MHD-STABILITY OF COLLISIONLESS ANISOTROPIC-PRESSURE PLASMAS CONFINED BY HIGH-CURVATURE MAGNETIC FIELD

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Magnetohydrodynamic (MHD) stability of finite-beta collisionless plasmas with anisotropic pressure is investigated. For simplicity we analyze Z-pinch-like magnetic configuration with internal conducting rod at the axis. This configuration corresponds to cylindrical model of dipole magnetic configuration, has important features inherent in magnetic systems with high field-line curvature, and is very convenient for an initial theoretical analysis. Axisymmetric flute-like modes are analyzed in the frame of one fluid anisotropic magnetohydrodynamics of Chew-Goldberger-Low (CGL). The stability criterion of flute-like modes and the corresponding families of marginally-stable (MS) pressure profiles are calculated and analyzed. Contrary to the flute-like modes, stability of non-axisymmetric Alfven modes strongly depends on redistribution of plasma energy along the field-lines. Therefore, perturbations of longitudinal and transversal plasma pressures are calculated from kinetic equation using path-integral method.

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1. INTRODUCTION

We consider the stability of static equilibrium of axisymmetric finiteβ ($\beta = 2p/B^2$) plasma column confined by pure azimuthal magnetic field (Z-pinch). All perturbations of plasma parameters and magnetic fields in various one-fluid MHD-like plasma models can be described in terms of plasma displacement ξ. Due to axial symmetry the displacement can be expanded into Fourier series $\xi = \sum \xi(r, z)\exp(im\theta)$, where each harmonic can be analyzed independently. Traditional ideal MHD-model with isotropic pressure (TMHD) predicts two classes of the most dangerous perturbations in such geometry [1, 2]: flute-like modes, which don't depend on θ ($m = 0$), and incompressible Alfven modes $m \geq 1$. Flute-like instability can develop at arbitrarily small β, while Alfven modes can become unstable and dominate over flute-like ones only when β is sufficiently large and exceeds a critical value $\beta_c \sim 1$. In this paper we analyze modifications of the above instabilities in more advanced and realistic MHD-like plasma models.

2. STABILITY OF FLUTE-LIKE MODES

Plasma uniformity along magnetic field lines and the absence of longitudinal particle and energy fluxes in this magnetic configuration guarantee that axisymmetric motions of collisionless anisotropic plasma can be correctly described by Chew-Goldberger-Low (CGL) magnetohydrodynamics [3]. In particular, the stability of flute-like modes in the CGL-model has to coincide with the result of semi-kinetic analysis by Kruskal-Oberman [4].

Unlike TMHD, CGL's pressure is diagonal tensor: $p_{ik} = p_\perp\delta_{ik} + (p_\| - p_\perp)b_ib_k$; $b = B/B$. The pressure components $p_\perp$, $p_\|$ satisfy the conditions of conservation of adiabats $s_\perp = p_\perp/\rho B$, $s_\| = p_\| B^2/\rho^3$ along fluid trajectories. i.e. $s_\perp$, $s_\|$ are the Lagrangian invariants [3]. This fact enables us to find variations of pressures $\delta p_{\perp\,CGL}(\xi)$, $\delta p_{\|\,CGL}(\xi)$ under plasma displacement ξ from arbitrary dynamical state [5]. These variations are used to derive energy principle for static equilibria, that guarantee the absence of linear instabilities when $\delta^2 W(\xi) \geq 0$ for any ξ. $\delta^2 W(\xi)$ can be found by means of direct variation of potential energy [6].

Axial symmetry leads to one-dimensional equilibrium:

$$p_\perp' + (p_\perp - p_\|)/r + B\,(r B)'/r = 0\,. \tag{1}$$

Here and below prime denotes the radial derivative. Now criterion of stability of flute-like modes can be written:

$$(p_\perp + p_\|)' + 3p_\perp + 4p_\| - (1+\beta_\perp)^{-1}(\beta_\perp + \beta_\|/2)^2B^2 \equiv (1+\beta_\perp)^{-1}(B^2 - p_\|)(\lambda s_\perp)' + B\,(\lambda^3 s_\|)'/r \geq 0, \tag{2}$$

where λ is the number of particles in specific flux-tube volume $U = r/B$. Similar to $s_\perp$ and $s_\|$ λ is Lagrangean invariant in axisymmetric plasma motions.

Margin of stability is determined by equations (1) and (2), but we need an additional one to close system.

Rather often it is assumed that plasma equilibrium is isotropic one $p_\perp = p_\| = p$. In this case

$$p = \mathrm{const}\; r^{-7/2}\,[1 + (a/r)^{3/2}]^{-9}\,. \tag{3}$$

In low β case $a \to 0$

$$p \sim r^{-7/2}\,, \tag{4}$$

and the pressure profile has the more steep decrease, than in TMHD $\gamma = 5/3$ ($p \sim r^{-10/3}$). This is in accordance with comparison theorem [7, p.12.4.].

