PLASMA TURBULENCE IN LOCALIZED SHEARED FLOWS

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Plasma configurations with strong velocity shear are considered taking into account magnetic shear. Ion temperature gradient driven drift instability and Kelvin-Helmholtz type instability are analyzed for such sheared plasma flows. Possible instability saturation mechanisms and estimation of particle diffusivity are discussed.
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1. INTRODUCTION

The objective of this work is to study instabilities and features of turbulence in localized plasma flows. Turbulence depends on strong velocity shear sufficiently in the case under consideration. Magnetic shear is taken into account, too. Ion temperature gradient driven drift instability (ITG instability) and Kelvin-Helmholtz type instability (KH instability) are of the interest in this work. These instabilities are considered in framework of non-local approach. Eigenfrequencies and eigenfunctions for ITG modes are calculated using wave equation resulting from gyrokinetic approach [1]. For KH instability we use wave equation [2] in low-frequency limit ($|\omega|<<\omega_{ci}$, ω_{ci} is ion cyclotron frequency). To estimate amplitudes of the wave modes and particle diffusivity the model of particle interactions with waves [3] can be used.

2. ITG MODES

For the calculation we use the slab model, where the magnetic field is given by

$$\mathbf{B} = B\left(\mathbf{e}_z + \frac{x}{L_s}\mathbf{e}_y\right), \qquad (1)$$

where B is the absolute value of the magnetic field, x is the “radial” distance from considered (resonant) surface, $L_s = B_z (dB_y / dx)^{-1}$ is scale length of the magnetic shear, $\mathbf{e}_z$ is the unity vector directed along the magnetic field, and $\mathbf{e}_y$ is the transversal unity vector. For localized modes in such a geometry the component of the wave vector parallel to the magnetic field is $k_\parallel = k_y x / L_s$, where k_y is the transversal component of the wave vector.

For the radial profile of the wave potential $\Phi(x)$ one can use the wave equation [1]

$$\frac{d^2\Phi}{dx^2} + Q(x,\omega)\Phi = 0 , \qquad (2)$$

where

$$Q(x,\omega) = -k_y^2 + \rho_s^{-2}\left(\frac{1-\tilde{\Omega}}{\tilde{\Omega}+K} - \frac{\bar{u}_z s_u s x}{\tilde{\Omega}(\tilde{\Omega}+K)} + \frac{s^2 x^2}{\tilde{\Omega}^2}\right), \qquad (3)$$

$\tilde{\Omega} = \dfrac{\tilde{\omega} - k_y u_{1y}}{k_y V_{*e}}$, $\tilde{\omega} = \omega - k_y u_{0y}$, ω is the complex frequency, $\mathbf{u}_1 = \mathbf{u}(x) - \mathbf{u}_0$, $\mathbf{u}(x)$ and $\mathbf{u}_0$ are the ion flow velocity and it’s value at x=0, $V_{*e} = k_B T_e /(eBL_n)$, $K = (1 + L_n / L_T)(T_i / T_e)$, $L_n = -n(dn/dx)^{-1}$ is the scale length of density gradient, $L_T = -T_i (dT_i / dx)^{-1}$ is the scale length of the ion temperature gradient, $\bar{x} = x\rho_s^{-1}$, $\rho_s = e^{-1}B^{-1}\sqrt{m_i k_B T_e}$, e is the electron charge, m_i is the ion mass, k_B is the Boltzmann constant, T_e is the electron temperature, T_i is the ion temperature, $s = L_n / L_u$, $L_u = u_{0y,z} (du_{y,z} / dx)^{-1}$ is the scale length of the ion velocity shear.

Eq. (3) for Q function obtained under the following condition:

$$k_y \rho_{Ti} \lesssim 1, \quad \delta_\perp \gtrsim \rho_{Ti}, \quad k_\parallel \upsilon_{Ti} < |\tilde{\omega}| ,$$

where ρ_{Ti} is the thermal ion gyroradius, $\delta_\perp$ is the radial width of the radial profile $\Phi(x)$, υ_{Ti} is the thermal velocity of the ions.

