INFLUENCE OF A NORMAL ELECTRIC FIELD ON SURFACE WAVE DYNAMICS

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This paper presents a study of the nonresonant parametric excitation of counter-propagating surface waves by a uniform in space and varying in time electric pump field, perpendicular to a planar plasma-dielectric interface. The criterion of the wave excitation has been derived and analyzed. Expressions for the growth rates in the linear stage of the instability are obtained, and the threshold amplitudes of the external electric field above which the parametric instability can occur are found. The spectrum of the excited waves is analyzed as well.
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1. INTRODUCTION

Intensive research on surface wave (SW) parametric instability in bounded plasma-like mediums dates from the 1970 – 1980s. These research were connected with the necessity to solve a problem of the energy input into the working volumes of plasma installations, including controlled fusion devices. Therefore, the main attention in these research was focused on the parametric excitation of SWs owing to the induced scattering of electromagnetic waves incident from a dielectric or vacuum area on a semibounded plasma. More recently, these research have received further development in studies of SWs excited under the irradiation of solid targets by intense, ultrashort laser pulses.

Other research direction on SW parametric instability is connected with their excitation by an external homogeneous electric pump field. Influence of the pump field lying on the boundary plane is full enough studied, while research on SW excitation in isotropic plasma installations with a high frequency electric field, oriented perpendicularly to the medium interface, are represented much more poorly. Our research is devoted to the analysis of a nonresonant parametric instability of two counter-propagating SWs in a cold unmagnetized plasma, which is immersed in a normal (to the medium interface) high frequency electric field.

2. LINEAR SWS

We consider a semibounded homogeneous dissipative plasma bounded by a dielectric. Let the z-axis be directed along the wave propagation direction, while the x-axis is perpendicular to the plasma-dielectric interface. The plasma occupies the half-space $x > 0$, whereas the dielectric occupies the $x < 0$ region. In the considered structure, the wavenumber k_z and frequency ω of SWs propagating along the plasma-dielectric interface are known to be connected by the following relation [1]

$$k_z^2 = k^2 \frac{\varepsilon_p \varepsilon_d}{\varepsilon_p + \varepsilon_d}. \qquad (1)$$

In this expression, $k = \omega/c$ is the vacuum wavenumber with c being the speed of light in vacuum. The dielectric permittivities of the mediums are denoted by ε_d for the dielectric and by $\varepsilon_p = 1 - \omega_{pe}^2/\omega^2$ for the plasma with the electron plasma frequency ω_{pe}, respectively.

According to linear dispersion relation (1), two counter-propagating SWs may exist in the considered structure with the same frequency. Their wavenumbers are $\pm k_z$. The fields of these waves can be represented in the following form

$$\mathbf{W}_\pm = \frac{1}{2}\left[\overline{\mathbf{W}}_\pm \exp(-i\omega t) + \overline{\mathbf{W}}^*_\pm \exp(i\omega t)\right] \exp(-\gamma t), \qquad (2)$$

where $\mathbf{W} = (E_x, E_z, H_y)$. The factor $\exp(-\gamma t)$ in expression (2) describes a linear attenuation of the SWs with a damping rate γ. The rate of collisional and resonant damping of the considered waves is [1]

$$\gamma = \frac{\nu}{2}\frac{\varepsilon_d(1-\varepsilon_p)}{\varepsilon_p^2+\varepsilon_d} - \sqrt{-\frac{\varepsilon_p^2}{\varepsilon_p+\varepsilon_d}}\,\frac{\pi\eta k\omega\varepsilon_p^2\varepsilon_d^2}{\varepsilon_d^2-\varepsilon_p^3-\varepsilon_p\varepsilon_d(1-\varepsilon_p)}, \qquad (3)$$

with $\nu \ll \omega$ being the electron collision frequency. Here, the parameter $\eta = (d\varepsilon_p/dx)^{-1}_{x=x_0}$ characterizes the plasma density inhomogeneity in the narrow non-uniform transition layer at the resonant point x_0, where $\varepsilon_p(x_0) = 0$.

