the partially clothed pions, nucleons and antinucleons (in particular, $\pi N \rightarrow \pi N$, $NN \rightarrow NN$ and $NN \leftrightarrow \pi NN$)

$$\begin{aligned} K(a_c) &= K_0(a_c) + K(NN \rightarrow NN) + K(\bar{N}\bar{N} \rightarrow \bar{N}\bar{N}) \\ &+ K(N\bar{N} \rightarrow N\bar{N}) + K(\pi N \rightarrow \pi N) + K(\pi\bar{N} \rightarrow \pi\bar{N}) \\ &+ K(\pi\pi \leftrightarrow N\bar{N}) + K(NN \leftrightarrow \pi NN) + K(\bar{N}\bar{N} \leftrightarrow \pi\bar{N}\bar{N}) \\ &+ K(N\bar{N} \leftrightarrow \pi N\bar{N}) + K(N\bar{N} \leftrightarrow \pi\pi\pi) + K(\pi N \leftrightarrow \pi\pi N) \\ &+ K(\pi\bar{N} \leftrightarrow \pi\pi\bar{N}) + \dots, \end{aligned} \quad (15)$$

where interactions between clothed nucleons (N), anti-nucleons (\bar{N}) and pions (π) have been separated out.

5.1. NUCLEON-NUCLEON INTERACTION OPERATOR

Along this guideline we derive the $NN \rightarrow NN$ interaction operator within the Yukawa model

$$\begin{aligned} K(NN \rightarrow NN) &= \int d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}'_1 d\mathbf{p}'_2 V_{NN}(\mathbf{p}'_1, \mathbf{p}'_2; \mathbf{p}_1, \mathbf{p}_2) \\ &\quad \times b_c^\dagger(\mathbf{p}'_1) b_c^\dagger(\mathbf{p}'_2) b_c(\mathbf{p}_1) b_c(\mathbf{p}_2), \\ V_{NN}(\mathbf{p}'_1, \mathbf{p}'_2; \mathbf{p}_1, \mathbf{p}_2) &= -\frac{1}{2} \frac{g^2 m^2}{(2\pi)^3} \frac{\delta(\mathbf{p}'_1 + \mathbf{p}'_2 - \mathbf{p}_1 - \mathbf{p}_2)}{\sqrt{E_{\mathbf{p}'_1} E_{\mathbf{p}'_2} E_{\mathbf{p}_1} E_{\mathbf{p}_2}}} \\ &\quad \times \bar{u}(\mathbf{p}'_1) \gamma_5 u(\mathbf{p}_1) \frac{1}{(p_1 - p'_1)^2 - \mu^2} \bar{u}(\mathbf{p}'_2) \gamma_5 u(\mathbf{p}_2). \end{aligned} \quad (16)$$

The corresponding relativistic and properly symmetrized NN quasipotential is given by

$$\begin{aligned} \tilde{V}_{NN}(\mathbf{p}'_1, \mathbf{p}'_2; \mathbf{p}_1, \mathbf{p}_2) &= -\frac{1}{2} \frac{g^2 m^2}{(2\pi)^3} \frac{\delta(\mathbf{p}'_1 + \mathbf{p}'_2 - \mathbf{p}_1 - \mathbf{p}_2)}{2\sqrt{E_{\mathbf{p}'_1} E_{\mathbf{p}'_2} E_{\mathbf{p}_1} E_{\mathbf{p}_2}}} \\ &\quad \times \bar{u}(\mathbf{p}'_1) \gamma_5 u(\mathbf{p}_1) \frac{1}{2} \left\{ \frac{1}{(p_1 - p'_1)^2 - \mu^2} \right. \\ &\quad \left. + \frac{1}{(p_2 - p'_2)^2 - \mu^2} \right\} \bar{u}(\mathbf{p}'_2) \gamma_5 u(\mathbf{p}_2) - (1 \leftrightarrow 2). \end{aligned} \quad (17)$$

Its distinctive feature is the presence of the covariant (Feynman-like) ‘‘propagator’’,

$$\frac{1}{2} \left\{ \frac{1}{(p_1 - p'_1)^2 - \mu^2} + \frac{1}{(p_2 - p'_2)^2 - \mu^2} \right\}. \quad (18)$$

On the energy shell of the NN scattering, that is

$$E_i \equiv E_{\mathbf{p}_1} + E_{\mathbf{p}_2} = E_{\mathbf{p}'_1} + E_{\mathbf{p}'_2} \equiv E_f, \quad (19)$$

this expression is converted into the genuine Feynman propagator which occurs upon evaluating the S -matrix in the g^2 -order.