We consider sheared flow velocity profiles

$$u_{y,z} = u_{0y,z} \exp\left(-\frac{x^2}{2b^2}\right), \qquad (4)$$

where b is the width of sheared flow.

To find eigenfrequencies and solve wave equation we use WKB method. Corresponding dispersion equation is

$$\int_{x_-(\omega)}^{x_+(\omega)} \sqrt{Q(x,\omega)}dx = (l + 1/2)\pi , \qquad (5)$$

where $x_-(\omega)$ and $x_+(\omega)$ are zeros of the Q function (turning points), l is radial mode number. WKB solution (radial eigenfunction) is $\Phi(x) = Q^{-1/4} \exp\left(\pm i \int \sqrt{Q}dx\right)$.

In Figs. 1 and 2 results of the calculations of the growth rate $\gamma = \mathrm{Im}(\omega)$ are presented. The scale of the growth rate in these figures is $\omega_s = V_{*e}\rho_s^{-1}$.

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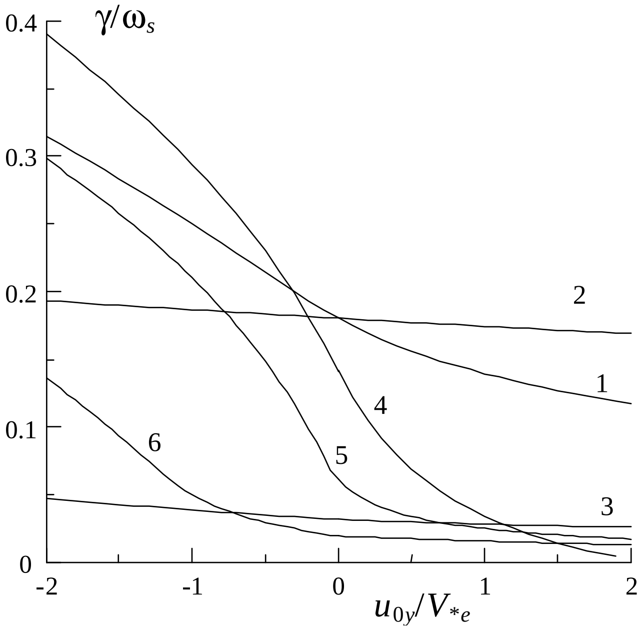

Fig. 1. Influence of perpendicular flow shear on ITG growth rate (no parallel flow shear):
1 – s = 0.1, K = 4, $k_y\rho_s$ = 0.5, b = 3ρ_s; 2 – s = 0.1, K = 4, $k_y\rho_s$ = 0.5, b = 10ρ_s; 3 – s = 0.1, K = 4, $k_y\rho_s$ = 0.1, b = 3ρ_s; 4 – s = 0.1, K = 4, $k_y\rho_s$ = 0.8, b = 3ρ_s; 5 – s = 0.01, K = 4, $k_y\rho_s$ = 0.5, b = 3ρ_s; 6 – s = 0.01, K = 2, $k_y\rho_s$ = 0.5, b = 3ρ_s

