INSTABILITY OF ION-CYCLOTRON WAVES IN A 2D MAGNETOSPHERIC PLASMA WITH ANISOTROPIC TEMPERATURE

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Dispersion equation is analyzed for field aligned ion-cyclotron waves in a two-dimensional (2D) magnetospheric plasma with circular magnetic field lines. The steady-state bi-Maxwellian distribution function is used to model the energetic protons in a hydrogen plasma at the geostationary orbit. As in the uniform magnetic field, the growth rate of the proton-cyclotron instability (PCI) in a 2D magnetospheric plasma is defined by the contribution of the energetic ions/protons to the imaginary part of the transverse permittivity elements. It is shown that the PCI growth rate in 2D axisymmetric magnetosphere can be significantly smaller than that is for the straight magnetic field case with the same macroscopic bulk parameters.

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1. INTRODUCTION

Cyclotron waves are an important constituent of plasmas in solar corona, solar wind and planetary magnetospheres. As it is known, energetic particles with anisotropic temperature can excite a wide class of cyclotron wave instabilities. Kinetic theory of such waves in the straight magnetic field plasmas is developed very well, see e.g. Refs. [1-6]. However, plasma models in the straight magnetic field are quite rough for planetary magnetospheres which are three-dimensional in the general case. As more suitable, the Earth's magnetosphere can be considered as a two-dimensional (2D) dipole magnetic field configuration. Another interesting 2D magnetospheric plasma model is a configuration with circular magnetic field lines, which is artificial but simpler and helpful to describe the principal wave processes in the Earth's magnetosphere. The dispersion equations for cyclotron waves in magnetospheric plasmas with dipole and circular magnetic field lines were derived in Ref. [7]. In this paper we analyze the dispersion characteristics of the electromagnetic ion-cyclotron (EMIC) waves in the hydrogen plasma confined in the last plasma model including the energetic protons with the bi-Maxwellian distribution function.

2. DISPERSION EQUATION FOR FIELD-ALIGNED CYCLOTRON WAVES

According to Ref. [7], the dispersion equations for field aligned cyclotron waves in magnetospheric plasmas with circular magnetic field lines can be rewritten by analogy with the straight magnetic field case in the form:

$$\left(\frac{n\pi c}{\omega R_0 L\theta_o}\right)^2 = 1 + 2\sum_{\sigma}\varepsilon^{n,n}_{l,(\sigma)}(L)\,, \qquad (1)$$

where σ denotes the particle species (electron, proton, heavy ions); n is the mode number along the geomagnetic field; $L = R/(R_0\cos\theta)$ is the non-dimensional L-shell parameter; θ is the geographical latitude; and the transverse permittivity elements are

$$\varepsilon^{n,n'}_l = \frac{\omega^2_{po}LR_0T_\parallel}{2\omega\pi^{1.5}\theta_o v_{T\parallel}T_\perp}\sum_{p=-\infty}^{\infty}\int_0^{\nu_o}(1-2\nu)^3 d\nu\int_0^{\infty}\frac{u^4A^n_{p,l}Y^{n'}_{p,l}}{pu - Z_l(\nu)}\times$$
$$\times\exp\left[-u^2\left(1-(1-2\nu)^2\left(1-\frac{T_\parallel}{T_\perp}\right)\right)\right]du. \qquad (2)$$

Here we have used the following definitions:

$$Y^n_{p,l}(u,\nu) = \int_{-\theta_t}^{\theta_t}\left[\frac{b(\theta)-1+T_\parallel/T_\perp}{\sqrt{1-(1-2\nu)^2b(\theta)}} + \frac{\pi n u v_{T\parallel}}{\omega R_0 L\theta_o}\left(1-\frac{T_\parallel}{T_\perp}\right)\right]\times$$
$$\times\left\{\cos\left[\frac{n\pi}{\theta_o}\theta - p\frac{2\pi}{\tau_b}\tau(\theta) - \frac{lR_0\omega_{co}}{L^2uv_{T\parallel}}C(\theta)\right]+\right.$$
$$\left.+(-1)^p\cos\left[\frac{n\pi}{\theta_o}\theta + p\frac{2\pi}{\tau_b}\tau(\theta) + \frac{lR_0\omega_{co}}{L^2uv_{T\parallel}}C(\theta)\right]\right\}d\theta,$$

