TRANSITION RADIATION OF THE CYLINDRICAL ELECTRON BUNCH AT THE SHARP VACUUM-PLASMA BORDER

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Transition radiation of the cylindrical electron bunch moving through the border between the vacuum and isotropic plasma semi-spaces is studied. Radiation patterns' frequency dependencies, integrated radiated power and total radiated energy are calculated.

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1. INTRODUCTION

Electromagnetic waves radiation excited by the electron beam that is injected into the ionospheric or magnetospheric plasma is one of the most interesting results of the space beam-plasma experiments [1]. In such experiments, splashes of electromagnetic radiation with the wideband frequency spectrum were observed at the moments of electron beam's fronts injection [2]. Transition radiation is one of the possible mechanisms of this radioemission. Particularly, the fronts of electron beam have a wideband spectrum. Due the linear transformation of the current density waves into electromagnetic waves on the plasma inhomogeneity, these fronts can excite the splashes of electromagnetic waves. This radiation is characterized by the frequencies' spatial selection effect [3] when each frequency is radiated mostly under its own angle. Transition radiation of the cylindrical electron bunch on the smooth plasma concentration profile was treated in [4], but only the time dependence of the total radiated power was calculated.

2. MODEL DESCRIPTION

The sharp border between the vacuum and isotropic plasma semi-spaces is considered. The cylindrical electron bunch with current density

$$\bar{j} = \begin{cases} \bar{e}_z j_0 \exp[i(\omega_0 t - \chi_0 z)], \text{ if } t \in [\frac{z - L/2}{v_0}, \frac{z + L/2}{v_0}] \text{ and } r \le a \\ 0, \quad \text{if} \quad t \notin [\frac{z - L/2}{v_0}, \frac{z + L/2}{v_0}] \text{ or } r > a \end{cases} \qquad (1)$$

moves normally across such border (L, a, v_0, j_0, ω_0 - length, radius, velocity, magnitude and frequency of electron bunch correspondingly; $\chi_0=\omega_0/v_0$; r,φ,z – cylindrical coordinates, z is a perpendicular to border).

3. CALCULATION OF THE RADIATED FIELD

One can expand expression (1) for the bunch current density in the Fourier integral for time t (frequency ω) and in the Fourier-Bessel integral for transversal coordinate r (transversal wavenumber χ). So the temporal and spatial spectrum of electron bunch current density will have the appearance of

$$j(\omega, \chi, z) = \frac{2 j_0 a}{(\omega_0 - \omega)\chi} J_1(\chi a) \sin\left[(\omega_0 - \omega)\frac{L}{2v_0}\right] \exp[-i\omega \frac{z}{v_0}], \qquad (2)$$

where $J_1(x)$ – first order Bessel function. Using this expression and Maxwell equations, one can obtain the spectrum of the eigen electromagnetic field of electron bunch:

$$E_z^{(e)} = \frac{-4\pi i\omega}{c^2(\chi^2 + \frac{\omega^2}{v_0^2}(1 - \varepsilon \frac{v_0^2}{c^2}))} j(\omega, \chi, z), \qquad (3)$$

where ε - plasma dielectric permittivity. Equations set for the radiation field components is formed according with the expression (3) and electromagnetic boundary conditions. So for the radiated field one can obtain:

$$E_{z1}^{(w)} = \frac{-E_{z1}^{(e)}(\alpha_2\varepsilon_1 + \gamma_1\varepsilon_2) + E_{z2}^{(e)}\varepsilon_2(\alpha_2 + \gamma_2)}{\alpha_2\varepsilon_1 + \alpha_1\varepsilon_2};$$

$$E_{z2}^{(w)} = \frac{E_{z1}^{(e)}\varepsilon_1(\alpha_1 - \gamma_1) + E_{z2}^{(e)}(\varepsilon_1\gamma_2 - \alpha_1\varepsilon_2)}{\alpha_2\varepsilon_1 + \alpha_1\varepsilon_2}, \qquad (4)$$

where $E_{z1}^{(w)}$ and $E_{z2}^{(w)}$ – z-components of the forward and backward radiated electromagnetic wave field correspondingly, and:

$$\alpha_{1,2} = \frac{1}{k_{z1,2}(1 - \varepsilon_{1,2}\frac{\omega^2}{k_{z1,2}^2 c^2})}; \quad \gamma_{1,2} = \frac{v_0}{\omega(1 - \varepsilon_{1,2}\frac{v_0^2}{c^2})};$$

$$k_{z1,2} = \sqrt{\frac{\omega^2}{c^2}\varepsilon_{1,2} - \chi^2} \qquad (5)$$

4. RADIATION PATTERNS IN DEPENDENCE ON FREQUENCY

Expressions (4-5) can be integrated by the transversal wavenumber χ so the radiated field dependence on radiation angle θ and frequency ω will be obtained. This integration is performed using stationary phase method. Corresponding density plots (darker color represents larger value, lighter - smaller) are shown on Fig.1-3.