According to the linear theory [1], spatial distribution of the SW fields is given by

$$\begin{gathered}
\text{in the plasma } (x>0)\\
\overline{E}_{\pm x} = \pm i\frac{k_z}{\kappa_p}E_\pm \exp(-\kappa_p x \pm i k_z z),\\
\overline{E}_{\pm z} = E_\pm \exp(-\kappa_p x \pm i k_z z),\\
\overline{H}_{\pm y} = i\frac{k\varepsilon_p}{\kappa_p}E_\pm \exp(-\kappa_p x \pm i k_z z),\\
\text{in the dielectric } (x<0)\\
\overline{E}_{\pm x} = \mp i\frac{k_z}{\kappa_d}E_\pm \exp(\kappa_d x \pm i k_z z),\\
\overline{E}_{\pm z} = E_\pm \exp(\kappa_d x \pm i k_z z),\\
\overline{H}_{\pm y} = -i\frac{k\varepsilon_d}{\kappa_d}E_\pm \exp(\kappa_d x \pm i k_z z),
\end{gathered} \qquad (4)$$

where E_+ and E_- are the E_z-field amplitudes both the waves, propagating in the positive and negative directions of the z-axis. Here, $\kappa_{p,d}^2 = k_z^2 - k^2\varepsilon_{p,d}$ characterize penetration depths of the wave fields into the plasma and dielectric, accordingly.

3. NONLINEAR DISPERSION RELATION

We study the parametric excitation of electromagnetic SWs by a uniform in space and variable in time electric field, oriented perpendicularly to the plasma-dielectric interface:

$$\mathbf{E}_0 = \frac{1}{2}\left[\overline{\mathbf{E}}_0 \exp(-i\omega_0 t) + \overline{\mathbf{E}}_0^* \exp(i\omega_0 t)\right], \qquad \overline{\mathbf{E}}_0 = (E_0, 0, 0). \tag{5}$$

To consider this field to be uniform in the plasma region, the values

$$q_x = \left|E_0^{-1}\partial E_0/\partial x\right|, \; q_z = \left|E_0^{-1}\partial E_0/\partial z\right|, \tag{6}$$

which characterize the inhomogeneity of amplitude E_0, must be much less than the respective SW values, $q_x \ll \kappa_p$, $q_z \ll k_z$, over the SW skin depth, $1/\kappa_p$.

The efficiency of SW excitation by field (5) is provided by the reciprocity of the waves under study and by the fulfilment of the spatial synchronism condition with the pump field for them: $0 = k_z + (-k_z)$. Note that, generally, the temporary synchronism of the three interacting waves, $\omega_0 = \omega + \omega$, is not obligatory [2]. In what follows, we consider nonresonant excitation of SWs, when its frequencies are equal to $\omega = \omega_0/2 + \Delta\omega$ with $\Delta\omega$ being a frequency detuning.

Starting from nonlinear Maxwell's equations and the equation of plasma electron motion in the fields of weakly nonlinear SWs we can write the following set of equations for the SW fields in the plasma region

$$\nabla \times \overline{\mathbf{E}}_\pm - ik\overline{\mathbf{H}}_\pm = 0, \tag{7}$$

$$\nabla \times \overline{\mathbf{H}}_\pm + ik\varepsilon_p\overline{\mathbf{E}}_\pm = (4\pi/c)\mathbf{J}_\pm, \tag{8}$$

where the right-hand side of Eq. (8) is governed by a nonlinear current, $\mathbf{J}_\pm$, to second order in the amplitudes of the excited waves

$$\mathbf{J}_\pm = i\frac{e^3 n_0 \exp(2i\Delta\omega t)}{2m^2\omega^2\omega_0}\left[\nabla(\overline{\mathbf{E}}_0\overline{\mathbf{E}}^*_\mp) - \frac{\omega^2}{\omega_{pe}^2}\overline{\mathbf{E}}_0(\nabla\overline{\mathbf{E}}^*_\mp)\right]. \tag{9}$$

The first item in this expression is a current density of the volume charges of the plasma, whereas the second one characterizes a current of the surface charges, induced at the dielectric border. It can be easily shown that, since, in the system under consideration, the electric pump field is perpendicular to the medium interface, the surface current to second order is identically equal to zero.

If we substitute the linear values of SW and pump fields (4), (5) into expression (9) for the nonlinear current, we obtain its components

$$J_{\pm x} = \frac{e^3 n_0 k_z}{2m^2\omega^2\omega_0}\times \times E_0 E^*_\mp \exp(-\kappa_p x \pm ik_z z + 2i\Delta\omega t), \tag{10}$$

$$J_{\pm z} = -i\frac{e^3 n_0 k_z^2}{2m^2\omega^2\omega_0\kappa_p}\times \times E_0 E^*_\mp \exp(-\kappa_p x \pm ik_z z + 2i\Delta\omega t). \tag{11}$$

Solving (7) and (8) together with (9), it is possible to get the following expressions for the fields of the nonlinear SWs in the plasma region

$$\overline{H}_{\pm y} = i\frac{k\varepsilon_p}{\kappa_p}C_\pm \exp(-\kappa_p x \pm ik_z z), \tag{12}$$

$$\overline{E}_{\pm x} = \pm i\frac{k_z}{\kappa_p}C_\pm \exp(-\kappa_p x \pm ik_z z) - \frac{i}{k\varepsilon_p}\frac{4\pi}{c}J_{\pm x}, \tag{13}$$

$$\overline{E}_{\pm z} = C_\pm \exp(-\kappa_p x \pm ik_z z) - \frac{i}{k\varepsilon_p}\frac{4\pi}{c}J_{\pm z}, \tag{14}$$

where $C_\pm$ are constants determined by boundary conditions.

