SPECTRUM OF SMALL-SCALE PLASMA STRUCTURES IN THE PHOTOSPHERE

Yu.V. Kyzyurov

Main Astronomical Observatory, National Academy of Sciences of Ukraine, Kiev, Ukraine,
e-mail: kyzyurov@mao.kiev.ua

In this report we consider possibility of formation of small-scale plasma structures in the turbulent flows of photospheric gas on the Sun and analyse dependence of their spectrum and intensity on height and the magnetic field strength. It was shown that in the height range 150–350 km the slope of the structure spectrum decreases with increasing the altitude. Under the weak magnetic field (B = 5 G), the intensity of plasma structures is unchanged with height. The increase in the magnetic field strength results in a rise in the structure intensity and in a decrease in the spectral slope.

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INTRODUCTION

The structure and dynamics of the solar photosphere are very important for better understanding of basic solar phenomena such as atmospheric energy transport, turbulent diffusion of magnetic field or chaotic excitation of solar oscillations. The degree of photospheric gas ionization is quite small [1, 2]. It means that electrically charged particles in the photosphere can be considered as passive contaminants embedded in motions of the gas. Results of observations clearly show that the photospheric flows include both organized and stochastic motions [3, 4]. The spectra associated with the random velocity fields obey power laws, which are close to the spectrum of Kolmogorov turbulence [4]. Turbulent motions of the gas have to result in formation of random plasma structures in the photosphere [5]. Parameters of the photosphere and turbulent mixing depend on height [1, 2, 6]. In addition there are regions with various magnetic field strengths in the photosphere [7]. It is important to analyse possible dependence of the spectrum and intensity of plasma structures generated in turbulent photospheric flows on the height and the magnetic field strength. This analysis is the aim of the report. The present consideration will be restricted to small-scale structures with length-scales smaller than the length-scale of the mean plasma density gradient.

BASIC EQUATIONS AND RELATIONS

To describe turbulent mixing in the solar photosphere (which is a slow process) a three-fluid model can be used. Since the charged particles are passive contaminants, they have no influence on motions of neutral gas and the gas velocity field $\mathbf{u}(\mathbf{x}, t)$ may be treated as a known function of position and time. The gas in the photosphere can be regarded as incompressible, $\nabla\mathbf{u}=0$. The behaviour of charged particles embedded in the gas flow can be described by the following set of equations [5]:

$$\partial N_s/\partial t + \nabla(N_s \mathrm{v}_s) = 0\,, \tag{1}$$

$$\tau_s^{-1}(\mathbf{v}_s - \mathbf{u}) = q_s\mathbf{E}/m_s + \Omega_s(\mathbf{v}_s \times \mathbf{b}) - \mathrm{v}_{Ts}^2 N_s^{-1}\nabla N_s\,, \tag{2}$$

where the variables are chosen as density N_s and velocity $\mathbf{v}_s$ for each species ($s\equiv i, e$), τ_s is a characteristic time of charged particle collisions with neutrals, q_s is the particle charge ($q_e=-q_i=-e$), $\Omega_s= q_sB/m_sc$ is the gyrofrequency, v_{Ts} is the thermal velocity, m_s is the particle mass, $\mathbf{b}=\mathbf{B}/B$ is the unit vector along the magnetic field $\mathbf{B}$, $\mathbf{E}$ is the electric field.

In the photosphere $\tau_i\Omega_i<<1$ and the assumptions of quasi-neutrality $N_e=N_i=N$ and isothermality $T_e=T_i=T_n=T$ are valid.

In the case of turbulent flows the gas velocity may be separated into mean and fluctuating parts $\mathbf{u}=\mathbf{u}_0+\mathbf{u}_1$ ($\mathbf{u}_0=\langle\mathbf{u}\rangle$, $\langle\mathbf{u}_1\rangle=0$, $u_1<u_0$). The same may be made for plasma density $N=N_0+N_1$ ($N_0=\langle N\rangle$, $\langle N_1\rangle=0$, $N_1< N_0$); N_1 represents plasma structures generated by the turbulent velocity field $\mathbf{u}_1$.