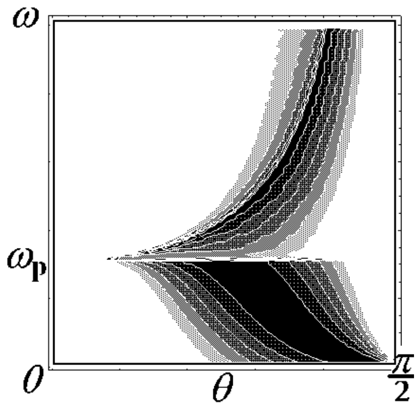

Fig.1. Radiation patterns into vacuum as a function of frequency

Plot on Fig.1 represents the transition radiation into vacuum. Radiation patterns in this case are strongly different on the frequencies above and below the Langmuir frequency of the plasma semi-space (ω_p). There is a sharp maximum in the band above ω_p that corresponds to the excitation of the border's quasi-eigen mode (Such a mode exists near the border, can not propagate in one of the semi-spaces, and emanates into another semi-space at the angle of the total reflection). Below the ω_p, radiation patterns have less sharp maximum that is formed as a result of the interference between the waves with different transversal wavenumbers.

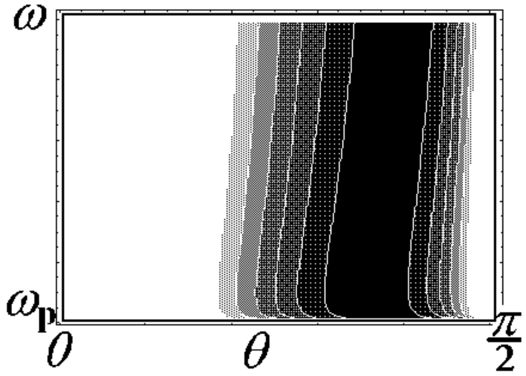

Fig.2. Radiation patterns into plasma as a function of frequency

Fig.2 shows the similar dependence for the radiation into plasma. This radiation takes place only in the frequency band above ω_p. Radiation patterns in this case have only the interference maximum that is relatively smooth.

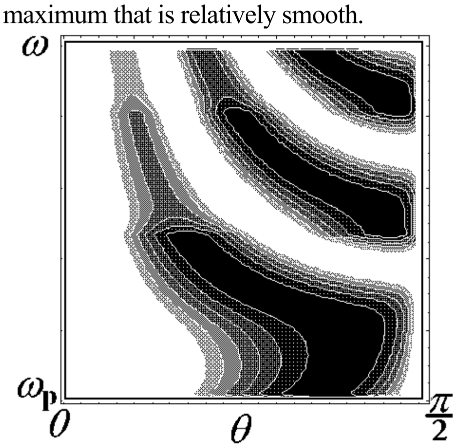

Fig.3. Radiation patterns into plasma in the wide frequency band

For wider frequency band, radiation patterns into plasma are shown on Fig.3. On high frequencies, these patterns contain the additional maxima due the interference effects. If the width of the bunch grows, corresponding frequencies become smaller. The number of the interference maxima grows when the ratio of the bunch radius to the radiated wavelength increases.

In all cases, angle of the most efficient radiation vary depending on frequency due the effect of spatial selection of the transition radiation [3].

5. TOTAL RADIATED POWER AND ENERGY

The energy flux of the transition radiation can be obtained via the integration of the dependencies that are shown above. This integration is performed numerically.

Form of the radiated pulse on different radiation angles for the radiation into vacuum is shown on Fig.4. This pulse keeps its form during the propagation. Sharp splashes on this figure correspond to the moments when the bunch fronts move through the border. Non-zero level of the radioemission between these splashes can be explained similarly to the radiation of the collapsing dipole in the case of the vacuum-metal border.

