CLOTHED PARTICLE REPRESENTATION IN QUANTUM FIELD THEORY: MASS RENORMALIZATION

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The method of unitary clothing transformations is applied in the model involving nucleon and neutral pion fields interacting via the regularized pseudoscalar Yukawa-type coupling. In this approach the mass counterterms are cancelled (at least, partly) by commutators of the generators of clothing transformations and the field interaction operator forming the pion and nucleon mass shifts expressed through the corresponding three-dimensional integrals whose integrands depend on certain covariant combinations of the relevant three-momenta. The property provides the momentum independence of mass renormalization. The conditions imposed upon the cutoff vertex function are specified.

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1. FIELD MODEL WITH REGULARIZED INTERACTION

Our departure point is the Hamiltonian

$$H(\alpha_0) = H_F(\alpha_0) + H_I(\alpha_0)$$
$$= H_F(\alpha_0) + M_{ren}(\alpha_0) + V(\alpha_0) + V_{ren}(\alpha_0), \qquad (1)$$

where α_0 – set of all creation and destruction operators of the "bare" particles with physical masses and couping constants [1]. In case of a spinor (fermion) field and a neutral pseudoscalar (meson) field one has

$$H_F(\alpha_0) = \int d\mathbf{k}\omega_{\mathbf{k}} a^\dagger(\mathbf{k}) a(\mathbf{k})$$
$$+\int d\mathbf{p} E_{\mathbf{p}} \sum_r \left[b^\dagger(\mathbf{p},r) b(\mathbf{p},r) + d^\dagger(\mathbf{p},r) d(\mathbf{p},r) \right], \qquad (2)$$

with the operators for mesons $a(\mathbf{k})$, nucleons $b(\mathbf{p},r)$, antinucleons $d(\mathbf{p},r)$ and their adjoint counterparts. The quantities $\mathbf{k}$, $\mathbf{p}$ and r are the particle momenta and the fermion polarization index. Relativistic physical energies are expressed as $E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m^2}$ and $\omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + \mu^2}$, where m and μ play role of physical (renormalized) masses. $M_{ren}(\alpha_0) = M_{ren,\,mes}(\alpha_0) + M_{ren,\,ferm}(\alpha_0)$ are usual mass counterterms for mesons and fermions, containing respective mass shifts $\delta\mu^2 = \mu_0^2 - \mu^2$ and $\delta m = m_0 - m$ where m_0 and μ_0 play role of trial (unrenormalized) masses. The one-particle operators in (2) satisfy the usual commutation relations

$$\left[a(\mathbf{k}),\, a^\dagger(\mathbf{k}') \right] = \delta(\mathbf{k} - \mathbf{k}');$$
$$\left\{ b(\mathbf{p},r), b^\dagger(\mathbf{p}',r') \right\} = \left\{ d(\mathbf{p},r), d^\dagger(\mathbf{p}',r') \right\} = \delta_{rr'}\delta(\mathbf{p} - \mathbf{p}')$$
$$\left\{ F_i(\mathbf{p},r), F_j^\dagger(\mathbf{p}',r') \right\} = \delta_{i,j}\delta_{r,r'}\delta(\mathbf{p} - \mathbf{p}');$$
$$F^\dagger(\mathbf{p},r) = \left(b^\dagger(\mathbf{p},r),\quad d(-\mathbf{p},r) \right). \qquad (3)$$

In our non-local extension of the Yukawa-type coupling the interaction operator is

$$V(\alpha_0) = \int d\mathbf{k}\, \hat{V}^{\mathbf{k}}\, a(\mathbf{k}) + H.c.;$$
$$\hat{V}^{\mathbf{k}} = \int d\mathbf{p}' d\mathbf{p} \sum_{r,r',i,j} F_i^{?}(\mathbf{p}',r')\, V_{i,j}^{\mathbf{k}}(\mathbf{p}',r';\mathbf{p},r) F_j(\mathbf{p},r);$$
$$V_{i,j}^{\mathbf{k}}(\mathbf{p}',r';\mathbf{p},r) = \frac{i}{(2\pi)^{3/2}} \frac{m}{\sqrt{2\omega_{\mathbf{k}} E_{\mathbf{p}'} E_{\mathbf{p}}}} \delta(\mathbf{p} + \mathbf{k} - \mathbf{p}')$$
$$\times \bar{U}_{i,l}(\mathbf{p}',r') g_{l,n}(\mathbf{p}',\mathbf{p},\mathbf{k})\, \gamma_5 U_{n,j}(\mathbf{p},r), \qquad (4)$$

with physical vertex functions $g_{i,j}(\mathbf{p}',\mathbf{p},\mathbf{k})$ (real Lorentz-scalar cutoff form-factors in each vertex). Dirac spinors u and v satisfy the conventional equations and form the matrix

$$U(\mathbf{p},r) = \begin{pmatrix} u(\mathbf{p},r) & 0 \\ 0 & v(-\mathbf{p},r) \end{pmatrix}. \qquad (5)$$