Integrating equation (7) and (8) over x between $-\delta$ and δ, and then letting δ tend to zero, one can obtain the boundary conditions for the SW fields

$$\left.\begin{aligned} \overline{H}_{\pm y}(x=-0) &= \overline{H}_{\pm y}(x=+0), \\ \overline{E}_{\pm z}(x=-0) &= \overline{E}_{\pm z}(x=+0). \end{aligned}\right\} \tag{15}$$

The continuity of $H_{\pm y}$ is a result of the absence of a surface charge current at the medium interface.

Applying boundary conditions (15) to fields in the plasma (12), (14), and dielectric (4), we can derive the constants $C_\pm$ characterizing amplitude values of weakly nonlinear SW fields (12)–(14)

$$C_\pm = E_\pm + \frac{ek_z^2\omega_{pe}^2}{2cm\kappa_p k\varepsilon_p\omega^2\omega_0}E_0E^*_\mp \exp(2i\Delta\omega t), \tag{16}$$

as well as the nonlinear dispersion relation for the considered SWs

$$k\left(\frac{\varepsilon_p}{\kappa_p} + \frac{\varepsilon_d}{\kappa_d}\right)E_\pm = -\frac{ek_z^2\omega_{pe}^2}{2cm\kappa_p^2\omega^2\omega_0}E_0E^*_\mp \exp(2i\Delta\omega t). \tag{17}$$

The left-hand side of Eq. (17) is a dispersion relation of the linear SWs, while its right-hand side is a response of the tangential component of the nonlinear current J_z, caused by the SW interaction with the pump field.

4. RESULTS AND DISCUSSION

Now we consider the SW parametric excitation by the pump field in the weak interaction framework, when $|\partial \ln E_\pm/\partial t| \ll |\omega|$. In this approach the dynamical equations [2], corresponding to nonlinear dispersion equation (17), for the excited wave amplitudes can be written as follows

$$\frac{\partial E_\pm}{\partial t} = i\alpha E_0 E^*_\mp \exp(2i\Delta\omega t), \tag{18}$$

$$\alpha = \frac{e\omega}{2cm\omega_0}\frac{\varepsilon_p(1-\varepsilon_p)\varepsilon_d^2}{(\varepsilon_p - \varepsilon_d)(\varepsilon_p^2 + \varepsilon_d)\sqrt{-(\varepsilon_p + \varepsilon_d)}},$$

where the coefficient α characterizes the efficiency of the SW interaction with the pump field.

The solution of equation (18) is threshold in nature:

$$E_\pm = [C_{1\pm}\,\mathrm{ch}(\beta t) + C_{2\pm}\,\mathrm{sh}(\beta t)]\exp(i\Delta\omega t), \tag{19}$$

$$\beta = \sqrt{\alpha^2|E_0|^2 - \Delta\omega^2}.$$

Here, the parameter β characterizes a growth of the SW amplitudes at their interaction with the pump field. The constants in solution (19) have the following form

$$C_{1\pm} = E_{\pm}(0) \exp\left(-i\phi_{\pm}(0)\right), \qquad (20)$$

$$C_{2\pm} = C_{1\mp} \frac{\alpha|E_0|}{|\beta|} \operatorname{sign} \sin[\phi_+(0) + \phi_-(0) - \phi_0],$$

where $\phi_{\pm}(0) = \arg E_{\pm}(0)$ and $\phi_0 = \arg E_0$.

