

# ABOUT ABSOLUTENESS OF DATA ON ELASTIC ELECTRON SCATTERING WITH $^{12}\text{C}$ NUCLEUS

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The results obtained in Mainz in 1982 year were check up. The analysis of data from this work was made at momentum transfer range  $q = 0.25 - 0.75 \text{ fm}^{-1}$  using the model independent form factor (the expansion of form factor in a power series of  $q^2$ ) and the form factor corresponding to the distribution of charge density in the shell model framework. We found a 3% systematical overestimation in Mainz data.

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## 1. INTRODUCTION

The results of electronuclear experiments are usually brought to absolute values by means of their normalization using especially precise (master) data from elastic electron scattering. These data are obtained from elastic electron scattering with  $^{12}\text{C}$  or  $^1\text{H}$  nuclei, and, sometimes, with  $^4\text{He}$  nucleus. During the experiment for the purpose of normalization in addition to measurements with the nucleus under study we also measure elastic electron scattering cross sections of one of the nuclei, for which we possess reference data. The obtained cross sections are reduced to the nucleus ground state form factor values  $F_{el}(q_i)$ . Using the found values  $F_{el}(q_i)$  we calculate the normalization factor

$$K_i = \frac{F_{el,0}^2(q_i)}{F_{el}^2(q_i)}, \quad (1)$$

where  $F_{el,0}(q_i)$  is the reference form factor;  $q_i$  is the momentum transferred to the nucleus.

The importance of the reference form factor precision in the processing of experimental data was shown in work [1] performed in Darmstadt. Earlier the root mean square radii for  $^4\text{He}$  to  $^{209}\text{Bi}$  nuclei (24 nuclei in all), normalized to measurements with  $^{12}\text{C}$  nucleus from [2, 3], were obtained in this laboratory. In view of uncertainties about the precision of data from [2, 3] new measurements of elastic electron scattering cross section of  $^{12}\text{C}$  nucleus were carried out in Darmstadt. Using this result the renormalization for all available data was performed and the revised values of charge radii were obtained.

The latest and, obviously, the most precise work on elastic electron scattering with  $^{12}\text{C}$  nucleus was carried out in Mainz lab [4]. These data were used in the processing of our measurements results. However, a question about the probability of a systematical error in  $^{12}\text{C}$  nucleus data from ref.[4] has arisen. The

present paper is dedicated to the study of the above mentioned problem.

## 2. DATA ANALYSIS

In Mainz lab the elastic electron scattering cross section measurements of  $^{12}\text{C}$  nucleus were carried out at  $q = 0.25 - 2.75 \text{ fm}^{-1}$ . However, the measurements, which the authors of this work consider absolute, were made at  $q = 0.25 - 0.75 \text{ fm}^{-1}$  (the rest of the measurement results were relative and standardized to these data). Below we shall only analyze data from the momentum transfer range  $q \leq 0.75 \text{ fm}^{-1}$ .

The data table of electron initial energies  $E_0$ , scattering angles  $\theta$  and elastic electron scattering cross sections  $d\sigma/d\Omega$  measured on  $^{12}\text{C}$  nucleus can be found in ref.[4]. To use the results of this work for the normalization procedure, it is necessary to find the squared form factor of nucleus ground state  $F_{el,0}^2(q_i)$  at different momenta transfer  $q_i$ . For this purpose:

- (a) let us transform  $E_0$ ,  $\theta$  and  $d\sigma/d\Omega$  values to the corresponding values of  $F_{el,0}^2(q_i)$  and  $q_i$ ;

- (b) let us select the analytical function  $F_{th}^2(q)$ , which will approximate the obtained  $F_{el,0}^2(q_i)$  in the momentum transfer range we are interested in. This is necessary to avoid measuring the form factors  $F_{el}$  at the same  $q_i$  value as reference form factors  $F_{el,0}(q_i)$  during normalization using eq.(1).