The vertex counterterm $V_{ren}(\alpha_0) \equiv V_0(\alpha_0) - V(\alpha_0)$ is determined by the formulae (4) but with $g_{l,n}$ substituted by $\delta g_{l,n} \equiv g_0 - g_{l,n}$ where g_0 - bare coupling constant. The CPT invariance of the total Hamiltonian leads to

$$g_{i,j}^*(\mathbf{p}',\mathbf{p},\mathbf{k}) = g_{i,j}(\mathbf{p}',\mathbf{p},\mathbf{k}),$$
$$g_{i,j}(\mathbf{p}',\mathbf{p},\mathbf{k}) = g_{i,j}(\mathbf{p},\mathbf{p}',-\mathbf{k}),\quad i \neq j, \qquad (6)$$
$$g_{2,2}(\mathbf{p}',\mathbf{p},\mathbf{k}) = g_{1,1}(\mathbf{p},\mathbf{p}',-\mathbf{k}).$$

2. CLOTHED PARTICLE REPRESENTATION

By definition, the one-bare-particle states $\left|\alpha_0^\dagger\Omega_0\right\rangle$ are the eigenstates of the free part of the Hamiltonian $H_F(\alpha_0)\left|\alpha_0^\dagger\Omega_0\right\rangle = E_0\left|\alpha_0^\dagger\Omega_0\right\rangle$. However, the same one-particle states are not the eigenstates of the total Hamiltonian $H(\alpha_0)\left|\alpha_0^\dagger\Omega_0\right\rangle \neq E\left|\alpha_0^\dagger\Omega_0\right\rangle$. Whether there exists a new set of creation/destruction operators α_c in terms of which $H_F(\alpha_c)|\alpha_c^\dagger\Omega\rangle = E|\alpha_c^\dagger\Omega\rangle$ and $H_c(\alpha_c)|\alpha_c^\dagger\Omega\rangle = E|\alpha_c^\dagger\Omega\rangle$? Greenberg and Schweber [2] assumed that such set of operators α_c, called "clothed", existed and was connected with the initial set of operators α_0 via the unitary transformation which kept S-operator intact

$$\alpha_0 = W(\alpha_c)\alpha_c W^\dagger(\alpha_c),\ WW^\dagger = W^\dagger W = 1,$$
$$W(\alpha_c) = e^{R(\alpha_c)},\ R(\alpha_c) = -R^\dagger(\alpha_c). \tag{7}$$

Applying this transformation to the total Hamiltonian operator, we find

$$H(\alpha_0) \equiv H_c(\alpha_c) = H_F(\alpha_c) + H_I(\alpha_c),$$
$$H_I(\alpha_c) = V(\alpha_c) + M_{ren}(\alpha_c) + V_{ren}(\alpha_c)$$
$$+\sum_{k=1}^{\infty}\frac{1}{k!}\left[R(\alpha_c), H_F(\alpha_c)\right]^k + \sum_{k=1}^{\infty}\frac{1}{k!}\left[R(\alpha_c), V(\alpha_c)\right]^k$$
$$+\sum_{k=1}^{\infty}\frac{1}{k!}\left[R(\alpha_c), M_{ren} + V_{ren}\right]^k, \tag{8}$$

where $[R,V]^k \equiv [R,[R,...[R,V]...]]$ with k-brackets.

Generator R has to be chosen in such a way that $H_c(\alpha_c)$ does not contain terms, called "bad", which prevent the one-particle states to be the eigenstates of the total Hamiltonian (e.g., M_{ren} and V), namely $H_I(\alpha_c)|\alpha_c^\dagger\Omega\rangle = 0$.