The way of derivation of $\Psi(\mathbf{k},\omega)$, the spatiotemporal spectrum of $\delta N= N_1/N_0$, from Eqs. (1), (2) is described in [5]. Length-scales of random gas motions were restricted to the inertial range of turbulence. In this range turbulence is homogeneous and isotropic one with known statistical properties. The spectrum tensor of the field $\mathbf{u}_1$ [4, 5, 8] is:

$$\Phi_{\alpha\beta}(\mathbf{k},\omega) = D_{\alpha\beta}(\mathbf{k})\tau_t(k)E(k)\cdot[4\pi^2(1+\omega^2\tau_t^2)]^{-1}, \tag{3}$$
$$k_0<k<k_\nu,$$

where $D_{\alpha\beta}=\delta_{\alpha\beta}-k_\alpha k_\beta/k^2$ is the projection operator, $\tau_t(k)=(\nu k^2+\varepsilon^{1/3}k^{2/3})^{-1}$ is the decay time of eddy with a length-scale k^{-1}, $E(k)=C_1\varepsilon^{2/3}k^{-5/3}$ is the energy spectrum function, k_0^{-1} is the basic energy input scale, $k_\nu=(\varepsilon/\nu^3)^{1/4}$ is the Kolmogorov dissipation wavenumber, ν is the kinematic viscosity of the gas, ε is the rate of turbulent energy dissipation per unit mass, the Kolmogorov constant C_1 is around 1.5 [9].

To obtain $\Psi(\mathbf{k},\omega)$ the only electric field $\mathbf{E}$ considered was that required to prevent charge separation (due to $\mathbf{E}$ electrons tend to follow ions). In addition a contribution of the mode interaction in the process of plasma structure generation was taking into account through the coefficient of turbulent diffusion K_T. For the structures with length-scales smaller than $L_N=N_0|\nabla N_0|^{-1}$, the length-scale of ∇N_0, the following expression was derived [5]

$$\Psi(\mathbf{k},\omega) = [4\pi^2(1+\omega^2\tau_k^2)(1+\omega^2\tau_t^2)]^{-1}\tau_t\tau_k^2 Q(\mathbf{k})\,, \tag{4}$$
$$L_N^{-1}<k<k_d,$$

here $\tau_k=(D_Ak^2+K_Tk^2)^{-1}=(D_Ak^2+\varepsilon^{1/3}k^{2/3})^{-1}$, D_A is the ambipolar diffusion coefficient, $Q(\mathbf{k})= [(\mathbf{n}\times\mathbf{k})^2/(L_Nk)^2+ +(\mathbf{b}\times\mathbf{k})^2/(\tau_i\Omega_i)^2]C_1\varepsilon^{2/3}k^{-11/3}$, $\mathbf{n}=L_NN_0^{-1}\nabla N_0$ is the unit along

∇N_0, $k_d = (\varepsilon/D_A^3)^{1/4}$ is the Oboukhov-Corrsin wavenumber known in the theory of passive scalar turbulent convection [9], in the present case it define the structure length-scale at which K_T=D_A.

From Eq.(4) we can obtain the spatial spectrum of δN

$$P_N(\mathbf{k}) = \int_{-\infty}^{\infty} \Psi(\mathbf{k},\omega)\,d\omega = [4\pi(1+\tau_t/\tau_k)]^{-1}\tau_t\tau_k Q(\mathbf{k}) . \quad (5)$$

Unlike [5] the inequality $D_A \neq \nu$ was taken into account in the present case. Using Eq.(5) a mean-square level of δN in the range (k_1, k_2) may be calculated

$$\left\langle \delta N^2 \right\rangle = \int P_N(\mathbf{k})\,d\mathbf{k} = S((k_2/k_d)^{4/3}) - S((k_1/k_d)^{4/3}) , \quad (6)$$

where

$$S(x) = \frac{3}{8} L_N^{-2} k_d^{-2} x^{-3/2}[(3+\mathrm{Pr})x - 2/3] +$$
$$+ \frac{3}{2}\frac{L_N^{-2}k_d^{-2}}{1-\mathrm{Pr}}[\mathrm{arctg}\, x^{1/2} - ((1+\mathrm{Pr})/2)^{5/2}\mathrm{arctg}(x(1+\mathrm{Pr})/2)^{1/2}] +$$
$$+ \frac{3}{8}\frac{\tau_i^2\Omega_i^2}{1-\mathrm{Pr}}[2\ln(x/(1+x)) - (1+\mathrm{Pr})\ln(x(1+\mathrm{Pr})/(2+x(1+\mathrm{Pr})))]$$

here Pr=ν/D_A is the diffusion Prandtl number.

