NONLINEAR DAMPING OF ELECTRON BEAM-DRIVEN LANGMUIR WAVES DUE TO LANGMUIR-KINETIC ALFVÉN-WHISTLER COUPLING IN THE SOLAR CORONA

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We present a coherent nonlinear theory of three-wave coupling involving Langmuir, kinetic Alfvén and whistler waves. The initial stage of the energy exchange among these modes and the following nonlinear temporal dynamics are studied. The role of pump depletion, dissipation and frequency mismatch in the nonlinear wave dynamics is analyzed. Depending on the relative damping rates of the waves, the initial Langmuir waves can be nonlinearly transformed either into whistlers, or into KAWs. The theory is applied to the Langmuir waves excited by electron beams in a diluted solar corona where the local electron-cyclotron frequency is higher than the local electron plasma frequency.
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1. INTRODUCTION

The nonlinear wave-wave interactions involving electron-beam driven Langmuir wave have been widely studied in context of generation mechanisms for the solar radio bursts. Large amplitude Langmuir waves can generate radio emission by nonlinear coupling to low-frequency MHD waves such as ion sound wave, whistler and shear Alfvén waves [1, 2]. We present here another relevant nonlinear parametric process for beam-driven Langmuir waves (L) in the solar corona, namely their decay into whistler (W) and kinetic Alfvén wave (KAW).

Linear theory of parametric instability for LDKAW+W has been investigated by Voitenko et al. 2003 [3]. In the linear theory the pump amplitude is assumed constant. In this paper we develop a coherent nonlinear theory of the process LDKAW+W, taking into consideration the effect of pump depletion. In addition, the roles played by dissipation and frequency mismatch in the nonlinear wave dynamics are analyzed numerically.

2. COUPLED WAVE EQUATIONS FOR LDKAW+W

We treat the nonlinear parametric coupling of three waves: an oblique pump Langmuir wave with frequency ω_L and wave vector $k_L = \{k_{Lx};0;k_{Lz}\}$; whistler wave with frequency ω_W and wave vector $k_W = \{0;0;k_{Wz}\}$ and a kinetic Alfvén wave with frequency ω_A and wave vector $k_A = \{k_{Ax};0;k_{Az}\}$ with $k_{Ax} >> k_{Az}$. For a three-wave coupling, the following resonant conditions should be satisfied:

$$\omega_L \approx \omega_W + \omega_A; \quad k_L = k_W + k_A. \qquad (1)$$

The resonant condition can be easily satisfied only when the local electron plasma frequency is smaller than the local electron-cyclotron frequency $(\omega_{pe} < \Omega_e)$, as we choose in our plasma model.

We adopt the two-fluid plasma description. The nonlinear system of coupled wave equations governing the three-wave process LDKAW+W is given by

$$D_L E_L = iC_{WA} E_{Ax} E_{Wx};$$
$$D_W E_{Wx} = -iC_{LA} E_L E^*_{Ax}; \qquad (2)$$
$$D_A E_{Ax} = -iC_{LW} E_L E^*_{Wx},$$

where the dispersion operators D_L, D_A, D_W are

$$D_L = i\nu_L\omega_L + \omega_L^2 - \frac{1}{2}\left[\left(\omega_{pe}^2 + k_L^2V_{Te}^2 + \Omega_e^2\right) - \sqrt{\left(\omega_{pe}^2 + k_L^2V_{Te}^2 + \Omega_e^2\right)^2 - 4\Omega_e^2\frac{k_{Lz}^2}{k_L^2}\left(\omega_{pe}^2 + k_L^2V_{Te}^2\right)}\right],$$

$$D_W = \omega_W^2 - c^2k_W^2 - \omega_{pe}^2\frac{\omega_W}{\omega_W - \Omega_e} + i\nu_W\omega_W,$$

$$D_A = \omega_A^2 - k_{Az}^2V_A^2K^2 + i\nu_A\omega_A.$$

The damping frequencies are

$$\nu_L = \left(\omega_{pe}^2/\omega_L^2\right)\nu_e; \quad \nu_W = \frac{\omega_{pe}^2\nu_e}{\left(\omega_W - \Omega_e\right)^2};$$

$$\nu_A = \sqrt{\frac{\pi}{8}}\omega_A\frac{\mu_s}{K}\frac{V_A}{V_{Te}} + 0.25\nu_e\frac{\chi_e}{1+\chi_e};.$$

Here ν_e electron damping, the dispersion function for the KAW K determines the wave phase velocity

$$K = \sqrt{\left(1+\mu_T\right)/\left(1+\chi_e\right)},$$

and the dispersive variables for the KAW are $\mu_T = k_{Ax}^2\rho_T^2$; $\chi_e = k_{Ax}^2\delta_e^2$; where $\rho_T = V_T/\Omega_p$; $\chi_e = c/\omega_{pe}$; here V_T, V_A and c are slow wave velocity, Alfvén velocity and velocity of light respectively.