3. BAD TERMS REMOVAL AND MASS RENORMALIZATION

It is convenient to separate several types of operators in $H_c(\alpha_c)$. "Transition" operators, $O_{t,g}^{(n)}$ and $O_{t,b}^{(n)}$ of g^n-order, consist of more than three creation/destruction operators of any species. "g" and "b" mark "good" operators which refer to the physical processes and "bad" operators which prevent the one-particle states to be the eigenstates of the total Hamiltonian, respectively. "Mass-" and "vertex-like" operators, $O_{M_r}^{(n)}$ and $O_{V_r}^{(n)}$ of g^n-order, which replicate the structures of the mass and vertex counterterms M_{ren} and V_{ren}, respectively. Assuming the latter expanded in powers of g $M_{ren} = \sum_{k=1}^{\infty} M_{ren}^{(2k)}$, $V_{ren} = \sum_{k=1}^{\infty} V_{ren}^{(2k+1)}$, we expect the mass and charge corrections to have the same expansions.

After the clothing

$$H_c(\alpha_c) = H_F(\alpha_c) + [R, H_F] + V(\alpha_c)$$
$$+M_{ren}(\alpha_c) + [R,V]$$
$$+V_{ren}(\alpha_c) + \frac{1}{2}[R,V]^2 + [R, M_{ren}] + ..., \tag{9}$$

the Hamiltonian contains bad terms of all orders in g.

Thus, the generator $R = \sum_{k=1}^{\infty} R^{(k)}$ has all orders in g and contains same operator structures as "bad" terms.

The primary interaction operator V consists of the g^1-order bad terms $H_b^{(1)} \equiv V$. Therefore, we are going to define the generator $R^{(1)}$ in the following way [3]:

$$H_b^{(1)} + \left[R^{(1)}, H_F\right] = 0. \tag{10}$$

Adopting this definition, we have

$$H_c(\alpha_c) = H_F(\alpha_c) + \frac{1}{2}\left[R^{(1)}, V\right] + M_{ren}^{(2)}$$
$$+\frac{1}{3}\left[R^{(1)}, V\right]^2 + \left[R^{(1)}, M_{ren}^{(2)}\right] + V_{ren}^{(3)} + \dots. \tag{11}$$

The g^2-order contribution to $H_c(\alpha_c)$ is $\frac{1}{2}\left[R^{(1)}, V\right] + M_{ren}^{(2)}$ with the decompositions

$$M_{ren}^{(2)}\left(\delta m^{(2)}, \delta\mu^{(2)}\right) = M_{ren,g}^{(2)}\left(b^\dagger b, d^\dagger d, a^\dagger a\right)$$
$$+M_{ren,b}^{(2)}\left(d^\dagger b^\dagger,\ a^\dagger a^\dagger, H.c.\right);$$
$$\left[R^{(1)}, V\right] = \left[R^{(1)}, V\right]_{M_r,g}\left(b^\dagger b, d^\dagger d, a^\dagger a\right)$$
$$+\left[R^{(1)}, V\right]_{M_r,b}\left(d^\dagger b^\dagger,\ a^\dagger a^\dagger, H.c.\right)$$
$$+\left[R^{(1)}, V\right]_{t,g}\left(b^\dagger b^\dagger bb, b^\dagger a^\dagger ba, etc\right)$$
$$+\left[R^{(1)}, V\right]_{t,b}\left(b^\dagger b a^\dagger a^\dagger, etc\right). \tag{12}$$

Following [3,4], to determine the mass shifts in the g^2-order we require

$$M_{ren,g}^{(2)} + \frac{1}{2}\left[R^{(1)}, V\right]_{M_r,g} = 0, \tag{13}$$

finding the g^2-order mass corrections in the form

$$\delta\mu^{22} = \frac{2g^2}{(2\pi)^3}\int\frac{d\mathbf{p}}{E_\mathbf{p}} g_{1,2}(\mathbf{p}-\mathbf{k},\mathbf{p},\mathbf{k})$$
$$\times g_{1,2}(\mathbf{p}-\mathbf{k},\mathbf{p},-\mathbf{k})\left\{1 + \frac{\mu^4}{4(pk)2-\mu^4}\right\};$$
$$\delta m = \frac{g^2}{4m(2\pi)^3}\left[I_1(p) + I_2(p)\right],$$
$$I_1(p) = \int\frac{d\mathbf{k}}{\omega_\mathbf{k}} g_{1,1}(\mathbf{p},\mathbf{p}-\mathbf{k},\mathbf{k}) g_{1,1}(\mathbf{p},\mathbf{p}-\mathbf{k},-\mathbf{k})$$
$$\times pk\left\{\frac{1}{\mu^2 - 2pk} - \frac{1}{\mu^2 + 2pk}\right\},$$
$$I_2(p) = \int\frac{d\mathbf{q}}{E_\mathbf{q}} g_{1,1}(\mathbf{p},\mathbf{p}-\mathbf{k},\mathbf{k}) g_{1,1}(\mathbf{p},\mathbf{p}-\mathbf{k},-\mathbf{k})$$
$$\times\left\{\frac{m^2 - pq}{2\left[m^2 - pq\right] - \mu^2} + \frac{m^2 + pq}{2\left[m^2 + pq\right] - \mu^2}\right\}, \tag{14}$$