The 1D spectrum of plasma structures in the turbulent photospheric flow that may be measured along **z**-direction may be obtained from Eq.(5) too:

$$P_N(k_z) = \int_0^{k_\zeta} k_\perp dk_\perp \int_0^{2\pi} P_N(\mathbf{k})\,d\varphi =$$
$$= \frac{1}{4}\int_0^{k_\zeta} (L_N^{-2} f(k_\perp, k_z, \theta_1) + \tau_i^2\Omega_i^2 k^2 f(k_\perp, k_z, \theta_2)) F(k) k^{-7} k_\perp dk_\perp \quad (7)$$

where $f(k_\perp,k_z,\theta)=k_\perp^2+k_\perp^2\cos^2\theta+2k_z^2\sin^2\theta$, θ_1 is the angle between **z** and **n**, θ_2 between **z** and **b**, $k_\zeta^2=k_d^2-k_z^2$, $k^2=k_\perp^2+k_z^2$, $F(k)=[(1+(k/k_d)^{4/3})\times(2+(k/k_d)^{4/3}+(k/k_\nu)^{4/3})]^{-1}$.

Eqs. (6), (7) give an opportunity to estimate changeability of small-scale plasma structures with changing the height and the magnetic field strength.

CHANGEABILITY OF PHOTOSPHERIC PLASMA STRUCTURES

To estimate changeability of the photospheric plasma structures we shall consider the case when **n** and **b** are in vertical direction, while the possible measurement direction **z** is horizontal. Then $\theta_1=\theta_2=\pi/2$ and Eq.(7) takes the form

$$P_N(k_z) = \frac{1}{4}\int_0^{k_\zeta} (L_N^{-2} + \tau_i^2\Omega_i^2 k^2)(k_\perp + 2k_z) F(k) k^{-7} k_\perp dk_\perp . \quad (8)$$

The plasma structures are analysed near heights of 150 and 350 km. The outer scale of turbulence k_0^{-1}= L_0=940 km is the same for both heights [6], and we suppose that $L_N \approx L_0$. The mean gas velocity on L_0 is u_0, and then $\varepsilon = u_0^3/L_0$. Parameters of the photosphere taken from [1,2,6] together with the calculated values of k_ν^{-1} and $k_d^{-1}$are presented in Table 1. Characteristics of plasma structures calculated with use of Eqs.(6), (8) and the value $\tau_i\Omega_i$ are shown in Table 2 (γ is the power index when $P_N(k_z)$ was approximated by a simple power law $k_z^{-\gamma}$). The limits of integration in Eq.(6) are $k_1 = 2\pi/L_m$, $k_2 = k_\nu$ (L_m=300 km).

Fig.1 shows the 1D spectrum $P_N(k_z)$ calculated with the use of Eq.(8) for h=150 km: line 1 is for the case of the magnetic field B=5 G, line 2 for B=250 G, a straight line is the power law $k_z^{-5/3}$. Fig.2 represents the same for h=350 km.

From the figures and Table 2 it seen that the small-scale plasma structures have to be sensitive to the change in both the height and the magnetic field strength. In the region with a weak magnetic field, dependence of the structure intensity on height is almost absent, though the spectral shape changes. An increase in the magnetic field provides a change in the rms fluctuation level and the spectral slope.

