SELF-SIMILAR SOLUTIONS OF MULTI-DIMENSIONAL NONLINEAR SCHRÖDINGER EQUATIONS

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The method of a choice of self-similar variables for the description of multi-dimension wave collapse (WC) evolution is offered. It is based on the requirement that self-similar variables should provide preservation of an average square of radius of a wave package. Proposed self-similar substitutions do not break convertibility of the initial equation and allow us to present an average square of width of a wave package in a universal kind. In offered self-similar variables a spherical-symmetric stretched in the one of directions WC is investigated.

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The wave collapse (WC) is a disastrous in time (explosive) increase of an energy density in a diminishing volume. It is one of the fundamental phenomenons in wave propagation in dispersive media.

There are many examples of WC formation. It is, first of all, collapse of Langmuir waves [1], which appears under intensive plasma heating by electromagnetic radiation or electron bunches. It is considered by many researchers as one of the main mechanism of an energy dissipation of Langmuir waves on accelerated electrons. Further, it is worth to mention the phenomenon of the light self-focusing in nonlinear environments, theoretically predicted by G.A. Askarjan [2] and experimentally confirmed by N.F. Pilipetskiy and A.F. Rustamov [3], self-focusing weakly nonlinear hydrodynamic perturbations in a static space dusty background [4]. This is not a full list of scenarios of WC formation. All these processes are described by one of fundamental equations of a nonlinear physics − a nonlinear Schrödinger equation (NSE) and are of considerable scientific and practical interest.

In the multidimensional geometry (two-dimensional and three-dimensional) WC, as phenomenon of a formation of a singularity for a finite time, plays the same fundamental role, as a soliton in the one-dimensional geometry. In the last case, their formation is conditioned by the balance of two opposite processes − dispersive diffusion of wave packet and its nonlinear focusing. Moreover, this balance is stable.

The balance is broken under conditions of the multidimensional WC (multidimensional NSE). The system becomes nonstable, and nonlinear processes define the dynamics of WC. Study of WC (conditions of appearing and a temporal dynamics of an "explosive" increasing the wave amplitude) is important since these issues are crucial for estimation of the collapse efficiency, as nonlinear mechanism of wave energy transformations.

It should be noted that the development of WC in different media is the subject of numerous researches. For instance, in [1, 5, 6] conditions of the WC origin have been discussed in details and criteria of choose of a type of self-similar solutions for the numerical study of NSE have been received. As a rule, the numerical calculations are required a lot of computing resources (CPU time and an operative memory) and also are connected with the development of complex multidimensional difference algorithms, which are unstable (do not provide the conservation of NSE integrals). Last fact proves to be true, for example, results of the work [6], where there is a big discrepancy of the specified value of initial amplitude (Fig.1,a) and received from Fig.2,a arithmetically. This discrepancy appears, in our opinion, because of there is no the conservation of NSE integrals in self-similar variables, which used in [6]. In our opinion the listed above lacks can be removed by the correct choice of self-similar variables which take into account internal properties of the system and provides the conversation of NSE integrals.

In the present paper the conditions of origin and temporary dynamics of the two-dimensional and three-dimensional WC are investigated on the basis NSE and consequences of its integrals.

1. THE NONLINEAR SCHRÖDINGER EQUATION AND ITS INTEGRALS

NSE for the wave function in dimensionless variables has the form [1, 5]:

$$i\psi_t + \Delta\psi + |\psi|^2 \psi = 0, \qquad (1)$$

where: Δ − is an Laplas operator, the lower index marks a time partial derivative.

The equation (1) is a convertible: at replacements $t \to -t$ and $\psi(\vec{r},-t) \to \psi^*(\vec{r},t)$, where the sign (*) marks a complex conjugation, the form of the equation does not change.

Under the theoretical description and analysis of the wave processes in a plasma, the equation (1) can be received from nonlinear wave equation for potential by averaging the initial system of the equations, describing nonlinear media on fast oscillations of the frequency $\omega(k_0)$, corresponding to the centre of the wave package, where k_0 − is the wave number. The analogue of NSE in hydrodynamics is the equation, describing propagation of the wave package on the surface of a water, which has been obtained by V.E. Zakharov and A.B. Shabbat [7].