6. EQUATIONS FOR BOUND AND SCATTERING STATES

The clothed one-particle states are eigenstates of H . There may be other H eigenstates, viz., the states with discrete values of the system mass. For the Yukawa model the corresponding states may be fermion-

fermion states (deuteron-like), etc. They appear with the following zeroth approximation (ZA)

$$K_{ZA} = K_2 + g^2 K_4^{(2)}, \quad (20)$$

which is created by adding to the two-operator (one-body) contribution

$$\begin{aligned} K_2 &= \int \omega_{\mathbf{k}} a_c^\dagger(\mathbf{k}) a_c(\mathbf{k}) d\mathbf{k} + \int E_{\mathbf{p}} \sum_r \left[b_c^\dagger(\mathbf{p}, r) b_c(\mathbf{p}, r) \right. \\ &\quad \left. + d_c^\dagger(\mathbf{p}, r) d_c(\mathbf{p}, r) \right] d\mathbf{p} \equiv K_\pi + K_N, \end{aligned} \quad (21)$$

the four-operator (two-body) contributions of the g^2 -order that arise, in particular, from the commutator $\frac{1}{2}[R, V]$. The operator K_{ZA} has an important property: it conserves the total number of clothed particles. Moreover, the Fock subspace spanned onto the two-particle $H_0(a_c)$ eigenstates can be divided into several sectors (the NN -, πN -sector, etc.) such that K_{ZA} leaves each of them to be invariant (see Subsection 4.2 and Appendix B of Ref. [2] for details). If we consider Now the K_{ZA} eigenstates that belong to the NN sector,

$$\begin{aligned} \Phi_{NN} &= \sum_{r_1, r_2} \int d\mathbf{p}_1 d\mathbf{p}_2 \Phi_{NN}(\mathbf{p}_1, r_1; \mathbf{p}_2, r_2) \\ &\quad \times b_c^\dagger(\mathbf{p}_1, r_1) b_c^\dagger(\mathbf{p}_2, r_2) \Omega, \end{aligned} \quad (22)$$

then $K_{ZA} \Phi_{NN}$ will be the state vector of the same sector so that the eigenvalue equation $K_{ZA} \Phi_{NN}^E = E \Phi_{NN}^E$ yields

$$[K_N + K(NN \rightarrow NN)] \Phi_{NN}^E = E \Phi_{NN}^E \quad (23)$$

in the sector. The corresponding equation for the WF $\Phi_{NN}^E(1; 2)$ looks as

$$\begin{aligned} (E - E_{\mathbf{p}_1} - E_{\mathbf{p}_2}) \Phi_{NN}^E(1; 2) \\ = \int d\mathbf{p}'_1 d\mathbf{p}'_2 \tilde{V}_{NN}(1, 2; 1', 2') \Phi_{NN}^E(1'; 2'). \end{aligned} \quad (24)$$

If we start with the same zeroth approximation to H , our description of the clothed $\pi\bar{N}$ and $\bar{N}\bar{N}$ states will be very similar to that given for πN and NN states. A different situation holds in the case of clothed fermion-antifermion and two-meson states, where one has to handle eigenstates of a mixed kind (see [2]). Nevertheless, the corresponding equations for the WF's can be solved in a nonperturbative way using the methods elaborated in the theory of nuclear reactions.