Let us transform the  $E_0$ ,  $\theta$  and  $d\sigma/d\Omega$  values to the values of  $F_{el,0}^2(q_i)$  and  $q_i$ , using well-known formulas

$$F_{el,0}^2 = \frac{d\sigma/d\Omega}{\sigma_{Mott}}, \quad (2)$$

$$q = \frac{2E_0}{\hbar c} \cdot \frac{\sin(\theta/2)}{\sqrt{\eta}} \cdot \xi, \quad (3)$$

where

$$\sigma_{Mott} = \left( \frac{Ze^2}{2E_0} \right)^2 \cdot \frac{\cos^2(\theta/2)}{\eta \cdot \sin^4(\theta/2)}$$

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is the scattering cross section on the nucleus with the charge  $Z$ ;

$$\eta = 1 + \frac{2E_0 \sin^2(\theta/2)}{M}$$

is the kinematical correction,  $M$  is the nucleus mass;

$$\xi = 1 + \frac{3}{2} \cdot \frac{Ze^2}{\sqrt{5/3} \cdot R_{rms} \cdot E_0}$$

is the correction, which takes into account the influence of the nucleus coulomb field on the incoming electron. Note that the formulas shown here are from ref.[4].

For approximation of the obtained values  $F_{el,0}^2(q_i)$  we use simple presentations of nucleus ground state form factor  $F_{th}^2(q)$ . As known, some of these presentations describe the data at small momenta transfer [5] well enough and allow to obtain the values of the root mean square radius  $R_{rms}$  with fairly good precision. Such is the expansion of form factor in a power series of  $q^2$ , which is

$$F_{th}^2(q) = 1 - \frac{1}{3} \cdot a \cdot q^2 + \frac{1}{60} \cdot b \cdot q^4, \quad (4)$$

as well as the form factor of the nucleus ground state corresponding to the distribution of charge density in the shell model framework. For  $^{12}\text{C}$  nucleus this form factor can be expressed as follows [6]

$$F_{th}^2(q) = \left(1 - \frac{c^2 \cdot q^2}{9}\right)^2 \cdot \exp\left(-\frac{d^2 \cdot q^2}{2}\right). \quad (5)$$

Here  $a$ ,  $b$ ,  $c$  and  $d$  are parameters of fitting related to the root mean square radius: in the case of the form factor expansion in a power series of  $q^2$  (eq.(4))  $R_{rms} = \sqrt{a}$ , and for the form factor with the distribution of charge density in the shell model framework (eq.(5))  $R_{rms} = \sqrt{\frac{2}{3}c^2 + \frac{3}{2}d^2}$ .

By definition

$$\lim_{q \rightarrow 0} F_{el}^2(q) = 1.$$

This approach was used in some of the first  $ee'$ -scattering works and in works with especially difficult conditions of measurements (for instance, the measurements of electron scattering on  $^3\text{H}$  nuclei implanted in titanium base [7]). Thus, a variable multiplier  $k$  was introduced in analytic presentation of

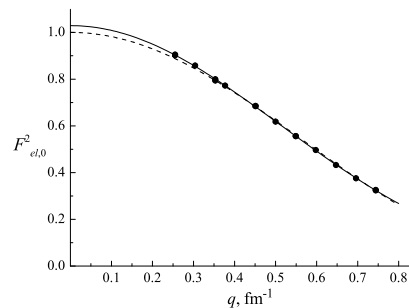
form factor which is fit to elastic electron scattering data. The  $k$  value which was obtained as a result of the fitting is precisely the normalization factor for absolutization of measured data. Using this experience, we shall write the expression for the fitting function as

$$F^2(q) = k \cdot F_{th}^2(q). \quad (6)$$

If there is no systematic deviation in the data under study, it is possible to assume the variable factor  $k = 1.0$ . Also, it is possible to leave the  $k$  factor as a variable parameter, however in this case we have to obtain its value close to 1.0 within the limits of the parameter errors.

The example of fitting eq.(4) to Mainz data with and without eq.(6) is shown in Fig.1. The statistical precision of the data is 0.45% – 0.49% therefore the errors boundaries aren't visible in the figure. The results of fitting the equations (4,5,6) to these data are shown in table.

Since the value of the parameter  $k$  appeared to be different from 1.0 approximately by 10 standard deviations, it is necessary to check whether the obtained result is dependent on the analysis conditions chosen. There are 16 experimental points in the examined momentum transfer range, and among them there are two points for each of  $q = 0.25; 0.35; 0.45; 0.55; 0.74 \text{ fm}^{-1}$ .



**Fig.1.** The squared form factor of  $^{12}\text{C}$  nucleus ground state. The closed circles are the values obtained from the data of ref. [4]; solid line is the fitting of eq.(4) with variable parameter  $k$  to these data; dashed is the same fitting with the fixed  $k = 1.0$

#### The result of fittings

	$k^{*)}$	$R_{rms}$	$\chi_i^2$	$k$	$R_{rms}$	$\chi_i^2$
power series in $q^2$	1.0	$3.07 \pm 0.05$	962.0	$1.026 \pm 0.003$	$2.42 \pm 0.01$	0.71
shell model	1.0	$2.33 \pm 0.03$	5.0	$1.029 \pm 0.003$	$2.45 \pm 0.05$	0.75

\*) The analysis with fixed value  $k = 1.0$  is shown in the left part of the table.