where $p = (E_\mathbf{p}, \mathbf{p})$, $q = (E_\mathbf{q}, \mathbf{q})$ and $k = (\omega_\mathbf{k}, \mathbf{k})$. To derive these expressions in the covariant form it is sufficient to put $g_{2,1}(\mathbf{p},\mathbf{p}-\mathbf{k},\mathbf{k}) = g_{1,2}(\mathbf{p}-\mathbf{k},\mathbf{p},-\mathbf{k})$ and $g_{2,1} = g_{1,1}$ which leads to $g_{2,1} = g_{1,2}$. The mass shifts derived are expressed through the three-dimensional integrals whose integrands depend on certain covariant combinations of the relevant three-momenta, providing the momentum independence of the mass renormalization. The present results prove to be equivalent to the results obtained by Dyson-Feynman technique if we put $g_{i,j} \equiv g$.

4. SOME GENERAL LINKS

Let us consider the momentum independence in question from a general point of view and look at the one-particle matrix elements

$$\langle \mathbf{k}'|S|\mathbf{k}\rangle = \langle \mathbf{k}'|\left[1 + S^{(1)} + S^{(2)} + ...\right]|\mathbf{k}\rangle, \quad (15)$$

to be definite with the spinless states $|\mathbf{k}\rangle = a^{\dagger}(\mathbf{k})|\Omega_0\rangle$ as in case of pion, where, e.g.,

$$\langle \mathbf{k}'|S^{(2)}|\mathbf{k}\rangle = -2\pi i\delta(\omega_{\mathbf{k}'} - \omega_{\mathbf{k}})\langle \mathbf{k}'|T^{(2)}(\omega_{\mathbf{k}})|\mathbf{k}\rangle, \quad (16)$$

with the second order T-operator

$$T^{(2)}(\omega_{\mathbf{k}}) = V(\omega_{\mathbf{k}} + i0 - H_F)^{-1} V. \quad (17)$$

To set links with previous discussions it is sufficient to note that on certain conditions the following relation

$$\frac{1}{2}\langle \mathbf{k}'|\left[R^{(1)}, V\right]|\mathbf{k}\rangle = \langle \mathbf{k}'|V(\omega_{\mathbf{k}} + i0 - H_F)^{-1} V|\mathbf{k}\rangle \quad (18)$$

holds within the model discussed if the pion mass $\mu < 2m$. In particular, it means that the propagator with the intermediate nucleon-antinucleon states in Eq. (18) is not singular for $\omega_{\mathbf{k}} > \mu$. Then, according to [3], the solution of equation (10) can be presented in integral form

$$R_1 = -i \lim_{\varepsilon \to 0+} \int_0^{\infty} dt V_D(t) e^{-\varepsilon t}, \quad (19)$$

and the proof of Eq. (18) becomes trivial.

Using the translational invariance of V, it is readily seen that

$$\langle \mathbf{k}'|V(\omega_{\mathbf{k}} + i0 - H_F)^{-1} V|\mathbf{k}\rangle = \frac{\delta(\mathbf{k}'-\mathbf{k})}{\omega_{\mathbf{k}}} G(\mathbf{k}). \quad (20)$$

Normally, $V = \int d\mathbf{x} V(\mathbf{x})$ where the interaction density $V(\mathbf{x})$ in the Dirac picture is the Lorentz scalar

$$U(\Lambda)V_D(x)U^{-1}(\Lambda) = V_D(\Lambda x). \quad (21)$$

Such introduction of interaction density may be inherent not only in a local field theory.