7. THE S -OPERATOR. AN EQUIVALENCE THEOREM FOR THE S MATRIX

After constructing the interaction operators in the CPR it becomes to be indispensable to express the conventional S -matrix through the clothed-particle interactions and states. In this respect, let me recall the definition

$$S = \lim_{t_2 \rightarrow +\infty} \lim_{t_1 \rightarrow -\infty} e^{iH_0 t_2} e^{-iH(t_2 - t_1)} e^{-iH_0 t_1} \quad (25)$$

of the S -operator $S \equiv S(a)$ for the decomposition $H = H_0(a) + H_I(a)$. Furthermore, let us introduce the S -operator

$$S_{cloth} = \lim_{t_2 \rightarrow +\infty} \lim_{t_1 \rightarrow -\infty} e^{iK_0 t_2} e^{-iK(t_2-t_1)} e^{-iK_0 t_1} \quad (26)$$

for the decomposition $H = K(a_c) = K_0(a_c) + K_I(a_c)$ or

$$S_{cloth} = \lim_{t_2 \rightarrow +\infty} \lim_{t_1 \rightarrow -\infty}$$

$$W_D(t_2) e^{iK_0 t_2} e^{-iH(a_c)(t_2-t_1)} e^{-iK_0(a_c)t_1} W_D^\dagger(t_1), \quad (27)$$

where $W_D(t) = \exp(iK_0 t) W \exp(-iK_0 t)$.

Since

$$\lim_{t \rightarrow \pm\infty} W_D(t) = 1 \text{ or } \lim_{t \rightarrow \pm\infty} R_D(t) = 0 \quad (28)$$

then

$$S_{cloth} = \lim_{t_2 \rightarrow +\infty} \lim_{t_1 \rightarrow -\infty} e^{iK_0(a_c)t_2} e^{-iH(a_c)(t_2-t_1)} e^{-iK_0(a_c)t_1}. \quad (29)$$

Matrix elements $\langle a^\dagger \dots \Omega_0 | S(a) | a^\dagger \dots \Omega_0 \rangle$ of $S = S(a)$ between the *bare* states $a^\dagger \dots \Omega_0$ with $H_0 \Omega_0 = 0$ and matrix elements $\langle a_c^\dagger \dots \Omega | S(a_c) | a_c^\dagger \dots \Omega \rangle$ of $S_{cloth} = S(a_c)$ between the *clothed* states $a_c^\dagger \dots \Omega$ with $K_0 \Omega = 0$ are equal to each other since the a_c -algebra with the vacuum Ω is isomorphic to the a -algebra with the vacuum Ω_0 . Thus, all what we need to ensure this equivalence is the asymptotic condition (28), i.e.,

$$\lim_{t \rightarrow \pm\infty} \exp[iH_0(a_c)t] R(a_c) \exp[-iH_0(a_c)t] = 0. \quad (30)$$

As it has been shown in [6] this condition may be fulfilled, at least, for the Yukawa model.

8. LINKS WITH THE IN(OUT) FORMALISM. REDUCTION TO CLOTHED PARTICLE STATES

In order to describe collisions bound systems it is pertinent to proceed with the S-matrix $S_{fi} = \langle f; out | i; in \rangle$ in the Heisenberg (H) picture, built of the in(out) states, e.g., for the reaction $\pi d \rightarrow NN$ with $S_{\pi d \rightarrow NN} = \langle NN; out | a_{in}^\dagger(k_\pi) | d \rangle$. Further, one introduces (to be definite for opposite-charged scalar particles) $A(k, t) = (f_k^*, \varphi)$, $B^\dagger(k, t) = (\varphi, f_k) = -(f_k, \varphi)$ with respect to the scalar product $(F_1, F_2) \equiv i \int d\mathbf{x} F_1^*(\mathbf{x}, x_0) \partial_0 F_2(\mathbf{x}, x_0)$ for the H-field operator

$$\varphi(x) = \varphi(\mathbf{x}, t) \equiv e^{iHt} \varphi_D(\mathbf{x}) e^{-iHt},$$

$$\varphi_D(\mathbf{x}) = \int d\mathbf{k} [A(k) f_k(\mathbf{x}) + B^\dagger(k) f_k^*(\mathbf{x})]. \quad (31)$$