From the equation (1) it is easy to get the first integral of the motion − a wave action or "number of the particles"[1], which is valid for the model of any dimensions:

$$N = \int_{-\infty}^{\infty} |\psi|^2 \, d\vec{r} = const > 0. \qquad (2)$$

This integral is proportional to the energy of the wave package.

The second motion integral can be received from the equation (1) in two- and three-dimensional cases and is a Hamiltonian of the system:

$$H = \int_{-\infty}^{\infty} |\nabla\psi|^2 \, d\vec{r} - \frac{1}{2}\int_{-\infty}^{\infty} |\psi|^4 \, d\vec{r} \equiv X - Y = const\,. \quad (3)$$

Besides, from equation (1) it is possible to get the functional

$$\langle r^2 \rangle = N^{-1} \cdot \int_{-\infty}^{\infty} r^2 \cdot |\psi|^2 d\vec{r}\,, \quad (4)$$

which defines the average value of the squared radius of the wave package.

However, dependency of this functional on a time has its specifics for two- and three-dimensional NSE. So, below, we study the temporary dynamics of multi-dimensional WC.

2. THE TWO-DIMENSIONAL WAVE COLLAPSE

In 1971 for two-dimensional NSE the Vlasov-Petrischev-Talanov criterion of the wave collapse have been found [8]. It was the fundamental result in theories of waves collapse. It was the first, rigorous result for nonlinear wave systems with dispersion. It was shown that a formation of a singularity for a finite time in media is possible, in spite of a linear dispersion of the waves, which, for instance, in linear optics prevents arising singularity points - focuses.

The criterion of a wave collapse follows from the relation for the second derivative in time of the average squared radius of a wave package:

$$\frac{d^2}{dt^2}\int_{-\infty}^{\infty} r^2 |\psi|^2 \, d\vec{r} = 8 \cdot H. \quad (5)$$

Since the Hamiltonian H is a constant, the equation (5) can be twice integrated:

$$\int_{-\infty}^{\infty} r^2 |\psi|^2 \, d\vec{r} = 4 \cdot H \cdot t^2 + C_1 \cdot t + C_2\,. \quad (6)$$

The constants C_1, C_2 are additional integrals of the motion, which are defined by initial data [8]:

$$C_2 = \langle r^2 \rangle\Big|_{t=0} > 0\,,\; C_1 = \frac{\partial}{\partial t}\langle r^2 \rangle\Big|_{t=0}\,.$$

From equation (6) one can obtain the criterion of Vlasov-Petrischev-Talanov, which states that in the system with the negative Hamiltonian $H < 0$, at arbitrary C_1 and C_2, average squared width of the wave package $\langle r^2 \rangle$ vanishes for a certain finite time t_0. This indicates existence of a singularity in the wave function $\psi(\vec{r},t)$.

In [9] on the basis of complicated numerical experiments the automodel type of dependency of the field on time $\psi(\vec{r},t)$ in vicinity of the singularity point has been obtained in the form

$$|\psi(r,t)| \to \frac{1}{f(t_0 - t)} R\left(\frac{r}{f(t_0 - t)}\right),\; f(0) = 0\,, \quad (7)$$

where the function $R(\xi)$ coincides with the form of a standard two-dimensional soliton.

In the present paper we show the method of the choice of the self-similar substitution, which allows us to study the properties of the wave function $\psi(\vec{r},t)$ on the stage of WC. For this, consider the equation (6) on the assumption that t_0 is a root of the right-hand part of (6):

$$\int_{-\infty}^{\infty} r^2 |\psi|^2 \, d\vec{r} = 4 \cdot H \cdot \left(t^2 - t_0^2\right). \quad (8)$$

The right-hand part of the eq. (8) takes positive magnitudes at $t < t_0$ only for the negative Hamiltonian $H < 0$. Consequently, as it was noted above, instability develops for the negative Hamiltonian (under $H > 0$ a destruction of the wave package occurs). In this regards it is convenient to rewrite the eq. (8) in such a view:

$$\int_{-\infty}^{\infty} r^2 |\psi|^2 \, d\vec{r} = 4|H| \cdot \left(t_0^2 - t^2\right). \quad (9)$$

To study the properties of the wave function we choose such a radial self-similar variable, which satisfies the constancy of the average squared radius. These requirements are satisfied by the substitution:

$$\xi = \frac{r}{\sqrt{t_0^2 - t^2}}\,,\; \psi'(\xi) = \psi(r,t)\sqrt{t_0^2 - t^2}\,, \quad (10)$$

where ξ – is a new self-similar variable and $\psi'(\xi)$ is a new wave function in new variables. Then the eq. (9) takes the form:

$$\int_{-\infty}^{\infty} \xi^2 |\psi'|^2 \, d\vec{\xi} = 4 \cdot |H|\,. \quad (11)$$

Since N and H are constants, not dependent on time values, from eq. (11) follows the constancy of the average squared radius of a wave package $\langle r^2 \rangle = 4|H|/N$.

