PLASMA WAKEFIELD EXCITATION PROVIDING HOMOGENEOUS FOCUSING OF ELECTRON BUNCHES

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Wakefield plasma lens, which focuses all relativistic electron bunches of the sequence identically and uniformly, are investigated analytically and by numerical simulation. The necessary conditions of such lens operation are the followings: length of the bunches is $\xi_b=q(\lambda/2)$, q=1, 2, …(λ is the plasma wavelength), the distance between bunches is $\Delta\xi=p\lambda$, p=1, 2, … All bunches have the same charge and the 1-st bunch has a half of this charge. It is shown that with the exception of 1-st bunch all other bunches are occurred in zero longitudinal wakefield and in uniform along bunch length focusing radial wakefield. In the case of inhomogeneous longitudinal distribution of electron bunch density the middle of bunches are focused slower in comparison with their edges.

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INTRODUCTION

The focusing of relativistic electron bunches by wakefield, excited in plasma, is very interesting and important (see, for example, [1-2]). The focusing of bunches by wakefield, excited in plasma by resonant sequence of relativistic electron bunches, is inhomogeneous. In [3] the mechanism of focusing by plasma wakefield, in which all bunches of sequence are focused identically and uniformly, has been proposed and numerically investigated. We analytically and numerically investigate by 2.5D code lcode [4] the longitudinal distribution of radial wake-force, excited by sequence of lengthy electron bunches in homogeneous plasma.

1. RESULTS OF SIMULATION

At first the wakefield excitation by resonant sequence of bunches is considered. We find the longitudinal E_z, radial E_r fields and focusing force F_r in the areas of location of bunches. For this purpose we use the theory developed in [1]. For the bunch of permanent density, the length of which equals $\xi_b=\lambda/2$, we get that E_z and E_r are proportional to $Z_{II}^{(\lambda/2)}(\xi)$ and $Z_\perp^{(\lambda/2)}(\xi)$

$$Z_{II}^{(\lambda/2)}(\xi)=(2/k)\sin(k\xi),\ Z_\perp^{(\lambda/2)}(\xi)=-(2/k)\cos(k\xi). \quad (1)$$

E_z in the middle of 1st bunch equals

$$Z_{II.1}^{(\lambda/2)}=(1/k)\int_0^{\pi/2} dx_0 \cos(k\xi-x_0)|_{k\xi=\pi/2}=(1/k). \quad (2)$$

One can see it is, as well as observed, in 2 times less than amplitude of the wakefield after 1st bunch.

Now we derive the fields into the 2nd resonant bunch,

$$Z_{II.2}^{(\xi)}(\xi)=(3/k)\sin(k\xi),\ Z_{\perp.2}^{(\xi)}(\xi)=(2/k)[1-2\cos(k\xi)]. \quad (3)$$

$Z_{II.2}^{(\xi)}(k\xi)$ changes from $Z_{II.2}^{(\xi)}(x=2\pi)=0$ to $Z_{II.2}^{(\xi)}(k\xi=2.5\pi)=Z_{II.2}^{(max)}=(3/k)$ and then again $Z_{II.2}^{(\xi)}(k\xi=3\pi)=0$. Thus $Z_{\perp.2}^{(\xi)}(k\xi)$ changes from $Z_{\perp.2}^{(\xi)}(x=2\pi)=-(2/k)$ to $Z_{\perp.2}^{(\xi)}(x=3\pi)=(6/k)$, reaching zero in the 1st half of bunch, where $\cos(x_a)=1/2$, $x_a=2\pi+\pi/3<2\pi+\pi/2$. I.e. longer (in $(\pi-x_a)/x_a=2$ times) part (back front) of bunch focuses in larger field E_r than 1st front (more short) of bunch defocuses (in 3 times less field E_r). In resonant case 1st bunch only focuses, and for other bunches (if length of bunch equals $\xi_b=\lambda/2$) back fronts (more long) are in larger focusing field E_r, than (more short) 1st fronts are in defocusing E_r. I.e. this lens is inhomogeneous.

In [3] wakefield plasma lens has been numerically simulated with homogeneous focusing force for the sequence of bunches, lengths of which equal $\xi_b=\lambda/2$, with the 1st bunch, charge of which in two times less than charges of other bunches $Q_1=Q_i/2$, i=2, 3, …, distance between bunches equals 1.5λ. From Fig. 1 one can see that in this case dips of electron plasma density, in which bunches are localized, wide and shallow, and humps are narrow and high. From Fig. 1 one can see that $E_z=0$ in the areas of location of bunches.