Table 1. Parameters of the solar photosphere

Parameter	h=150 km	h=350 km
T, K	5180	4670
N_n, m^{-3}	5.05×10^{22}	1.01×10^{22}
N_e, m^{-3}	6.04×10^{18}	1.12×10^{18}
m_i, a.u.m.	25	26.3
u_0, km/s	1.1	2.05
k_ν^{-1}, cm	10.4	20
k_d^{-1}, cm	1.33	2.5

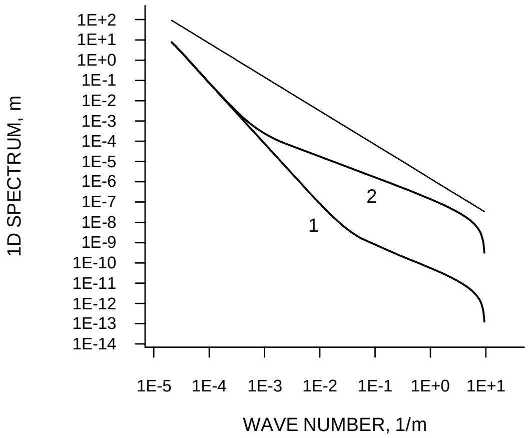

Fig.1. Spectrum $P_N(k_z)$ at h=150 km

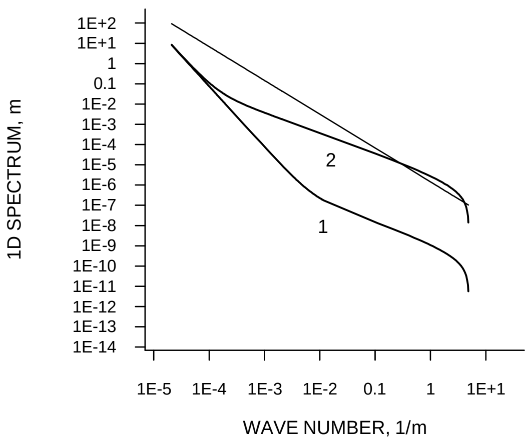

Fig.2. Spectrum $P_N(k_z)$ at h=350 km

Table 2. Characteristics of plasma structures

h, km	B, G	$\tau_i\Omega_i$	$<\delta N^2>^{1/2}$, %	γ
150	5	1.98×10^{-5}	2.5	2.22
150	250	9.9×10^{-4}	2.6	1.41
350	5	9.4×10^{-5}	2.5	1.94
350	250	4.7×10^{-3}	2.8	1.23

CONCLUSIONS

An analytic expression for the 1D spectrum of the plasma structures in a turbulent flow of photospheric gas Eq.(7) as well as the formula for estimation of the RMS level of their intensity Eq.(6) were presented in the report.

Using the expressions it was shown that in the height range 150–350 km the slope of the structure spectrum decreases with increasing the altitude. Under the weak magnetic field (B=5 G), the intensity of plasma structures is unchanged with height. The increase in the magnetic field strength results in a rise in the structure intensity and in a decrease in the spectral slope.

The obtained results seem to be important for better understanding of basic solar phenomena, such as generation of the random component of magnetic field or chaotic excitation of solar oscillations.

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СПЕКТР МЕЛКОМАСШТАБНЫХ ПЛАЗМЕННЫХ СТРУКТУР В ФОТОСФЕРЕ

Ю.В. Кызьюров

Рассматривается возможность формирования мелкомасштабных плазменных структур в турбулентных потоках фотосферного газа на Солнце и анализируется зависимость их пространственного спектра и интенсивности от высоты и напряженности магнитного поля. Показано, что в интервале высот 150-350 км наклон спектра рассматриваемых структур с увеличением высоты уменьшается. При слабом магнитном поле (B=5 Гс) интенсивность плазменных структур с высотой не меняется. Увеличение напряженности магнитного поля приводит к росту интенсивности структур и уменьшению наклона спектра.

СПЕКТР ДРІБНОМАСШТАБНИХ ПЛАЗМОВИХ СТРУКТУР У ФОТОСФЕРІ

Ю.В. Кизьюров

Розглядається можливість формування дрібномасштабних плазмових структур в турбулентних потоках фотосферного газу на Сонці та аналізується залежність їх просторового спектра та інтенсивності від висоти та напруженості магнітного поля. Показано, що в інтервалі висот 150–350 км нахил спектра структур, що розглядаються, із збільшенням висоти зменшується. За умов слабкого магнітного поля (B=5 Гс) інтенсивність плазмових структур з висотою не змінюється. Збільшення напруженості магнітного поля веде до зростання інтенсивності структур та зменшення нахилу спектра.