Fig.5 shows the similar dependencies for the case of the radiation into plasma. In this case the sharp splashes that correspond the bunch fronts are also present. Contrary the previous, these pulses are blurring during the propagation as a result of plasma dispersion.

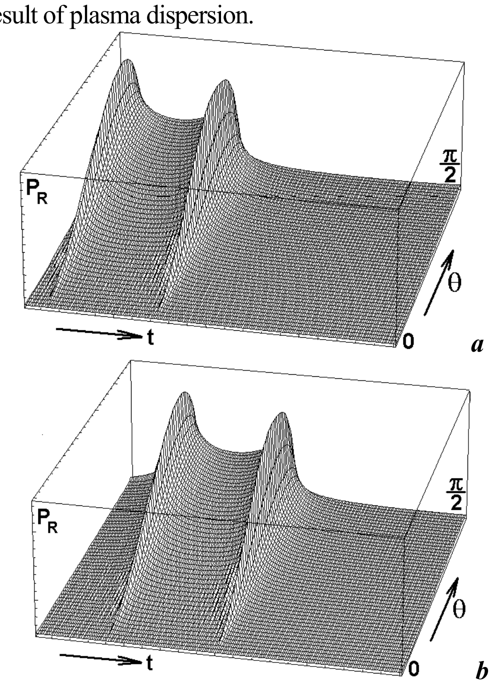

Fig.4. Energy flux of the integrated transitional radiation into vacuum as a function of time and angle (distance from border in case b is larger than in case a)

Level of the radiated power between the front splashes is smaller in comparison with the vacuum case (at the time when blurring is not yet sufficient) because most of the power of collapsing dipole-type radiation lies in the spectral band below the Langmuir frequency ω_p.

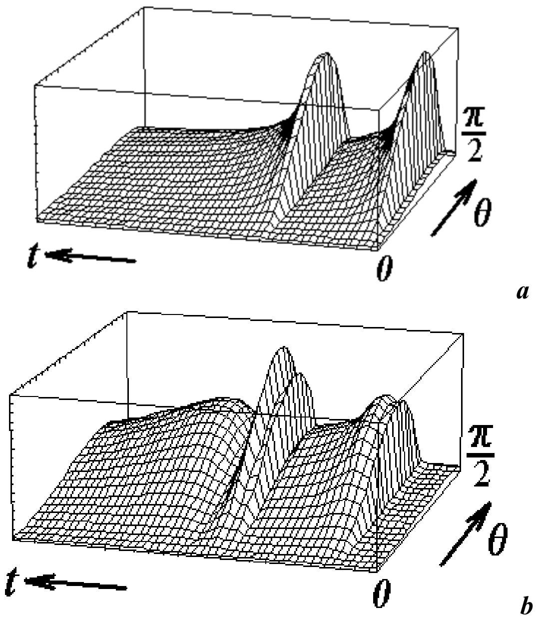

Fig.5. Energy flux of the integrated transitional radiation into plasma as a function of time and angle (distance from border in case b is larger than in case a)

To obtain the total radiated energy, the integral of radiated power over all angles and time was calculated. Fig. 6-8 show the dependencies of this total energy upon the model parameters.

The dependence of the total energy upon the bunch length is shown on Fig.6. For the case of the radiation into vacuum (Fig.6a) this energy grows when the length increases.

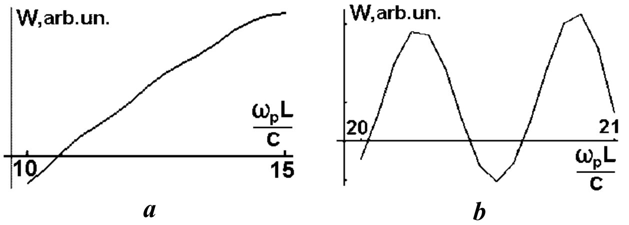

Fig.6. Total radiated energy as a function of bunch length (a – radiation into vacuum, b - into plasma)

That fact can be explained as the integral of the collapsing dipole-type radiation between the front splashes. Contrary, for the radiation into plasma (Fig.6b) most of the radiated power is contained in the front pulses so average level of the radiated energy remains almost constant. Both dependencies have non-monotonic component that can be explained as the interference between the radiation from bunch fronts. This effect is more efficient for the radiation into plasma because in that case more spectral components are superposable.