Marginally-stabile (MS) pressure profile for flute-like modes in TMHD corresponds to homogeneity of equilibrium entropy function S [1,2]. Computer simulations [2] have shown that turbulence tends to maintain the MS-state S = const with characteristic relaxation time τ_r. In our case we should expect that the similar relaxation will lead to conditions λ = const, $s_\perp$ = const, and $s_\|$ = const. As a result, we obtain the following turbulent relaxed pressure profiles:

$$\tilde{I} = I_{tot}/(c\sqrt{\pi})\,, \quad p_\perp = [r_0\tilde{I}^2/2r(r + r_0)^2][1 - (r_{cr}/r)^2]^2\,,$$

$$p_\| = [2\tilde{I}^2 r_{cr}^2/r^3(r + r_0)][1 - (r_{cr}/r)^2], \tag{5}$$

where I_{tot} – total current in inner conductor and plasma column, $r_0 = 2\,p_\perp r/B^2$ =const, $r_{cr}^{\ 2} = p_\| r^3/ 2B\,\tilde{I}$ =const . At low β

$$p_\perp \sim r^{-3},\ p_\| \sim r^{-4} \ . \qquad (6)$$

Pressure profiles, corresponding to both cases and to TMHD, in double logarithmic scale are shown at Fig. 1.

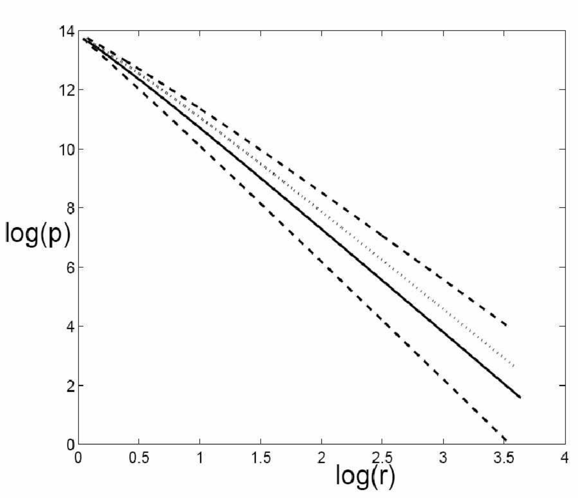

Fig.1. Characteristic pressure profiles in double logarithmic scale. Solid line - case of isotropization $p_\perp = p_\| = p$, upper and lower dashed lines - $p_\perp$ and $p_\|$ respectively, dotted line - TMHD

The characteristic time of turbulent relaxation can be comparable with the time of pressure isotropization. In this case the isotropization can appreciably modify the turbulent plasma dynamics.

3. STABILITY OF ALFVEN MODES

Stability of nonaxisymmetric Alfven modes in anisotropic collisionless plasmas appreciably depends on longitudinal redistribution of plasma energy. Near the instability threshold the growth-rates as well as the characteristic turbulent frequencies are less than the bounce frequencies of longitudinal particle motion. The CGL model is not applicable in this case. Therefore, we use kinetic path-integral method [8] to calculate the perturbation of particle distribution function f (t, **r**, **v**)and to find the corresponding pressure perturbations. In this case, under the assumption of high bounce frequencies, perturbations of all relevant plasma parameters can be expressed in terms of $\xi_\perp$. Pressure tensor remains diagonal, but behaviour of $p_\perp$ and $p_\|$ differs that in CGL. In other words, the CGL adiabats fail their applicability. This is a direct consequence of nonhydrodynamic longitudinal particles fluxes. We obtain

$$\delta p_\perp(\xi) = \delta p_{\perp\,CGL}(\xi) + (p_\perp^{\ 2}/p_\|)\,(\mathrm{div}\,\xi_\perp - \xi_r/r)\,,$$

$$\delta p_\|(\xi) = \delta p_{\|\,CGL}(\xi) - 2\,p_\perp(\mathrm{div}\,\xi_\perp - \xi_r/r)\,, \qquad (7)$$

and $\xi_\|$ is such, that

$$\mathrm{div}\,\xi = (\mathrm{div}\,\xi_\perp - \xi_r/r)(1 - p_\perp/p_\|). \qquad (8)$$

Note, that in the case of isotropic pressure $p_\perp = p_\|$ nonaxisymmetric modes are incompressible as in TMHD.