Let's find key parameters of the module of wave function $|\psi'(\xi)|$ starting from the expression (11).

For this purpose we suppose, that the module of wave function is set in the kind of Gauss package:

$$|\psi'(\xi)| = A \cdot \exp(-B^2 \cdot \xi^2)\,, \quad (12)$$

where A and B – are the amplitude and radial width of the wave package respectively. Then, substituting (12) in equality (11), we shall receive the following ratio between parameters of wave function:

$$B = A^{\frac{1}{2}} / 2 \cdot (2 \cdot |H|)^{\frac{1}{4}}\,. \quad (13)$$

In a Fig.1 the dependences of relative amplitudes of wave functions on dimensionless time t/t_0 in various points of space r_i (i=1, 2, 3) for parameters of wave function $A = 1;\, H = -0{,}06;\, B = 0{,}85$ are shown.

From Fig.1 one follows, that near the singularity point (the curve 1) the initial amplitude of the wave function grows in the explosive-like manner, i.e. WC develops. At moving from the singularity point, the amplitude of the wave function grows slower in time (a curve 2), and then vanishes. At the some distance from the singularity (a curve 3) the wave function is a monotonous decreasing function of a time.

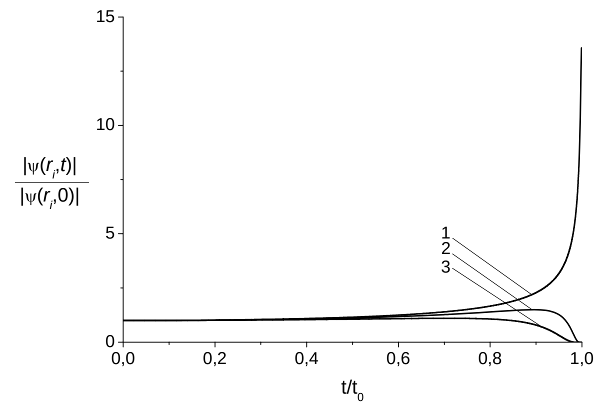

Fig.1. The dependence of the relative amplitude of the self-similar solution of two-dimensional NES on a time in different points of the space: 1 – $r_1 = R1/R_0 << 1$; *2 –* $r_2 = 10\, R1/R_0$; *3 –* $r_3 = 50\, R1/R_0$, *here* R_0 *– is a typical size of the wave package*

Thus, for the first time, the method of the choice of self-similar variables for the analysis of the two-dimensional WC development is offered. It is in choise of self-similar variables in term of which requirement the conservation of the average squared radius of the wave package should hold. Such self-similar variables provide transition to such a system of frame where WC is absent.

3. THE SPHERICAL-SYMMETRIC WAVE COLLAPSE

Let's consider a spherical-symmetric WC. In this case the expression for the second derivative in time of the average square of radius of the symmetric wave package (wave function does not depend on polar coordinates) takes the form [5]

$$\frac{d^2}{dt^2} \int_{-\infty}^{\infty} r^2 |\psi|^2 \, d\vec{r} = 4\left(2H - Y\right), \qquad (14)$$

where Hamiltonian H is an invariant.

For the development of the spherical-symmetric WC in time the evaluation was received [5]:

$$N \cdot \left\langle r^2 \right\rangle < 4H \cdot t^2 + C_1 \cdot t + C_2, \qquad (15)$$

where C_1 and C_2 are constants of integration.