$$A^n_{p,l}(u,\nu) = \int_{-\theta_t}^{\theta_t}\left\{\cos\left[\frac{n\pi}{\theta_o}\theta - p\frac{2\pi}{\tau_b}\tau(\theta) - \frac{lR_0\omega_{co}}{L^2uv_{T\parallel}}C(\theta)\right]+\right.$$
$$\left.(-1)^p\cos\left[\frac{n\pi}{\theta_o}\theta + p\frac{2\pi}{\tau_b}\tau(\theta) + \frac{lR_0\omega_{co}}{L^2uv_{T\parallel}}C(\theta)\right]\right\}\frac{b(\theta)\,d\theta}{\sqrt{1-(1-2\nu)^2b(\theta)}}$$

$$C(\theta) = \frac{\sqrt{2}(1-\nu)}{(1-2\nu)^{1.5}}\Pi\left(\alpha(\theta),\nu,\nu\right) - \frac{F\left(\alpha(\theta),\nu\right)}{\sqrt{2}(1-2\nu)^{1.5}}\,,$$

$$\alpha(\theta) = \arcsin\left(\sqrt{\frac{1-2\nu}{2(1-\nu)-\sin^2\theta}}\sin\theta\right),$$

$$\tau(\theta,\nu) = \sqrt{\frac{2(1-\nu)^2}{1-2\nu}}\Pi\left(\alpha(\theta),\frac{-\nu}{1-2\nu},\nu\right) - \sqrt{\frac{1-2\nu}{2}}F\left(\alpha(\theta),\nu\right)$$

$$\tau_b(\nu) = \sqrt{8(1-2\nu)}\,\Pi\left(\frac{\pi}{2},2\nu,\nu\right),$$

$$\Pi(\alpha,\delta,\nu) = \int_0^{\alpha}\frac{d\eta}{(1-\delta\sin^2\eta)\sqrt{1-\nu\sin^2\eta}}\,,$$

$$F(\alpha,\nu) = \int_0^{\alpha}\frac{d\eta}{\sqrt{1-\nu\sin^2\eta}}\,, \quad b(\theta) = \frac{1}{\cos^4\theta}\,,$$

$$\bar{b} = \frac{4}{\tau_b}\int_0^{\theta_t}\frac{b(\theta)}{\sqrt{1-(1-2\nu)^2b(\theta)}}d\theta\,, \qquad u = \frac{v}{v_{T\parallel}}\,,$$

$$\theta_t(\nu) = \arcsin\sqrt{2\nu}\,, \qquad \omega_b = \frac{2\pi v_{T\parallel}}{R_0L\tau_b}\,, \qquad \omega^2_{po} = \frac{4\pi Ne^2}{M}\,,$$

$$Z_l(\nu) = \frac{1}{\omega_b}\left(\omega + l\frac{\omega_{co}}{L^3}\bar{b}\right),\ \nu_o = \frac{L^2-1}{2L^2},\ \theta_o = \arccos\left(\frac{1}{L}\right).$$

Note, that Eq. (2) describes the contribution of any kind of the trapped particles to $\varepsilon^{n,n}_{l,(\sigma)}$. The corresponding expressions for electrons and ions can be obtained from (2) by replacing the temperatures $T_\parallel$ and $T_\perp$, density N, mass M, charge e by the electron $T_{\parallel e}$, $T_{\perp e}$, N_e, m_e, e_e and ion $T_{\parallel i}$, $T_{\perp i}$, N_i, M_i, e_i parameters, respectively.