The coupling coefficients for the kinetic Alfvén and whistler wave and their detail derivation are given in [3]. The nonlinear dispersion equation for the pump Langmuir wave can be derived using the Poisson's law and continuity equation. The coupling coefficient for the electrostatic Langmuir wave is given by

$$C_{WA} = k_L \frac{e}{m_e}\frac{1}{Y^2}\frac{z^2-b^2}{z^2-d_1}\frac{z}{b-\bar{\omega}_W}\frac{\mu_s}{1+\mu_i}\times$$

$$\left\{1+\frac{zb}{z^2-b^2}\left[\beta+\left(1-\frac{b}{z}\right)\frac{\bar{k}_{Wz}}{\bar{\omega}_W}s_A\frac{V_A}{V_{Te}}K-\frac{b}{z}\left(\beta-\frac{m_e}{m_i}(1+\mu_T)\right)\right]\right]+$$

$$\left.+s_A\frac{1}{z}\frac{V_{Te}}{V_A K}\frac{Z}{\mu_s}\left(1+s_A\frac{V_A}{V_{Te}}K\frac{\bar{k}_{Wz}}{\bar{\omega}_W}\frac{\bar{\omega}_W-b}{b}\right)(1+\mu_T)\right\}$$

where

$$Y = k_L\lambda_{De},\, Z = k_{LZ}\lambda_{De},\, X = k_{Lx}\lambda_{De},\, s_A = k_{Az}/|k_{Az}|$$

$$\bar{k}_{Wz} = k_{Wz}\lambda_{De};\bar{\omega}_W = \frac{\omega_W}{\omega_{pe}};\, z = \frac{\omega_L}{\omega_{pe}};\, b = \frac{\Omega_e}{\omega_{pe}}$$

3. INITIAL STAGE OF THE DECAY

The initial stage of the parametric decay process LDKAW+W with the Langmuir wave acting as the pump is governed by Eqs. (2), assuming $\nu_{L,A,W}=0$, $E_L =$ constant and $|E_L| >> |E_W|,|E_A|$. The rate of the exponential growth (growth rate), written in non-dimensional form is:

$$\frac{\gamma_{NL}}{\Omega_p} = \sqrt{\frac{1}{4}M^2\frac{BC}{b^2\left(2z(z-b)^2+b\right)z}W_L}\,,$$

where

$$W_L = \frac{|E_L|^2}{4\pi n_0 T_e}\,;\; B = \frac{X^2}{Y^2}\left(\frac{z}{b(1+\mu_T)}f - Y^2\right\}$$

$$C = X^2 - s_A\frac{b^3}{M}\frac{V_{Te}}{V_A}\frac{b^2-z^2}{z^3}KZ\,.$$

Here $f = \omega_A/\Omega_p$, $M = m_i/m_e$.

The dependence of the nonlinear growth rate on the wavenumbers of the Langmuir waves for antiparallel propagating KAWs (s_A=-1) is found numerically for typical coronal parameters (see Fig. 1 and [3] for the details).

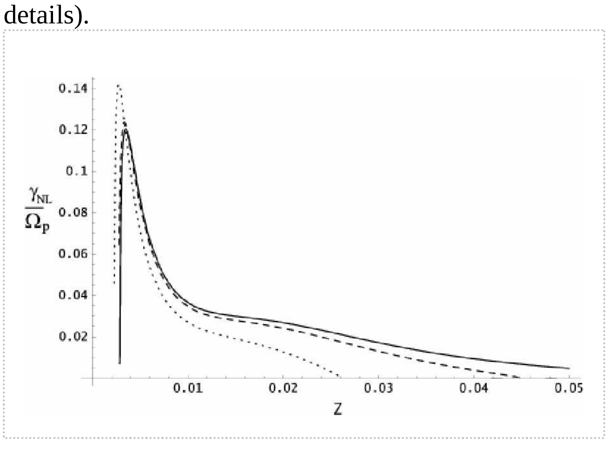

Fig.1. Nonlinear growth rate of the decay LDKAW+W. The normalized Langmuir wave energy is $W_L=10^{-4}$. Parameter b=1.04(solid line); 1.1(dash line) and 1.4(dot line)

The nonlinear growth rate strongly increases with perpendicular wavenumber of the KA/Langmuir wave and critically depends on the parameter *b* in range $k_{LZ}\lambda_{De}$ *> 0.01*: it is larger for *b > 1*, but quickly decreases with increasing *b*. So, the general tendency is that the faster electron beams in *b>1* regions are most efficient for producing of LAW events. However, even in the region where *b* deviates significantly from 1, the decay is fast when the parallel wavenumbers of Langmuir wave are reduced. The reducing of the parallel wavenumbers can occur due to the density variations along magnetic field lines and/or to the presence of low-frequency waves.