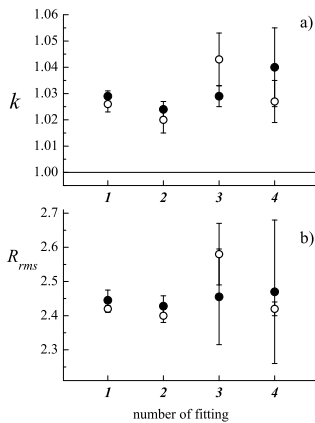
To verify whether the dependence of the obtained result on the selection of fitting range is possible, we made a number of fittings: **1** – all 16 points at  $q = 0.25 - 0.75 \text{ fm}^{-1}$ ; **2** – 13 points at  $q = 0.35 - 0.75 \text{ fm}^{-1}$ ; **3** – 8 points at  $q = 0.25 - 0.45 \text{ fm}^{-1}$  and **4** – 8 points at

$q = 0.50 - 0.75 \text{ fm}^{-1}$ . The results of this analysis are shown in Fig.2.

### 3. DISCUSSION AND CONCLUSIONS

First of all, it is necessary to note that in the case of the fitting with the fixed value  $k = 1.0$  we ob-

tained the improper  $\chi_i^2$  ( $\chi^2$  per degree of freedom), while in the case of the fitting with variable parameter  $k$ ,  $\chi_i^2 \approx 0.7$  (see table 1). As to the obtained values  $R_{rms}$ , within the limits of errors the identical values of this magnitude were found for two different presentations of form factor (eq.(4) and eq.(5)) and variable  $k$ . The values  $R_{rms}$  obtained in this case are close to 2.456 – the value of the root mean square radius of  $^{12}\text{C}$  nucleus (this value is the weighted mean of the results from a series of works [4, 8]). In case  $k$  being fixed, there is considerable discrepancy in the values of  $R_{rms}$ . Figure 2 shows that the values of variable multiplier  $k$  and  $R_{rms}$  which is obtained in this case within the limits of its errors does not depend on the selection of the fitting range.



**Fig.2.** The results of the fittings of eq.(6) with using eq.(4) (open circles) and with using eq.(5) (close circles) to the different ranges of data. The horizontal scale the represents numbers of the fitting variants (see text). a)  $k$  is the normalization factor; b)  $R_{rms}$  is the root mean square radius

Thus, the application of traditional methods of the data processing to measured results of ref. [4]

gives the acceptable values of  $\chi_i^2$  and  $R_{rms}$ . However, this results in the discrepancy in (2.6 – 2.9)% with the normalization of Mainz data (the absolute precision of the data is 0.4%, according to ref. [4]). We consider that this discrepancy in the normalization should be taken into account using the data of work [4] as master data.

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## ОБ АБСОЛЮТНОСТИ ДАННЫХ УПРУГОГО РАССЕЯНИЯ ЭЛЕКТРОНОВ НА ЯДРЕ $^{12}\text{C}$

А.Ю. Буки, И.С. Тимченко

Пересматриваются результаты измерений упругого рассеяния электронов на ядре  $^{12}\text{C}$ , выполненные в Майнце (1982 г.). Анализ данных этой работы проводится в диапазоне переданных импульсов  $q = 0.25 - 0.75 \text{ фм}^{-1}$  с разложением форм-фактора по степеням  $q^2$ , а также с использованием оболочечной модели распределения плотности заряда в ядре. Найдено 3% систематическое завышение результатов измерений рассматриваемой работы.

## ПРО АБСОЛЮТНІСТЬ ДАНИХ ПРУЖНОГО РОЗСІЯННЯ ЕЛЕКТРОНІВ НА ЯДРІ $^{12}\text{C}$

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Переглядаються результати вимірювань пружного розсіяння електронів на ядрі  $^{12}\text{C}$ , що були виконані в Майнці (1982 р.). Аналіз даних цієї роботи проводиться в діапазоні переданих імпульсів  $q = 0.25 - 0.75 \text{ фм}^{-1}$  з розкладанням форм-фактора по степенях  $q^2$ , а також з використанням оболонкової моделі розподілу щільності заряду в ядрі. Знайдено 3% систематичне завищення результатів вимірювань даної роботи.