Our dispersion equation is suitable to analyze the instabilities of both the right-hand (if l=1) and left-hand (if l=-1) polarized waves. Further, Eq. (1) should be resolved numerically for the real and imaginary parts of the wave frequency, $\omega = \mathrm{Re}\,\omega + i\,\mathrm{Im}\,\omega$, to define the instability conditions. As it is well known, in the straight magnetic field case, the squared refractive index of the EMIC waves (l = -1) in the hydrogen plasma is defined by the expression

$$\left(\frac{k_\parallel c}{\mathrm{Re}\,\omega}\right)^2 \approx \frac{\Omega_{pp}^2}{\Omega_{c0}(\Omega_{c0} - \mathrm{Re}\,\omega)}, \qquad (3)$$

where $\Omega_{pp}^2 = 4\pi N_p e^2 / M_p$ is the squared proton plasma frequency, $N_p = N_c + N_h$; N_c and N_h are the densities of the cold and hot protons, respectively;

$$\Omega_{c0} = \frac{eB(L,0)}{M_p c} = \frac{\omega_{co}}{L^3}$$

is the cyclotron frequency of the L-shell protons at the equatorial plane. Since the parallel wavenumber $k_\parallel$ is connected with the mode numbers n as $k_\parallel = n\pi / [R_0 L \theta_o]$, for plasmasphere with circular magnetic field lines, so that the mode numbers can be estimated as

$$n \approx \frac{R_0 L \theta_o \Omega_{pp}\,\mathrm{Re}\,\omega}{\pi c \sqrt{\Omega_{c0}(\Omega_{c0} - \mathrm{Re}\,\omega)}}. \qquad (4)$$

Where as the increment (decrement) of the EMIC waves in a hydrogen plasma γ, if $\gamma = \mathrm{Im}\,\omega << \mathrm{Re}\,\omega$, is defined by the expression

$$\frac{\gamma_s}{\Omega_{c0}} \approx -2\frac{\mathrm{Re}\,\omega\,(\Omega_{c0} - \mathrm{Re}\,\omega)^2}{\Omega_{pp}^2 (2\Omega_{c0} - \mathrm{Re}\,\omega)} \mathrm{Im}\,\varepsilon_{-1,h}, \qquad (5)$$

where

$$\mathrm{Im}\,\varepsilon_{-1,h} = \frac{\Omega_{ph}^2 \Omega_{c0} \sqrt{\pi}}{2(\mathrm{Re}\,\omega)^2 k_\parallel v_{T\parallel h}} \left[\frac{\mathrm{Re}\,\omega}{\Omega_{c0}} - \left(1 - \frac{\mathrm{Re}\,\omega}{\Omega_{c0}}\right)\left(\frac{T_{\perp h}}{T_{\parallel h}} - 1\right)\right] \times$$

$$\times \exp\left[-\left(\frac{\mathrm{Re}\,\omega - \Omega_{c0}}{k_\parallel v_{T\parallel h}}\right)^2\right]$$

By index 'h' we denote the plasma parameters for the resonant hot protons. As follows from Eqs. (3), the proton cyclotron instability (PCI) of EMIC waves, $\gamma > 0$, is possible if $\mathrm{Im}\,\varepsilon_{-1,h} < 0$, i.e., if $T_{\perp h} > T_{\parallel h}$.

3. NUMERICAL RESULTS

Now, let us compare the PCI growth rates in the plasmas confined in the straight magnetic field γ_s, and in the 2D magnetosphere with circular magnetic field lines γ_c. For simplicity, there are considered the hydrogen plasmas at the geostationary orbit, L=6.6, including the cold electrons with N_e=11 cm^{-3}, cold protons with N_c=10 cm^{-3}, and energetic protons with N_h=1 cm^{-3}. The parallel and transverse temperatures of the energetic protons are equal to $T_{\parallel h} = 10$ keV and $T_{\perp h} = 30$ keV, whereas the temperature of the cold particles is small and isotropic. In this case, the mode numbers n of the field aligned EMIC waves can be defined by Eqs. (3), (4); the corresponding dependence $n(\omega)$ is plotted in Fig. 1.