4. NONLINEAR TEMPORAL DYNAMICS OF THE COUPLED MODES

We now take into account the effect of pump depletion and study the nonlinear temporal behavior of Eqs. (2). Following the same steps as in [4] we get the dynamical system for the process LDKAW+W:

$$\begin{aligned}
\partial_\tau F_L &= 2(F_L F_A F_W)^{1/2}\cos\varphi - \nu'_L F_L\,,\\
\partial_\tau F_A &= -2(F_L F_A F_W)^{1/2}\cos\varphi - \nu'_A F_A\,,\\
\partial_\tau F_W &= -2(F_L F_A F_W)^{1/2}\cos\varphi - \nu'_W F_W\,,\\
\partial_\tau\varphi &= \delta + \left[-\left(\frac{F_A F_W}{F_L}\right)^{1/2} + \left(\frac{F_L F_W}{F_A}\right)^{1/2} + \left(\frac{F_L F_A}{F_W}\right)^{1/2}\right]\sin\varphi,
\end{aligned} \qquad (3)$$

here the amplitude F_α and phase φ_α are real variables in adopted polar representation $a_\alpha = \eta_\alpha F_\alpha^{1/2}\exp i\varphi_\alpha$, $\tau = \omega_{ch}t$, $\delta = \Delta/\omega_{ch}$, $\nu'_\alpha = \nu_\alpha\omega_\alpha/(\omega_{ch}\partial_\omega D)$, $\varphi=\varphi_L-\varphi_A-\varphi_W$. The normalization parameters $\eta_{L,A,W}$ are given by

$$\eta_L = \omega_{ch}\left(\frac{\partial_\omega D_W\partial_\omega D_A}{c_{LW}c_{LA}}\right)^{1/2},\; \eta_W = \omega_{ch}\left(\frac{\partial_\omega D_L\partial_\omega D_A}{c_{WA}c_{LW}}\right)^{1/2},$$

$$\eta_A = \omega_{ch}\left(\frac{\partial_\omega D_L\partial_\omega D_W}{c_{LA}c_{WA}}\right)^{1/2}\exp(-i\Delta t),$$

where $\Delta = \omega_L - \omega_A - \omega_W$ is the frequency mismatch. In nondissipative case with perfect frequency matching (δ=0) the solution of system (3) has periodic wavetrain form, therefore the decay process LDKAW+W represents periodic conversion of the energy of Langmuir pump wave into the energy of kinetic Alfvén and whistler wave. The finite frequency mismatch ($\delta\neq0$) diminishes the efficiency of energy transfer.

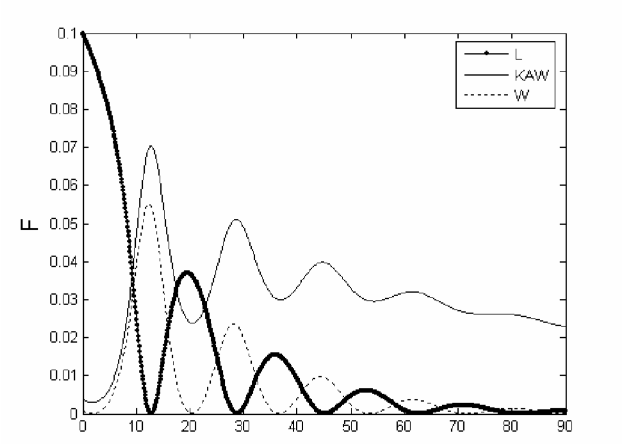

Fig.2. Nonlinear waveforms for the case of dissipative waves ($\nu'_A = 0.01$, ν'_L =0.04, ν'_W =0.07) with finite frequency mismatch (δ=0.01); ω_{ch}=1 s^{-1}