Taking into account the linear SW attenuation (3), the necessary condition of their excitation can be written as

$$|E_0| > |E_0|_{th} = \sqrt{(\gamma^2 + \Delta\omega^2)/\alpha^2}. \qquad (21)$$

This condition imposes a restriction on the minimum value of the pump field amplitude, above which the SW excitation is possible, $|E_0|_{th}$. Under smaller amplitudes of the pump field, the SW damping dominates over the growth of their amplitudes due to the parametric instability and results in a decreasing of both the SW amplitudes in time. When the pump field exceeds threshold value (21), a simultaneous growth of both the SW amplitudes appears. This growth is characterized by the nonlinear rate

$$\gamma_{NL} = \sqrt{\alpha^2|E_0|^2 - \Delta\omega^2} - \gamma. \qquad (22)$$

Thus, an increase in the pump field amplitude, $|E_0|$, as well as a decrease of the linear damping rate, γ, leads to an increase of the nonlinear growth rate, γ_{NL}. The maximum growth rate is reached in the case of resonant excitation ($\Delta\omega = 0$) [2], when both SWs are excited with the frequencies $\omega = \omega_0/2$.

The numerical analysis (fig.1) shows that threshold value (21) decreases, as the frequency ω_0 increases or the frequency detuning, $\Delta\omega$, decreases. Thus, the considered pump field can excite a spectrum of the SWs (fig.1). A width of this spectrum, $\Delta\omega_+(E_0) - \Delta\omega_-(E_0)$, is determined by the values $\Delta\omega_{\pm}(E_0)$, at which $|E_0|_{cr} = |E_0|$ and the nonlinear growth rate γ_{NL} vanishes.

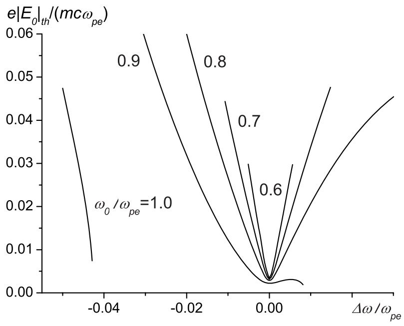

Fig.1. Influence of the pump field frequency, ω_0, and frequency detuning, $\Delta\omega$, on the threshold amplitude, $|E_0|_{th}$, for the plasma with $\nu/\omega_{pe} = 0.001$ and $(d\omega_{pe}^2/dx)^{-1}_{x=x_0}\omega_{pe}^3/c = 0.001$, bounded by fused silica with $\varepsilon_d = 3.78$

According to the results shown in fig.1, an increase in the frequency and amplitude of the pump field leads to an increase of the excited SW spectrum width, until the half-frequency of the pump field, $\omega_0/2$, comes close to the maximum value of the SW frequency, $\omega_{max} = \omega_{pe}/\sqrt{1+\varepsilon_d}$, above which the SWs do not exist [1]. It explains behavior of the curve in fig.1 corresponding to $\omega_0/\omega_{pe} = 1.0$. At that value of the parameter ω_0/ω_{pe}, the half-frequency of the pump field, $\omega_0/2$, exceeds the maximum SW frequency, ω_{max}, (in the presented calculations, $\omega_{max} \approx 0.457\ \omega_{pe}$). As a result, the excitation of the SWs with frequencies above the half-frequency of the pump field, $\omega > \omega_0/2$, becomes impossible. Further increase of ω_0 results in a decreasing of the SW spectrum, until the frequency $\omega_0/2 + \Delta\omega_-(E_0)$ reaches the value ω_{max}, after which the excitation of any SWs becomes impossible.

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ВЛИЯНИЕ НОРМАЛЬНОГО ЭЛЕКТРИЧЕСКОГО ПОЛЯ НА ДИНАМИКУ ПОВЕРХНОСТНЫХ ВОЛН

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Исследовано нерезонансное параметрическое возбуждение встречных поверхностных волн однородным в пространстве и переменным во времени электрическим полем накачки, перпендикулярным плоской границе плазма-диэлектрик. Получен и проанализирован критерий возбуждения поверхностных волн. Найдены инкременты роста поверхностных волн на начальной стадии неустойчивости, а также пороговые значения амплитуды внешнего электрического поля, выше которых возможно развитие параметрической неустойчивости. Изучен спектр возбуждаемых волн.

ВПЛИВ НОРМАЛЬНОГО ЕЛЕКТРИЧНОГО ПОЛЯ НА ДИНАМІКУ ПОВЕРХНЕВИХ ХВИЛЬ

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Досліджено нерезонансне параметричне збудження зустрічних поверхневих хвиль однорідним у просторі та змінним у часі електричним полем накачки, яке є перпендикулярним до планарної межі плазма-діелектрик. Отримано та проаналізовано критерій збудження поверхневих хвиль. Знайдено інкременти росту поверхневих хвиль на початковій стадії нестійкості, а також порогові значення амплітуди зовнішнього поля накачки, при перевищенні яких можливий розвиток параметричної нестійкості. Вивчено спектр хвиль, що збуджуються.