Then, exploiting Eq. (18) and the representation

$$(\omega_{\mathbf{k}} + i0 - H_F)^{-1} = -i \lim_{\varepsilon \to 0+} \int_0^{\infty} dt e^{i(\omega_{\mathbf{k}} + i\varepsilon - H_F)t}, \quad (22)$$

one can show that

$$G(\mathbf{k}) = -i(2\pi)^3 \lim_{\varepsilon \to 0+} \int_0^{\infty} dt e^{-\varepsilon t} \int d\rho \frac{\delta(\mathbf{k}'-\mathbf{k})}{\omega_{\mathbf{k}}}$$
$$\times \langle \Omega_0|a(k)V_D\left(\frac{1}{2}\rho\right)V\left(-\frac{1}{2}\rho\right)a^{\dagger}(k)|\Omega_0\rangle. \quad (23)$$

Here, as in [3], we are addressing the operators $a(k) = \sqrt{\omega_{\mathbf{k}}}\, a(\mathbf{k})$ that meet the covariant commutation rules

$$\left[a(k), a^{?}(k)\right] = \omega_{\mathbf{k}}\delta(\mathbf{k}' - \mathbf{k}). \quad (24)$$

This results in appearance of a typical combination

$$\frac{\delta(\omega_{\mathbf{k}'} - \omega_{\mathbf{k}})\delta(\mathbf{k}' - \mathbf{k})}{\omega_{\mathbf{k}}} C(k), \quad (25)$$

in the correspondent S-matrix element and one only needs to remember the relativistic invariance property

$$\frac{\langle k'|S|k\rangle}{\sqrt{\omega_{\mathbf{k}'}\omega_{\mathbf{k}}}} = \frac{\langle \Lambda k'|S|\Lambda k\rangle}{\sqrt{\omega_{\Lambda\mathbf{k}'}\omega_{\Lambda\mathbf{k}}}}. \quad (26)$$

This consideration gives us a possible (probably, general) way when finding the momentum independence of mass shifts within this three-dimensional formalism, at least, in first nonvanishing order in the coupling constant.

5. CONCLUSION

We have demonstrated here how the mass shifts in the system of interacting pion and nucleon fields can be calculated by the use of the clothed particle representation. The respective mass counterterms are compensated and determined directly in the Hamiltonian. We are dealing with the coincidence of the two divergent quantities: one of them is determined by the nucleon mass renormalization one-loop integral, while the other stems from the commutator $[R,V]$. We are trying to overcome this drawback by means of the introduction of the cutoff functions in momentum space. Such functions have certain properties conditioned by the basic symmetry requirements imposed upon the theory.

The procedure described above has an important feature, viz., the mass renormalization is made simultaneously with the construction of a new family of quasipotentials (Hermitian and energy independent) between the physical particles (the quasiparticles of the method). Explicit expressions for the quasipotentials can be found in [1].

By using a comparatively simple analytical means, we could show that the three-dimensional integrals, which determine the pion and nucleon renormalizations in the second order in the coupling constant g, can be written in terms of the Lorentz invariants composed of the particle three-momenta. In other words, these integrals are independent of the particle momenta.

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ПРЕДСТАВЛЕНИЕ ОДЕТЫХ ЧАСТИЦ В КВАНТОВОЙ ТЕОРИИ ПОЛЯ: ПЕРЕНОРМИРОВКА МАССЫ

В.Ю. Корда, А.В. Шебеко

Метод унитарных одевающих преобразований применен в модели квантовой теории поля, в которой нуклонное и нейтральное пионное поля взаимодействуют посредством регуляризованной псевдоскалярной связи типа Юкавы. В этом подходе массовые контрчлены частично сокращаются с коммутаторами генераторов одевающих преобразований и операторов взаимодействия, формируя пионные и нуклонные сдвиги массы. Найденные величины выражаются трехмерными интегралами от некоторых ковариантных комбинаций соответствующих импульсов частиц, что обеспечивает независимость рассчитанных поправок от импульсов. Определены условия, налагаемые на обрезающие вершинные функции, осуществляющие регуляризацию связи полей.

ЗОБРАЖЕННЯ ОДЯГНЕНИХ ЧАСТИНОК В КВАНТОВІЙ ТЕОРІЇ ПОЛЯ: ПЕРЕНОРМУВАННЯ МАСИ

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Метод унітарних одягаючих перетворень застосовано в моделі квантової теорії поля, в якій нуклонне і нейтральне піонне поля взаємодіють через регуляризований зв’язок типу Юкави. В цьому підході масові контрчлени частково скорочуються з комутаторами генераторів одягаючих перетворень і операторів взаємодії, що формує піонні та нуклонні зсуви мас. Знайдені величини подаються тривимірними інтегралами від деяких коваріантних комбінацій відповідних імпульсів частинок, що забезпечує незалежність розрахованих поправок від імпульсів. Визначено умови, що висуваються до вершинних функцій, які здійснюють регуляризацію зв’язку полів.