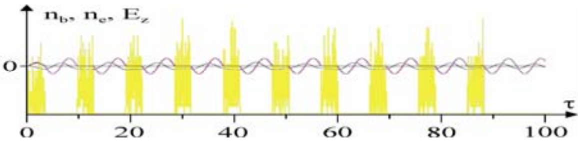

Fig. 1. Longitudinal distribution of density n_b of sequence of bunches (yellow), longitudinal wakefield E_z (red), δn_e (grey) and coupling factor $<E_z>$ of bunches with E_z (black), excited by sequence of 10 bunches. t is normalized on ω_{pe}

From Fig. 2 one can see that in the areas of bunches location F_r does not approximately depend on a longitudinal coordinate.

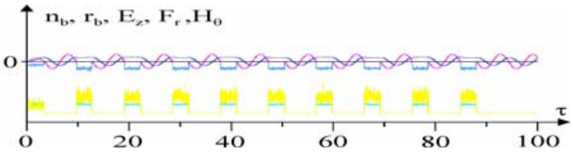

Fig. 2. n_b (yellow), radiuses r_b (blue), E_z (red), wake radial force F_r (dark blue), magnetic-field H_θ (pale blue)

In the areas of location of bunches the radial field of their volume charge is compensated by radial field, appearing as a result of shift of several number of plasma electrons from the areas of bunches location. $E_z=0$ in the areas of bunches location, except 1st one (see Fig. 2).

Thus screening of bunches takes place due to that wide (≈λ) dips of plasma electron density $\delta n_e<0$ and narrow (≈λ/2) humps of $\delta n_e>0$ appear (see Fig. 1). I.e.

plasma in the vicinity of bunches is positively charged. Due to inertness of electrons screening of 1st bunch is realized only in its end, where dip of density $\delta n_e<0$ is not flat, as for 2nd bunch, but approximately three-cornered.

For the bunch of permanent density and $\xi_b=\lambda/2$ longitudinal $Z_{II}^{(\lambda/2)}(\xi)$ and transversal $Z_{\perp}^{(\lambda/2)}(\xi)$ fields are equal (1). Now we derive the wakefield into 2nd bunch

$$Z_{II.2}^{(\xi)}(\xi)=(2/k)\sin(k\xi)+2\int_0^\xi d\xi_0\cos[k(\xi-\xi_0)+3\pi]=0 \quad (4)$$

As well as in numeral simulation, $E_z=0$ got in the areas of bunches location.

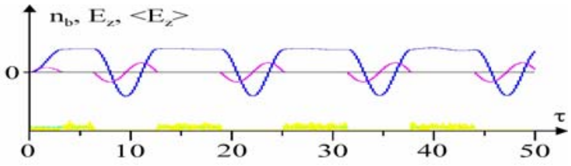

Fig. 3. n_b (yellow), r_b (blue), E_z (red), F_r (dark blue)

Now we consider wakefield plasma lenses for the sequence of bunches for three cases of their lengths at the interbunch gap equal to $\Delta\xi=\lambda$. The bunch-precursor of half-charge is used. We connect it with next (1st) bunch. 1st case: $\xi_b=\lambda$ (Fig. 3), 2nd case: $\xi_b=1.5\lambda$ (Figs. 4, 5), 3rd case: $\xi_b=2\lambda$ (Figs. 6–8). From Figs. 3–5, and 7, 8 one can see that in the areas of bunches location $E_z=0$, $F_r\approx$const.

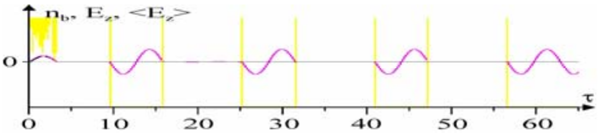

Fig. 4. n_b (yellow), E_z (red), $<E_z>$ (black)

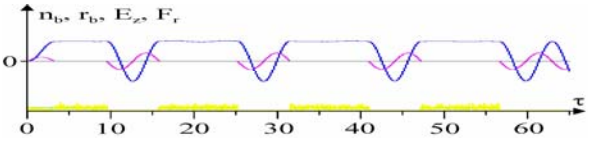

Fig. 5. n_b (yellow), r_b (blue), E_z (red), F_r (dark blue)

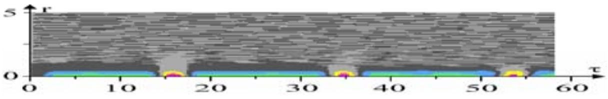

Fig. 6. Spatial distribution of δn_e in the wakefield

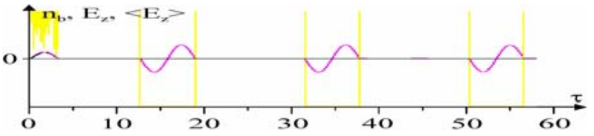

Fig. 7. n_b (yellow), E_z (red), $<E_z>$ (black)

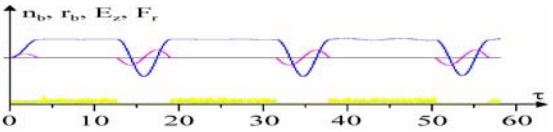

Fig. 8. n_b (yellow), r_b (blue), E_z (red), F_r (dark blue)

Now we consider the wake plasma lens for the bunches of identical charge with lengths equal to $\lambda/2$ and the interbunch gap equal to λ (Figs. 9–11).