Here $f_k(\mathbf{x})$ are the respective plane waves (strictly speaking, the wave-packet-like solutions of the Klein-Gordon equation with positive frequencies). Now, considering the similarity transformations $A(k) \rightarrow A_c(k) = W^\dagger A(k) W$ and $B(k) \rightarrow B_c(k) = W^\dagger B(k) W$ and employing the LSZ prescription

$$\lim_{t \rightarrow \pm\infty} \langle \Phi | A^\dagger(k, t) | \Psi \rangle = \langle \Phi | A_{in}^\dagger(k) | \Psi \rangle \quad (32)$$

to be valid for any normalizable states Φ and Ψ , one can show (see [6]) that the *one-meson in-state*

$$|k; in \rangle \equiv A_{in}^\dagger(k) \Omega = \lim_{t \rightarrow -\infty} A^\dagger(k, t) \Omega = A_c^\dagger(k) \Omega \quad (33)$$

and the *two-meson in-states*

$$\begin{aligned} |k_1, k_2; in \rangle &\equiv A_{in}^\dagger(k_1) A_{in}^\dagger(k_2) \Omega \\ &= \lim_{t \rightarrow -\infty} e^{i(H-k_1^0-k_2^0)t} A_c^\dagger(k_1) A_c^\dagger(k_2) \Omega. \end{aligned} \quad (34)$$

Its trivial consequence is,

$$|k_1, k_2; in \rangle \neq A_c^\dagger(k_1) A_c^\dagger(k_2) \Omega.$$

Similar relations with $t \rightarrow +\infty$ can be derived for the out-states. Moreover, such limits in the distant past and future are equivalent to the ‘‘Møller’’ operators

$$\Omega^{(\pm)}(E) = \pm i \lim_{\varepsilon \rightarrow +0} \frac{\varepsilon}{E \pm i\varepsilon - H}.$$

For instance, we find that

$$|k_1, k_2; in \rangle = \Omega^{(+)}(k_1^0 + k_2^0) A_c^\dagger(k_1) A_c^\dagger(k_2) | \Omega \rangle. \quad (35)$$

Such a time-independent representation via the Hamiltonian resolvent is closely connected with the approaches typical to the nonrelativistic quantum theory (see [5,6]).

9. CONCLUDING REMARKS

i) I have tried to show a possible way in finding bridges between the description of some bound and scattering states in QFT and the approach traditional for the nuclear physics. Of course, in spite of a similarity between nonrelativistic quantum mechanics and our clothing procedure, the latter gives rise to the new non-conserving clothed-particle-number interactions (quasipotentials).

ii) The method of unitary clothing transformations enables us to obtain (in a combination with nonperturbative recipes of QFT) a number of relations helpful both for the evaluation of reaction amplitudes and state vectors.

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МЕТОД УНИТАРНЫХ ОДЕВАЮЩИХ ПРЕОБРАЗОВАНИЙ В КВАНТОВОЙ ТЕОРИИ ПОЛЯ: ПРОБЛЕМА СВЯЗАННЫХ СОСТОЯНИЙ И S -МАТРИЦА

А.В. Шебеко

С помощью переформулирования квантовой теории поля в терминах так называемых “одетых” частиц и взаимодействий между ними рассмотрена проблема поиска простейших собственных состояний полного гамильтониана H . В этом подходе H и другие операторы, имеющие глубокий физический смысл, такие как генераторы Лоренц бустов и плотность тока, которые изначально зависят от операторов рождения и уничтожения “голых” частиц, выражаются через операторы “одетых” частиц. Переход к представлению “одетых” частиц осуществляется с помощью определенного унитарного (“одевающего”) преобразования без изменения исходного гамильтониана. Показано, как можно вычислять S -матрицу в новом представлении.

МЕТОД УНІТАРНИХ ОДЯГАЮЧИХ ПЕРЕТВОРЕНЬ В КВАНТОВІЙ ТЕОРІЇ ПОЛЯ: ПРОБЛЕМА ЗВ'ЯЗАНИХ СТАНІВ І S -МАТРИЦЯ

О.В. Шебеко

За допомогою переформулювання квантової теорії поля в термінах так званих “одягнених” частинок і взаємодій між ними розглядено проблему пошуку простіших власних станів повного гамільтоніану H . У цьому підході H та інші оператори, що мають глибоке фізичне значення, такі як генератори Лоренц бустів і густина струму, котрі початково залежать від операторів народження і знищення “голих” частинок, подаються через оператори “одягнених” частинок. Перехід до зображення “одягнених” частинок здійснюється за допомогою певного унітарного (“одягаючого”) перетворення без зміни початкового гамільтоніану. Показано, як можна обчислювати S -матрицу в новому зображенні.