The lack of the criterion (15) is in its representation as an inequality. Let's receive more strict condition of the spherical-symmetric WC development. For this we enter the scale transformation (stretching) of the space $r \to \alpha \cdot r$, where α – is a factor of resemblance. The law of transformation of the wave function $\psi\left(\vec{r}, t\right)$ in this case we define from condition of the conservation of the integral N:

$$\psi\left(\alpha \cdot \vec{r}, t\right) = \alpha^{-\frac{3}{2}} \psi\left(\vec{r}, t\right). \qquad (16)$$

Then, supposing in (14) $r \to \alpha \cdot r$ and considering the law of the wave function transformation (16) it is easy to get the criterion of the development spherical-symmetric collapse in suitable for analysis king as a result of simple calculation

$$N \cdot \left\langle r^2 \right\rangle = \frac{12}{7} H \cdot t^2 + C_1' \cdot t + C_2', \qquad (17)$$

where C_1' and C_2' are constants of the integration.

The expression (17) is received in suggestion of the small sprain of the space: the factor of the resemblance is given in the kind of $\alpha \equiv 1 + \varepsilon$, where ε – is a small value, in the limit tending to zero ($\varepsilon \to 0$), and expression (14) is decomposed with a small parameter series by ε with the subsequent allocation of members of the equal order. Here necessary to note that expression (17) does not contradict with more general inequality (15).

The view of the criterion (17) allows us to study the three-dimensional WC in analogy with the two-dimensional one. The following self-similar variables and the amplitude satisfy to requirements of the conservation of the average squared radius of the wave package:

$$\xi = \left(\frac{3}{7}\right)^{-1/2} \frac{r}{\sqrt{t_0^2 - t^2}}, \psi'\left(\xi\right) = \left(\frac{3}{7}\right)^{3/4} \psi\left(r, t\right)\left(t_0^2 - t^2\right)^{3/4}. \qquad (18)$$

The eq. (17) taking into account of (18) becomes:

$$\int_{-\infty}^{\infty} \xi^2 \left|\psi'\right|^2 d\vec{\xi} = 4\left|H\right|, \qquad (19)$$

and completely coincides with the equation for a two-dimensional WC.

Starting from an self-similar form of the wave function (18), it is possible to make a conclusion that for a case $D = 3$ the collapse develops faster, than for $D = 2$, where D – is a dimension of the space. Besides the chosen form of self-similar replacements for $D = 2; 3$ keeps convertibility of the equation (1).

As it can be seen from derived results the offered self-similar substitution allows us to present the average square of the radius of the wave package in universal king: both for the two-dimensional and the three-dimensional (spherical symmetric) case the average square of radius of a wave package is constant and is equally expressed through the Hamiltonian of the systems.

Acting in the same way as it is made for two-dimensional WC, we result the expression describing a relation between the amplitude and width of a wave package for the spherical-symmetric WC:

$$B = \frac{(3\sqrt{\pi})^{\frac{1}{5}}}{2\sqrt{2}} \cdot \left|H\right|^{-\frac{1}{5}} \cdot A^{\frac{2}{5}}. \qquad (20)$$

Thus, the WC in the spherical-symmetric case, as well as in two-dimensional one, is formed for negative Hamiltonian, but with other values of the wave package parameters (12).

The dependences of relative amplitudes of wave functions on dimensionless time t/t_0 in various points of space r_i (i=1, 2, 3) for a case $A = 1; H = -0{,}188$; $B = 0{,}69$ are similar to Fig.1, but amplitude in spherical-symmetric case is more in two times than in two-dimensional one at $t/t_0 \to 1$.

4. THREE-DIMENSIONAL AXIAL-SYMMETRIC WAVE COLLAPSE

For analysis of the three-dimensional axial-symmetric wave package choose the element of the volume in expressions (2)-(4) in the view $d\vec{r} = rdrdzd\varphi$ (the wave function depends also on the coordinate z). In this case the radial self-similar variable and the self-similar amplitude providing the requirements of the constancy of the average square of the radius of the wave package $\left\langle r^2 \right\rangle$ are defined by expressions:

$$\xi = \frac{r}{\sqrt{t_0^2 - t^2}}, \; \eta = \frac{z}{t_0^2 - t^2},$$
$$\psi'(\xi, z) = \psi(r, z, t) \cdot (t_0^2 - t^2) \; . \qquad (21)$$