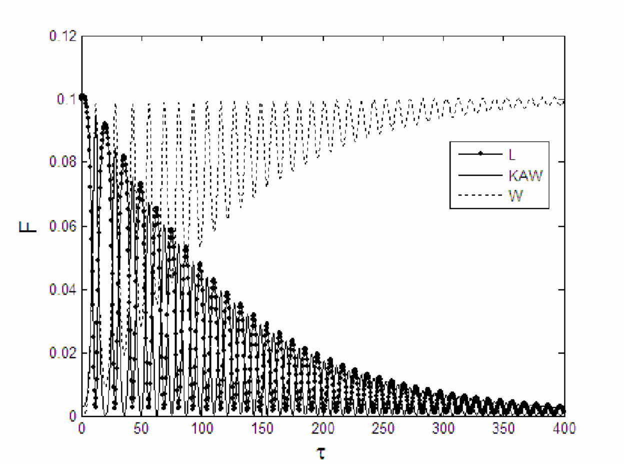

Fig.3. Nonlinear waveforms for the case of dissipative waves ($\nu'_A = 0.02$, $\nu'_L \approx 1.2\times10^{-6}$, $\nu'_W \approx 2.1\times10^{-6}$) with finite frequency mismatch ($\delta=0.1$); $\omega_{ch}=\Omega_p=3.3\times10^4\ s^{-1}$

Figs. 2, 3 gives the examples of numerical solutions of Eqs. (3) with both dissipation and frequency mismatch for the case of dissipation of the KAW due to electron-ion collisions and due to Landau damping on Maxwellian electrons, respectively. The plasma parameters are the same as in Fig.1. It is seen that in the first case the two high-frequency waves (Langmuir and whistler waves) follow similar temporal damping profiles and the initial energy of Langmuir wave is mostly converted into the kinetic Alfvén wave. In case of dissipation of KAWs due to Landau damping we observe a complete conversion of the initial Langmuir wave into whistler wave.

CONCLUSIONS

In this paper we investigate the nonlinear three-wave coupling involving Langmuir, kinetic Alfvén and whistler wave. By accounting for the finite wave damping we find a complete conversion of the initial Langmuir wave into whistler (or into KAW for smaller ratio of the KAW/whistler damping rates). The results are applied to the beam-driven Langmuir waves deduced from the observations. Our study suggests that the nonlinear decay of Langmuir wave energy into KAWs and whistlers can provide an efficient sink for low-dispersive Langmuir waves excited by fast electron beams in the solar corona when the electron plasma frequency is lower than the electron gyrofrequency. Such conditions can be satisfied in the thin (~10km) underdense filaments guided by magnetic filed lines which are connected to the low-temperature patches at the coronal base. At the same time, this nonlinear process may play a role also in the auroral zone of the Earth's magnetosphere, where Langmuir-Alfvén-whistler events are registered in-situ by satellites [5].

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НЕЛИНЕЙНОЕ ЗАТУХАНИЕ ЛЕНГМЮРОВСКИХ ВОЛН ВСЛЕДСТВИЕ ЛЕНГМЮР-АЛЬФВЕН-ВИСТЛЕР ВЗАИМОДЕЙСТВИЯ В СОЛНЕЧНОЙ КОРОНЕ

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Представлена когерентная нелинейная теория трехволнового взаимодействия ленгмюровской, кинетической альфвеновской волн и вистлера. Изучена начальная стадия энергообмена между данными волнами и его последующая временная динамика. Проанализировано влияние изменения амплитуды волны накачки, влияние диссипации и частотного сдвига на нелинейную волновую динамику. В зависимости от относительного затухания волн начальные легмюровские волны могут быть нелинейно трансформированы или в вистлеры, или в кинетические альфвеновские волны. Результаты теории применены для ленгмюровской волны, которая возбуждается электронным пучком в разреженной корональной плазме, где локальная электронно-циклотронная частота больше, чем локальная электронная плазменная частота.

НЕЛІНІЙНЕ ЗАТУХАННЯ ЛЕНГМЮРІВСЬКИХ ХВИЛЬ ВНАСЛІДОК ЛЕНГМЮР-АЛЬФВЕН-ВІСТЛЕР ВЗАЄМОДІЇ В СОНЯЧНІЙ КОРОНІ

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Представлена когерентна нелінійну теорія трихвильової взаємодії, що включає ленгмюрівську, кінетичну альфвенівську хвилі та вістлер. Вивчена початкова стадія енергетичного обміну між даними хвилями та його наступна часова динаміка. Проаналізовано вплив зміни амплітуди хвилі накачки, вплив диссипації та частотного зсуву на нелінійну хвильову динаміку. В залежності від відносного затухання хвиль, початкові ленгмюрівські хвилі можуть бути нелінійно трансформовані або в вістлери, або в кінетичні альфвенівські хвилі. Результати теорії застосовані для ленгмюрівських хвиль, що збуджуються електронним пучком в розрідженій корональній плазмі, де локальна електронно-циклотронна частота більша за локальну електронну плазмову частоту.