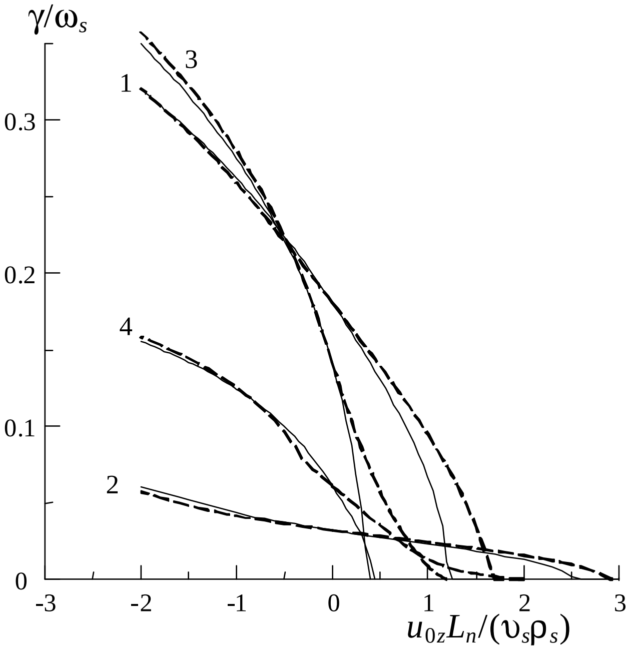

Fig. 2. Influence of parallel flow shear on ITG growth rate (no perpendicular flow shear):
1 – s = 0.1, K = 4, $k_y\rho_s$ = 0.5; 2 – s = 0.1, K = 4, $k_y\rho_s$ = 0.1; 3 – s = 0.1, K = 4, $k_y\rho_s$ = 0.8; 4 – s =0.01, K = 4, $k_y\rho_s$ = 0.5
———— – b = 10ρ_s, – – – – – – b = 3ρ_s

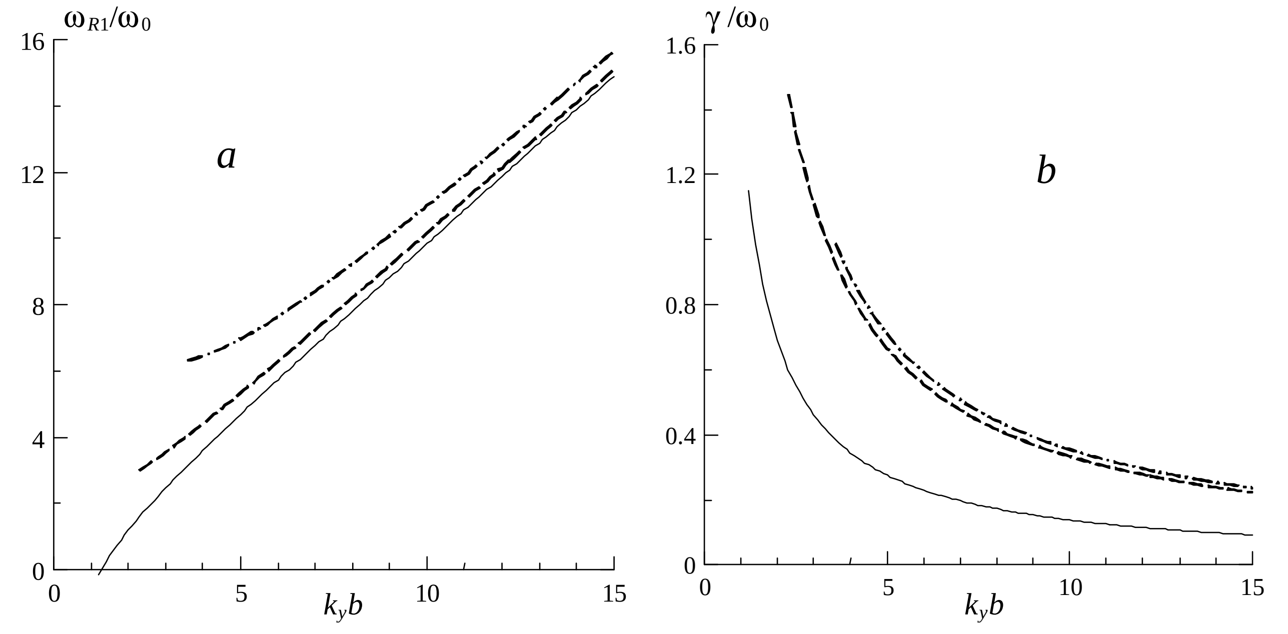

Fig. 3. Real frequencies (a) and growth rates (b) of KH modes l = 0 (————), l = 1 (– – – – –) *and* l = 2 (– - – - – - –)

3. KH MODES

In transport barriers strong sheared radial static electric field $E_r < 0$ is usually observed [4, 5]. In this case perpendicular sheared velocity $u_y = -E_r / B > 0$ leads to sufficient dumping of ITG turbulence (see Fig. 1). But, Kelvin-Helmholtz type instability can be induced by strong **E**×**B** velocity shear.