The eq. (1) in terms of the variables (21) takes the form:

$$it(t_0^2 - t^2)((\xi \frac{\partial \psi'}{\partial \xi} + 2\eta \frac{\partial \psi'}{\partial \eta})(t_0^2 - t^2) + 2\psi') + + \frac{1}{\xi} \frac{\partial}{\partial \xi} \xi \frac{\partial \psi'}{\partial \xi}(t_0^2 - t^2) + \frac{\partial^2 \psi'}{\partial \eta^2} + \psi' |\psi'|^2 = 0. \qquad (22)$$

Since in the chosen self-similar variable and amplitude (21) the average square of radius of a wave package is a constant, it is possible to put $\partial \psi'(\xi,\eta) / \partial \xi = 0$ in the equation (22), i.e. to consider, that wave function $\psi'(\xi,\eta) = \psi'(\xi_*,\eta)$ does not depend on a radial self-similar variable ξ . Here $\xi = \xi_*$ is an arbitrary point in the region of existence of the wave function.

Besides, from a constancy of integral (2) follows, that in eq. (22) parameter t will be equal to zero.
Then the eq. (22) is converted into the one-dimensional stationary nonlinear Schrödinger equation:

$$\frac{d^2 \psi'(\xi_*, \eta)}{d\eta^2} + \psi'(\xi_*, \eta) \cdot |\psi'(\xi_*, \eta)|^2 = 0. \qquad (23)$$

The solution of the equation (23) in general case is expressed through the Weierstraß's function of the imaginary argument. However for the analysis of the WC development in an axial direction it is enough to find the solution in a class of real functions. In this case the solution of the equation (23) in terms of variable z is expressed through the elliptic Jacobi's function $cn(x,k)$ [10]:

$$\psi'(\xi_*, z) = \psi'(\xi_*, 0) \cdot cn(\psi'(\xi_*, 0) \cdot \frac{z}{t_0^2 - t^2}, \frac{1}{\sqrt{2}}), \qquad (24)$$

where: $\psi'(\xi_*, 0)$ – the amplitude, $k = 1/\sqrt{2}$ -the module.

Choose the boundary conditions for the wave function (24) in axial direction, for instance, as a requirement of vanishing the amplitude of the wave function at the value of the argument corresponding to the first period of the elliptic Jacobi's function:

$$\psi'(\xi_*, z_i) = \psi'(\xi_*, 0) \cdot cn(h_i, \frac{1}{\sqrt{2}}) = 0, \qquad (25)$$

where: $i = 1,2$;

$h_{1,2} = \psi'(\xi_*, 0) \cdot \frac{z_{1,2}}{t_0^2 - t^2} = \pm K(\frac{1}{\sqrt{2}}) = \pm 1,8541$; $K(k)$ is the Legendre's elliptic integrals of the first kind.

Then from expression (25) follows, that the borders of a wave package on z-axis (zeros of the Jacobi's function) $z_i = \pm 1,8541 \cdot (\psi'(\xi_*, 0))^{-1} \cdot (t_0^2 - t^2)$ are shifted with the time to the beginning of coordinates, i.e. the axial WC develops.

Two possible scenarios of the WC development follow from the expression (24).

The first scenario is realized in the case when the characteristic longitudinal size of a wave package exceeds the radial (the sphere stretched along the axis z). Then the dynamics of development of the axial WC which described by expression (24) will be prolonged until the axial half-width of WC will coincide with its radius. In this case WC passes in a spherical-symmetric stage and the degree of singularity increases from 1/2 up to 3/4.

The second scenario can develop when the characteristic longitudinal size of the wave package along the z – axis less then the radial one. In this case according to expression (24) wave package having originally a slightly compressed sphere will be transformed into a «presolar disk» [11].

The above mentioned statement about development of the WC in the axial direction proves to be true in the next experimental facts on which we stop below.

Fig.2. Formation of the star system NGC 1333-IRAS 4B

The Fig.2 shows the artistic interpretation (in inverse black and white color) of the embryonic star, called NGC 1333-IRAS 4B (figured in center of image) [12], which have been observed by the NASA's Spitzer Space Telescope. NGC 1333-IRAS 4B is located in a pretty star-forming region, which is approximately 1,000 light-years away in the constellation Perseus. Its central stellar embryon is still "feeding" off the material collapsing around it and growing in size. The cold toroidal cocoon, including ice, surrounds the embryonic star which is in a state of the axi-symmetric WC.

Microwave Radiation [13] is quite corresponds to the concept of the axial-symmetric three-dimensional WC, when the radial rate of expansion of the Universe differ from the axial one.