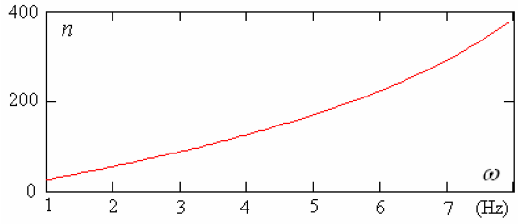

Fig. 1. Dependence of the mode numbers on the wave frequency for EMIC waves in a hydrogen plasma

The PCI growth rate γ_s for EMIC waves in the straight magnetic field plasma we estimate, as usually, by Eq. (5). As for γ_c, for EMIC waves in the magnetospheric-like plasma with circular magnetic field lines, we use the similar expression :

$$\frac{\gamma_c}{\Omega_{c0}} \approx -2\frac{\mathrm{Re}\,\omega\,(\Omega_{c0} - \mathrm{Re}\,\omega)^2}{\Omega_{pp}^2 (2\Omega_{c0} - \mathrm{Re}\,\omega)} \mathrm{Im}\,\varepsilon_{-1,h}^{n,n}, \qquad (6)$$

where

$$\mathrm{Im}\,\varepsilon_{-1,h}^{n,n} = \sum_{p=1}^{\infty} \frac{\Omega_{ph}^2 L R_0 T_\parallel}{2\omega\sqrt{\pi}\,\theta_o v_{T\parallel} T_\perp p^5} \int_0^{\nu_o} (1-2\nu)^3 A_{p,-1}^n\left(\frac{Z_{-1,h}}{p}, \nu\right)$$

$$\times Y_{p,-1}^n\left(\frac{Z_{-1,h}}{p}, \nu\right) Z_{-1,h}^4 \exp\left[-\frac{Z_{-1,h}^2}{p^2}\left(1 - (1-2\nu)^2\left(1 - \frac{T_\parallel}{T_\perp}\right)\right)\right] d\nu$$

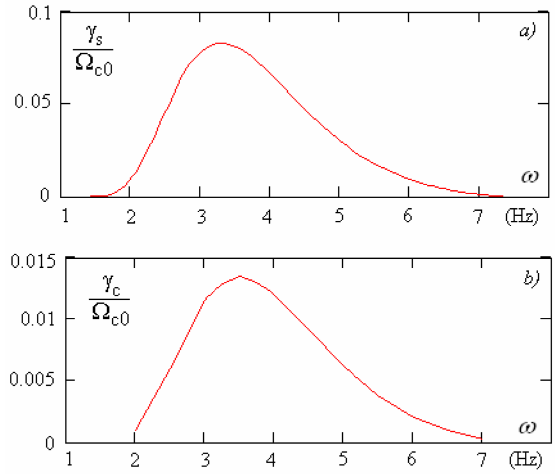

Fig. 2. The PCI growth rates versus ω for EMIC waves in the hydrogen plasmas confined in the straight uniform magnetic field (a) and in the 2D magnetosphere (b)

The PCI growth rates versus ω are presented in Fig. 2*a* for EMIC waves in the straight magnetic field plasma by Eq. (5), and in Fig. 2*b* for EMIC waves in the 2D magnetosphere-like plasma with circular magnetic field lines. The computations of γ_c are carried out in the interval $2\,\mathrm{Hz} \le \omega \le 7\,\mathrm{Hz}$, whereas the minimal gyrofrequency of the protons at L=6.6 is closed to $\Omega_{c0} \approx 11\,\mathrm{Hz}$. As shown in Fig. 2*a* and Fig. 2*b*, the instability of EMIC waves is possible for both plasma models in the frequency range $\omega < \Omega_{c0}$. It should be noted that the proton-cyclotron instability is impossible for EMIC waves in the frequency range $\Omega_{c0} < \omega < \Omega_{c0} b(\theta_o)$, where $\Omega_{c0} b(\theta_o)$ is the maximal gyrofrequency of the protons at the given L-shell magnetic field line.

As one can see, the dependence $\gamma_s(\omega)$ and $\gamma_c(\omega)$ on the wave frequency ω are similar; however, $\gamma_c(\omega)<<\gamma_s(\omega)$ under the same bulk parameters. The ratio $\gamma_s/\gamma_c \propto 4\div 10$ versus ω for considered magnetospheric-like plasmas is presented in Fig. 3. This dependence is not linear; the difference is very large (by factor 10) for EMIC waves in the range of ω ~ 2Hz and is smaller (by factor 4) in the range of high frequencies ω ~ 7Hz .