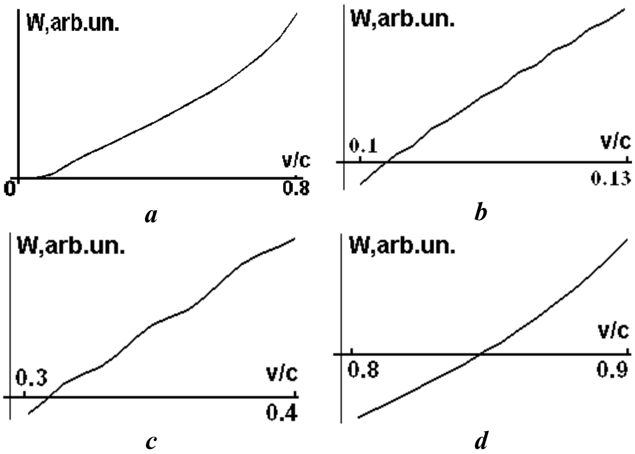

Fig.7. Total radiated energy into vacuum as a function of bunch velocity

The dependence of the total energy of the transition radiation into vacuum upon the bunch velocity is shown on Fig. 7. Particularly, Fig. 7a shows this dependence in the wide band of velocities. Total radiated energy grows when the bunch velocity increases. Partial plots (Fig. 7b-d) represent this dependence in more narrow bands. At the relatively small velocities (Fig. 7b) the non-monotonic component of that dependence has a small and short pulsations. The length and magnitude of these pulsations increases at the average bunch velocities (Fig. 7c) because the wavelength on given frequency also increases. In the relativistic band these pulsations become insignificant relatively to the fast growth of average radiated energy (Fig. 7d).

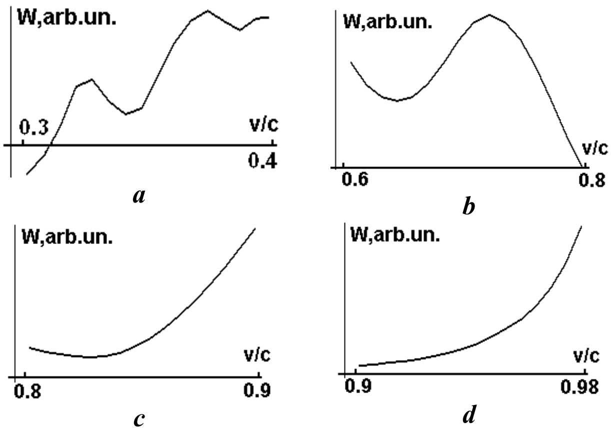

Fig.8. Total radiated energy into plasma as a function of bunch velocity

Similar dependencies for the case of the radiation into plasma are shown on Fig. 8, where average radiated energy and length of pulsations also increases with velocity growth. Pulsations' magnitude in this case is larger in comparison with the radiation into vacuum because the interference between the radiation from bunch fronts is more efficient in plasma.

6. CONCLUSIONS

The transition radiation of the electron bunch on the sharp vacuum-plasma border has the radiation patterns those are strongly dependent on frequency. The character of these patterns is significantly different above and below the Langmuir frequency of plasma that can be useful for this plasma diagnostics. There is the collapsing dipole-type radiation into vacuum between the front splashes. The total radiated energy dependence upon the parameters has the non-monotonic component that is related to the interference between the transition radiation of bunch fronts.

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ПЕРЕХІДНЕ ВИПРОМІНЮВАННЯ ЦИЛІНДРИЧНОГО ЕЛЕКТРОННОГО ЗГУСТКУ НА МЕЖІ ПОДІЛУ ВАКУУМ-ПЛАЗМА

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Досліджено перехідне випромінювання циліндричного електронного згустку, що проходить через межу поділу вакууму та ізотропної плазми. Розраховано залежність діаграми спрямованості випромінювання від його частоти, а також інтегральну потужність та повну енергію випромінювання

ПЕРЕХОДНОЕ ИЗЛУЧЕНИЕ ЦИЛИНДРИЧЕСКОГО ЭЛЕКТРОННОГО СГУСТКА НА ГРАНИЦЕ РАЗДЕЛА ВАКУУМ-ПЛАЗМА

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Исследовано переходное излучение цилиндрического электронного сгустка, проходящего через границу раздела вакуума и изотропной плазмы. Рассчитана зависимость диаграммы направленности излучения от его частоты, а также интегральная мощность и полная энергия излучения.