In summary we shall note, that the time dynamics of the WC in the offered self-similar variables (10), (18), (21) is convertible, the same as the initial equation (1). Therefore, it is quite probable, that WC can be not only a final state of evolution of a wave package, but also the initial one. Besides the anisotropy of the Relic In conclusion it is necessary to note that the present theory gives only follow the dynamics of self-focusing singularities until the condition of weak nonlinearity of NSE is valid. It is possible, that the further research of the formed system as a result of development three-dimensional WC will serve as a key to understanding of the formation of the star systems, galaxies and the origin of the solar system [13]. But these questions require special studies.

CONCLUSIONS

The method of a choice of self-similar variables for the description of the development of multidimensional WC is proposed. It consists of choice of the self-similar variables which should provide preservation of an average square of radius of a wave package. Such self-similar variables mean the transition to such a frame in which WC is absent.

The suggested self-similar substitutions in two- and three-dimensional cases (axi-symmetric and spherical – symmetric collapses) allow us to write an average square of a wave package radius in a universal kind. Both for the two-dimensional and the three-dimensional (spherical symmetric) case the average square of radius of a wave package is constant and is equally expressed through the Hamiltonian of the system. In physical variables (radius, time) the rate of development spherical - symmetric WC is higher than for two –dimensional one. At the development axial-symmetric three-dimensional WC the explosive increase in amplitude of wave function occurs along an axis of the system. Thus two scenarios of the development of WC are possible.

The first scenario is realized in that case when the characteristic longitudinal size of a wave package exceeds the radial (the sphere stretched along the axis z). Then the dynamics of development of the axial WC will be prolonged until axial half-width WC will coincide with its radius. In this case WC passes in a spherical – symmetric stage and the radial degree of singularity increases from 1/2 up to 3/4.

The second scenario can develop when the characteristic longitudinal size of the wave package less then the radial (a sphere compressed along z -axis). In this case wave package having originally a form of a slightly compressed sphere will be transformed into a «presolar disk».

The time dynamics of the WC in the offered self-similar variables is convertible, the same as the initial NSE equation. Therefore, it is quite probable, that WC can be not only a final state of evolution of a wave package, but also the initial one. Probably, the theory of the Big Bang and the Expansion of the Universe are the confirmation of it. Besides the anisotropy of the Relic Microwave Radiation is quite corresponds to the concept of the axial-symmetric three-dimensional WC, when the radial rate of expansion of the Universe differ from the axial one.

The present theory gives only follow the dynamics of self-focusing singularities until the condition of weak nonlinearity of NSE is valid. It is possible, that the further research of the system formed as a result of the development WC will serve as a key for understanding of the formation of the star systems, galaxies and the origin of the solar system. But these questions require special studies.

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АВТОМОДЕЛЬНЫЕ РЕШЕНИЯ МНОГОМЕРНЫХ НЕЛИНЕЙНЫХ УРАВНЕНИЙ ШРЕДИНГЕРА

С.Ф. Скоромная, В.И. Ткаченко

Предложен метод выбора автомодельных переменных для описания развития многомерного волнового коллапса (ВК). Он заключается в том, что искомые автомодельные переменные должны обеспечивать сохранение среднего квадрата радиуса волнового пакета. Предложенные автомодельные подстановки не нарушают обратимости исходного уравнения и позволяют представить средний квадрат радиуса волнового пакета в универсальном виде. В предложенных автомодельных переменных исследована временная динамика сферически-симметричного, растянутого вдоль одного из направлений ВК.

АВТОМОДЕЛЬНІ РІШЕННЯ БАГАТОВИМІРНИХ НЕЛІНІЙНИХ РІВНЯНЬ ШРЕДИНГЕРА

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Запропоновано метод вибору автомодельних змінних для опису розвитку багатовимірного хвильового колапсу (ХК). Він полягає в тім, що шукані автомодельні змінні повинні забезпечувати збереження середнього квадрата радіуса хвильового пакета. Запропоновані автомодельні підстановки не порушують оборотності вихідного рівняння й дозволяють представити середній квадрат радіуса хвильового пакета в універсальному виді. У запропонованих автомодельних змінних розглянута часова динаміка сферично-симетричного, розтягнутого уздовж одного з напрямків ХК.