Wave equation for KH eigenfunctions has a form Eq. (2) and it can be solved using WKB theory. In low-frequency limit Q function is

$$Q(x,\omega) = -k_y^2 + \frac{V_E''(x)}{\omega / k_y - V_E(x)}, \qquad (6)$$

where $V_E(x)$ is **E**×**B** drift velocity.

Here we consider the following flow velocity profile: $V_E = V_0 + \Delta V_E (1 - x^2 / b^2)$ inside flow ($|x| < b$), and $V_E = V_0 = const$ outside flow ($|x| > b$).

Results of the calculations of the real frequency

$\omega_{R1} = \mathrm{Re}(\omega - k_y V_0)$ and the growth rate $\gamma = \mathrm{Im}(\omega)$ are presented in Fig. 3. Scale the of real frequency an the increment is $\omega_0 = \Delta V_E / b$. These results obtained for the case when radial profile is centered in shear layer. Radial structure of low-frequency KH modes was considered in [6].

For $k_y b >> 1$ turnpoints are fare from the boundaries of the flow shear layer. They rich boundaries if $k_y b$ decrease down to some minimal value $(k_y b)_{\min} \sim 1$. For $k_y b < (k_y b)_{\min}$ there are no solutions with given flow velocity profile.

4. PARTICLE DIFFUSIVITY

In this chapter we estimate particle diffusivity and saturation level in framework particle-waves interaction calculations [3].

For steady-state turbulence one can use the instability growth and dumping rates balance:

$$\gamma = (k_y^2 + \delta_\perp^{-2}) D_\perp + \frac{1}{2} k_y < \Delta u_y >, \quad (7)$$

where $D_\perp$ is the radial particle diffusivity,

$$< \Delta u_y > \approx \frac{1}{2} \Delta r^2 \left\{ \frac{k_y}{|\omega_R|} \left[\frac{1}{r} \frac{d(ru_y)}{dr} \right]^2 + \frac{1}{r} \frac{d^2(ru_y)}{dr^2} \right\} \quad (8)$$

is perpendicular drift velocity averaged over particle drift orbit, $\Delta r = \sqrt{6\pi D_\perp / |\omega_R|}$ is the radial diffusion shift of the particle during oscillation period, ω_R is real frequency (taking into account Doppler shift), r is the radial coordinate.

One can suppose for strong **E**×**B** shear case that KH instability in dominant source of turbulence. Using calculation results (presented in Fig. 3) and Eq. 7 we estimate maximal collisionless diffusivity as $D_\perp \sim 10^{-3} \Delta V_E b$ at $k_y b \sim 1$.

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ТУРБУЛЕНТНОСТЬ ПЛАЗМЫ В ЛОКАЛИЗОВАННЫХ ШИРОВЫХ ТЕЧЕНИЯХ

А.Ю. Чирков

Рассматриваются плазменные конфигурации с сильным широм скоростей, а также с учетом магнитного шира. Для таких течений плазмы анализируются ионно-температурно-градиентная неустойчивость и неустойчивость типа Кельвина-Гельмгольца. Обсуждаются механизмы насыщения неустойчивостей и оценки коэффициента диффузии.

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А.Ю. Чирков

Розглядаються плазмові конфігурації із сильним широм швидкостей, а також з обліком магнітного шира. Для таких течій плазми аналізуються іонно-температурно-градієнтна нестійкість і нестійкість типу Кельвіна-Гельмгольца. Обговорюються механізми насичення нестійкостей і оцінки коефіцієнта дифузії.