The energy principle is obtained using linearized equation of motion. Stability criterion for all m ≥ 1 takes the following form:

$$A(\beta_\perp)\,\beta_\|^{\ 2} + 2B(\beta_\perp)\,\beta_\| + C(\beta_\perp) \le 0\,. \qquad (9)$$

Maximum $\beta_\| = 0.5$ at $\beta_\perp = 0$, while maximum $\beta_\perp = 0.35$ at $\beta_\| = 0.3$. In isotropic case maximum β = 0.34, while TMHD with β= 5/3 gives $\beta_{max} = 0.4$.

Formal use of CGL in this case leads to criterion, which is similar to (8), but is much more weak. Both regions of stability are shown on the Fig. 2.

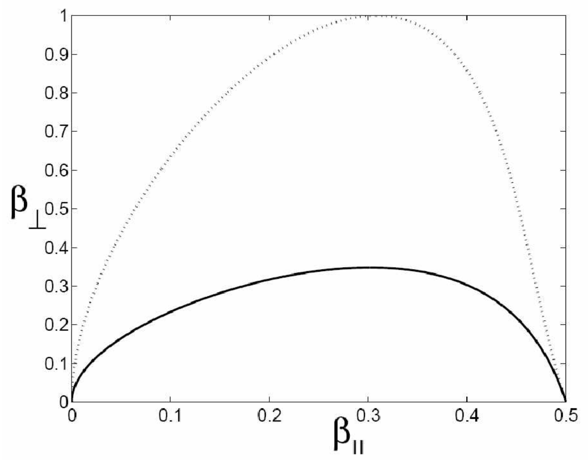

Fig.2. Regions of stability with respect to nonaxisymmetric modes (below curves); solid line – realistic model, dotted line - CGL

CONCLUSIONS

Stability criterion of flute like-modes and the corresponding families of marginally-stable (MS) pressure profiles are calculated and analyzed. Possible variants of MS plasma state formation are considered taking into account an expected turbulent relaxation and self-organization.

Stability of nonaxisymmetric Alfven modes is investigated. We have shown, that particle fluxes, arising under low-frequency perturbations, are responsible for stability criterion, which is more stringent than in CGL and TMHD both.

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МГД-УСТОЙЧИВОСТЬ БЕССТОЛКНОВИТЕЛЬНОЙ ПЛАЗМЫ С АНИЗОТРОПНЫМ ДАВЛЕНИЕМ, УДЕРЖИВАЕМОЙ МАГНИТНЫМ ПОЛЕМ БОЛЬШОЙ КРИВИЗНЫ

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Исследована магнитогидродинамическая (МГД) устойчивость бесстолкновительной плазмы с анизотропным давлением и конечной β. Для простоты мы анализировали систему типа Z-пинча с внутренним проводником на оси, которая соответствует цилиндрической модели дипольной магнитной конфигурации, обладает важными чертами, присущими магнитным системам с полем большой кривизны, и очень удобна для начального теоретического исследования. Осесимметричные желобковые моды рассматривались нами в рамках одножидкостной анизотропной гидродинамики Чу-Голдбергера-Лоу (ЧГЛ). Получен критерий устойчивости желобковых мод и рассчитаны и проанализированы соответствующие семейства гранично-устойчивых (ГУ) профилей давления. В противоположность желобковым модам, устойчивость неосесимметричных альфвеновских мод существенно зависит от перераспределения энергии плазмы вдоль магнитных силовых линий. Поэтому возмущения продольного и поперечного давления плазмы вычисляются из кинетического уравнения с помощью метода интегрирования по траекториям.

МГД-СТІЙКІСТЬ БЕЗЗІШТОВХУВАЛЬНОЇ ПЛАЗМИ З АНІЗОТРОПНИМ ТИСКОМ, ЩО УТРИМУЄТЬСЯ МАГНІТНИМ ПОЛЕМ ВЕЛИКОЇ КРИВИЗНИ

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Досліджена магнітогідродинамічна (МГД) стійкість беззіштовхувальної плазми з анізотропним тиском і кінцевою β. Для простоти ми аналізували систему типу Z-пінча з внутрішнім провідником на осі, що відповідає циліндричній моделі дипольної магнітної конфігурації, має важливі риси, властиві магнітним системам з полем великої кривизни, і дуже зручна для початкового теоретичного дослідження. Вісесиметричні жолобкові моди розглядалися нами в рамках однорідинної анізотропної гідродинаміки Чу-Голдбергера-Лоу (ЧГЛ). Отримано критерій стійкості жолобкових мод і розраховані і проаналізовані відповідні сімейства гранично-стійких (ГУ) профілів тиску. На противагу жолобковим модам, стійкість невісесиметричних альфвенівських мод істотно залежить від перерозподілу енергії плазми уздовж магнітних силових ліній. Тому збурювання подовжнього і поперечного тиску плазми обчислюються із кінетичного рівняння за допомогою методу інтегрування по траєкторіях.