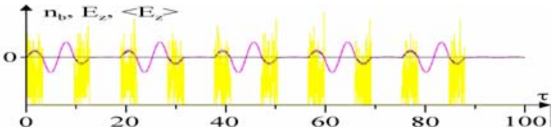

Fig. 9. n_b (yellow), E_z (red), $<E_z>$ (black)

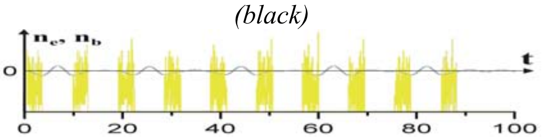

Fig. 10. n_b (yellow) u δn_e (grey)

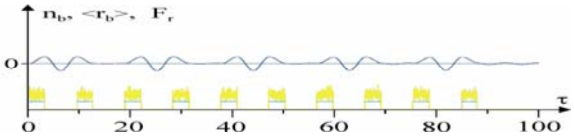

Fig. 11. n_b (yellow), r_b (blue), F_r (dark blue)

If all bunches are identical and they are placed through 1.5λ, all bunches are focused identically, but inhomogeneously along bunches (see Figs. 10, 11). Odd number bunches are decelerated, and the even number bunches are accelerated (see Fig. 9).

We consider the wake plasma lens for the bunches of identical charges with lengths and interbunch gap equal to λ (Figs. 12, 13). Both longitudinal and radial fields between bunches are equal to zero, and the fields in every bunch are equal to

$$Z_{II1}^{(\xi)}(\xi)=(1/k)\sin(k\xi),\ Z_{\perp1}^{(\xi)}(\xi)=(2/k)\sin^2(k\xi/2). \quad (5)$$

The radial field has one sign along bunch. Therefore bunches are focused, however on fronts of bunches $F_r=0$. The longitudinal field is oscillated between E_0 in the forehand of bunch and $-E_0$ in the back-end of bunch. Therefore the front half of bunch is decelerated, and the back is accelerated.

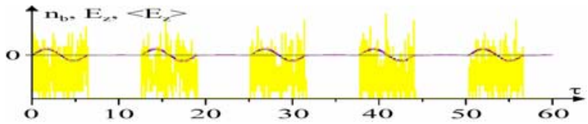

Fig. 12. n_b (yellow), E_z (red), $<E_z>$ (black)

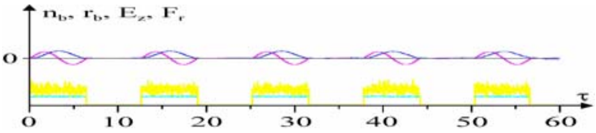

Fig. 13. n_b (yellow), r_b (blue), E_z (red), F_r (dark blue)

The results of numeral simulation for the case of length of bunches, equal to 2λ, are presented in Fig. 14.

We consider the case of electron density distribution along bunch according to $\sin^2(k\xi)$ (Figs. 15, 16). Bunches are placed through 1.5λ. The charge of 1st bunch is less than charges of other bunches in 2 times. From Fig. 16 one can see that 1st front of bunch is accelerated, and the back is decelerated.

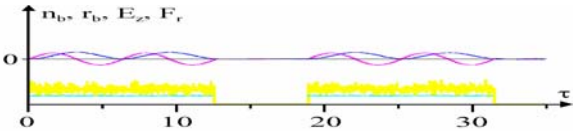

Fig. 14. n_b (yellow), r_b (blue), E_z (red), F_r (dark blue)

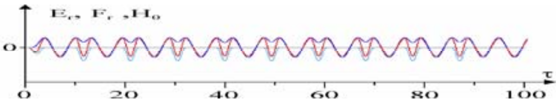

Fig. 15. E_r (red), F_r (dark blue), H_θ (pale blue), excited by sequence of 10 inhomogeneous bunches

From Fig. 15 one can see that fronts of bunches are focused stronger than their centers.