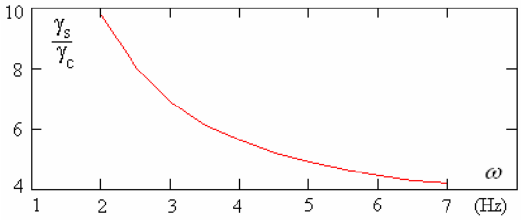

Fig. 3. The ratio γ_s/γ_c versus ω for EMIC waves in the hydrogen plasmas

The large difference between γ_s and γ_c is connected with the fact that the wave-particle interaction in the straight magnetic field plasma is more effective since the resonant particles move along the uniform magnetic field line with the constant parallel velocity and interact permanently (in time) with the wave according to the well known resonance condition $\omega-\Omega_{c0}=k_{\|}v_{\|}$. As for 2D magnetospheric plasmas, since $v_{\|}\neq const$ for the trapped particles, there is another wave-particle resonance condition involving the particle energy, pitch angle, cyclotron and bounce frequencies. As a result, the trapped particle bouncing between the reflection points only part of the bounce-time can interact effectively with the wave.

CONCLUSIONS

Dispersion equation is analyzed for EMIC waves in a hydrogen magnetospheric plasma with circular B-field lines. As in the straight B-field plasmas, the growth rate of PCI is defined by the contribution of the resonant particles to the imaginary part of the transverse permittivity elements. The comparison of the growth rates is carried out for EMIC waves in the hydrogen plasmas with the straight and circular B-field lines under the same macroscopic bulk parameters at the geostationary orbit L=6.6. It is shown that the PCI growth rate in the 2D magnetosphere is much less than it is in the straight uniform B-field case. Of course, the similar approach can be used to analyze the dispersion characteristics of the EMIC waves in 2D magnetospheric multi-ions plasmas with dipole and circular magnetic field lines including the protons and heavy ions (He^+, O^+) with the temperature anisotropy.

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НЕУСТОЙЧИВОСТЬ ИОННО-ЦИКЛОТРОННЫХ ВОЛН В 2D-МАГНИТОСФЕРНОЙ ПЛАЗМЕ С АНИЗОТРОПНОЙ ТЕМПЕРАТУРОЙ

Н.И. Гришанов, Н.А. Азаренков

Проанализировано дисперсионное уравнение ионно-циклотронных волн, распространяющихся параллельно геомагнитному полю в двумерной (2D) магнитосферной плазме с круговыми силовыми линиями удерживающего магнитного поля. Бимаксвелловская функция распределения использована при моделировании распределения энергичных протонов в водородной плазме вблизи геостационарных орбит. Как и в однородном магнитном поле, инкремент нарастания протонно-циклотронной неустойчивости (ПЦН) в 2D магнитосфере определяется вкладом энергичных протонов в мнимую часть поперечной диэлектрической проницаемости. Показано, что инкремент ПЦН в 2D аксиально-симметричной магнитосфере может быть значительно ниже, чем для плазмы в прямом магнитном поле с теми же макроскопическими параметрами.

НЕСТІЙКІСТЬ ІОННО-ЦИКЛОТРОННИХ ХВИЛЬ У 2D-МАГНІТОСФЕРНІЙ ПЛАЗМІ З АНІЗОТРОПНОЮ ТЕМПЕРАТУРОЮ

М.І. Гришанов, М.О. Азарєнков

Проаналізовано дисперсійне рівняння для іонно-циклотронних хвиль, які поширюються вздовж геомагнітного поля у двовимірній (2D) магнітосферній плазмі з силовими лініями магнітного поля у вигляді кола. Бімаксвелівська функція розподілу використана при моделюванні розподілу енергійних протонів у водневій плазмі поблизу геостаціонарних орбіт. Як і у випадку однорідного магнітного поля, інкремент зростання протонно-циклотронної нестійкості (ПЦН) у 2D магнітосфері визначається внеском енергійних протонів в уявну частину поперечної діелектричної проникливості. Доведено, що інкремент ПЦН у 2D аксіально-симетричній магнітосфері може бути значно нижчим, ніж для плазми у прямому магнітному полі з тими ж самими макроскопічними параметрами.