If the electrons of lengthy ($\xi_b=\lambda/2$) bunch are distributed according to $\sin^2(k\xi)$, $0<\xi_0<\lambda/2$, then after 1st bunch we get

$$Z_{II}^{(\sin 2)}(\xi)=(4/3k)\sin(k\xi). \qquad (6)$$

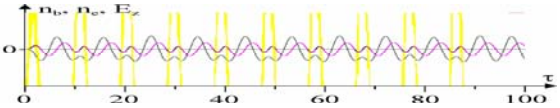

Fig. 16. n_b of 10 inhomogeneous bunches (yellow), n_e (grey), E_z (red)

Now we derive the radial field

$$Z_\perp^{(\sin 2)}(\xi)=(4/3k)\cos(k\xi) \qquad (7)$$

We obtain the field in the center of 1st bunch

$$Z_{II.1}^{(\sin 2)}=\int_0^{\lambda/4} d\xi_0\cos[k(\xi-\xi_0)]\sin^2(k\xi_0)|_{k\xi=\pi/2}=(2/3k). \quad (8)$$

One can see that, as well as observed, the field in the center of 1st bunch is in 2 times less than amplitude of the wake field after the 1st bunch.

For the fields into the 2nd bunch we derive

$$Z_{II.2}^{(\sin 2)}(\xi)=(2/3k)\sin(2k\xi). \qquad (9)$$

As well as observed, into the 2nd bunch period is in 2 times shorter and amplitude is in 2 times less than after the 1st bunch.

$$Z_{\perp .2}^{(\sin 2)}(\xi)=-(4/3k)+(2/3k)\sin^2(k\xi). \quad (10)$$

At the change into the 2nd bunch $3\pi<k\xi<4\pi$ on the edges (at $k\xi=3\pi$ and $k\xi=4\pi$) of 2nd bunch focusing is the strongest (the force is equal to $-(4/3k)$) and in the middle of the 2nd bunch (at $x=3\pi+\pi/2$), focusing is the weakest (force is in 2 times less $-(2/3k)$).

CONCLUSIONS

It has been shown that all bunches of the sequence can be focused identically and uniformly under the conditions: all bunches lengths are equal to $q\lambda/2$, q=1, 2, 3, .., the distance between them equals $p\lambda$, p=1, 2, 3, .., the charge of 1-st bunch equals a half of the charges of the other bunches. It has been shown that only 1-st bunch is in finite $E_z\neq0$. Other bunches are in zero longitudinal electrical wakefield $E_z=0$. Hence the 1-st bunch loss the energy for excitation of wakefield, which amplitude is constant along the sequence. Radial wake force F_r in regions, occupied by bunches, is constant along bunches length.

In the case of inhomogeneous longitudinal distribution of electron bunch density the middle of bunches are focused slower than edges.

If all bunches are identical and they are placed over 1.5λ, then bunches are focused identically, but not uniformly along bunch length.

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ВОЗБУЖДЕНИЕ КИЛЬВАТЕРНОГО ПОЛЯ В ПЛАЗМЕ, ОБЛАДАЮЩЕГО ОДНОРОДНОЙ ФОКУСИРОВКОЙ ЭЛЕКТРОННЫХ СГУСТКОВ

В.И. Маслов, И.Н. Онищенко, И.П. Яровая

Аналитически и численным моделированием исследуется кильватерная плазменная линза, в которой все релятивистские электронные сгустки последовательности фокусируются одинаково и однородно. При этом необходимо, чтобы сгустки имели длины, равные $\xi_b=q(\lambda/2)$, q=1, 2, …, скважность между сгустками была равна $\Delta\xi=p\lambda$, p=1, 2,…, заряд 1-го сгустка был в два раза меньше зарядов всех остальных сгустков. Показано, что только 1-й сгусток находится в конечном продольном электрическом кильватерном поле $E_z\neq0$. Другие сгустки находятся в $E_z=0$. Радиальная кильватерная сила F_r в областях расположения сгустков постоянна вдоль сгустков. В случае неоднородного продольного распределения плотности электронных сгустков их середины фокусируются медленнее, чем фронты.

ЗБУДЖЕННЯ КІЛЬВАТЕРНОГО ПОЛЯ В ПЛАЗМІ, ЯКЕ ОДНОРІДНО ФОКУСУЄ ЕЛЕКТРОННІ ЗГУСТКИ

В.І. Маслов, І.М. Онищенко, І.П. Ярова

Аналітично і чисельним моделюванням досліджується кільватерна плазмова лінза, в якій усі релятивістські електронні згустки послідовності фокусуються однаково і однорідно. При цьому необхідно, щоб згустки мали довжини, рівні $\xi_b=q(\lambda/2)$, q=1, 2, ..., шпаруватість між згустками була рівною $\Delta\xi=p\lambda$, p=1, 2,…, заряд 1-го згустка був в два рази менше зарядів усіх інших згустків. Показано, що тільки 1-й згусток знаходиться в кінцевому поздовжньому кільватерному електричному полі $E_z\neq0$. Інші згустки знаходяться в $E_z=0$. Радіальна кільватерна сила F_r в областях розташування згустків постійна уздовж згустків. У разі неоднорідного поздовжнього розподілу щільності електронних згустків їх середини фокусуються повільніше, ніж фронти.