TOPOLOGICAL GROUNDS FOR THE FAST KINEMATIC DYNAMO ORIGIN IN A KNOTTED THERMONUCLEAR REACTOR

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The paper deals with the topological grounds for the possibility of effective fast kinematic dynamo origin in so-called knotted thermonuclear reactors. On the basis of the utilization of the topologic spirality invariant the additional arguments in favour of plasma strings stability raise in such reactors are given. It is supposed that in the reactors with cavities like linking torical knots plasma confinement characteristics may be essentially improved that would allow carrying out thermonuclear synthesis.

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INTRODUCTION

The paper [1] deals with the possible constructions of so-called knotted thermonuclear reactors where according to the preliminary estimations the origin of poloidal-toroidal magnetic surfaces capable to quite an effective high-temperature plasma confinement is possible. Naturally, to prove this it is necessary to trace the evolution of very strong mutual-each other stipulated distributions of hydrodynamic $\vec{v}(t)$ velocities and $\vec{B}(t)$ magnetic field in such "knotted plasma". For this, at least, we must solve the following equation system of the ideal magnetic hydrodynamics (see, for example, [2]):

$$\frac{\partial\vec{v}}{\partial t} = -(\vec{v},\nabla)\vec{v} + (\operatorname{rot}\vec{B})\times\vec{B} - \nabla p, \quad \operatorname{div}\vec{v} = 0,$$
$$\frac{\partial\vec{B}}{\partial t} = \operatorname{rot}(\vec{v}\times\vec{B}), \quad \operatorname{div}\vec{B} = 0, \tag{1}$$

since in the first approach plasma can be treated non-compressible and carrying a "frozen" divergenceless magnetic field $\vec{B}(t)$ (from the mathematical point of view it means that any motion of the medium effects the magnetic field as diffeomorphism preserving its field lines mutual position) in spite of a very high temperature (of some million degrees) of plasma medium (the coefficients in these equations are normalized by a proper choice of measurement units). Here in the first equation the pressure gradient ∇p is determined simply by $\operatorname{div}\partial\vec{v}/\partial t = 0$ condition. However this problem can be treated from topologic point of view as well.

1. THE POSSIBILITY OF THE EFFECTIVE KINEMATIC DYNAMO ORIGIN IN KNOTTED PLASMA VOLUMES

Without solving non-linear equations (1) given in addition in quite a complicated knotted M region, let's try to substantiate the origination of exponentially increasing magnetic field in the conditions of a fast kinematic dynamo (so the velocity field $\vec{v}(t)$ is called) by means of a number of topologic characteristics. In addition the system is called kinematic dynamo equation

$$\frac{\partial\vec{B}}{\partial t} = \operatorname{rot}(\vec{v}\times\vec{B}) + \eta\Delta\vec{B}, \quad \operatorname{div}\vec{B} = 0, \tag{2}$$

where η – a magnetic viscosity being reverse to so-called Reynolds magnetic number $R_m = \eta^{-1}$, Δ – Laplacian-Beltrami operator on M. Evidently, that kinematic dynamo equation (2) is got from a complete non-linear system of magnetic hydrodynamics (1) by means of throwing back a reverse action of the magnetic field $\vec{B}(t)$ on the velocity field $\vec{v}(t)$, which is realized on the initial stage of the magnetic field increase accordingly with the dependence

$$\vec{B}(t) = \vec{B}(0)\exp(\lambda t), \tag{3}$$

in which a real part of λ value must be positive for all quite big Reynolds numbers. Further increase of the magnetic field causes a so-called self-congruent theory, i.e. a velocity field is found along with a magnetic field from a complete system of equations of magnetic hydrodynamics [2]

$$\frac{\partial\vec{v}}{\partial t} = -(\vec{v},\nabla)\vec{v} + (\operatorname{rot}\vec{B})\times\vec{B} + \nu\Delta\vec{v} - \nabla p,$$
$$\operatorname{div}\vec{B} = \operatorname{div}\vec{v} = 0,$$
$$\frac{\partial\vec{B}}{\partial t} = \operatorname{rot}(\vec{v}\times\vec{B}) + \eta\Delta\vec{B}, \tag{4}$$

where ν – a kinematic viscosity. Besides, a magnetic field (and, hence a magnetic energy) will increase at the expense of plasma medium kinetic energy pumping. Evidently, the more complicated the topology of such field is the more effective this process occurs, i.e. the dependence (3) may take place at quite large times as well which in a complicated way depend on parameters η and ν, comprising non-linear equation systems (4). Qualitatively it is analyzed by means of spirality topologic invariant which direct consideration is treated below.

2. THE ANALYSIS OF THE MAGNETIC FIELD ENERGY BY MEANS OF SPIRALITY

One of the most important topologic characteristics of such complicated fields (such as, for example, magnetic fields $\vec{B}(t)$ inside knotted thermonuclear reactors [1]) is a spirality κ (the word "spirality" was introduced into a magnetic hydrodynamics in [3])

$$\kappa(\vec{B}) = \int_M \vec{B}\cdot\vec{A}dV, \tag{5}$$

where $\vec{A}$ – a vector-potential of the field of $\vec{B}(t)$, satisfying $\nabla\times\vec{A}=\vec{B}$, $\nabla\cdot\vec{A}=0$ expressions. The spirality (5) is both the measure of twisting and the measure of linking of magnetic field lines. Really, as it is shown in [4, 5], it is proportional to a linking coefficient lk of any two field lines of $\vec{B}(t)$ field, i.e.

$$\kappa(\vec{B}) = \mathrm{lk}\cdot Q^2, \tag{6}$$

where Q – a field $\vec{B}(t)$ string through any section of a knotted volume. By means of (6) one may determine a lower border of $\vec{B}(t)$ [5] magnetic field energy:

$$E \ge C\left|\kappa(\vec{B})\right|, \tag{7}$$

where C – a positive constant, depending on a form and dimension of a compact M region. Since the spirality (6) is a global topologic invariant (Hopf invariant) regarding arbitrary diffeomorphisms of magnetic field (see [4]), the correlation (7) in an ideal medium is fulfilled precisely and in a resistive medium changes very slowly with a global time of a magnetic diffusion τ_d (for example, it occurs in astrophysics [6, 7]).

It should be noted that the spirality (6) is called a self-spirality as it is a measure of twisting and writhing of a magnetic stream tube. The coefficient of a self-linking lk in this case is Calugareanu $\mathrm{lk}=\mathrm{Wr}+\mathrm{Tw}$ invariant, where Wr – a writhing coefficient, Tw – a twisting coefficient (see, for example, [5]). In many constructions of tokamaks and stellarators the spirality increase $\kappa(\vec{B})$ of a magnetic field $\vec{B}(t)$ is realized at the expense of Wr coefficient increase (see, for example, [8, 9]). In knotted thermonuclear reactors proposed in [1], the spirality could be increased both at the expense of Wr and Tw coefficients.

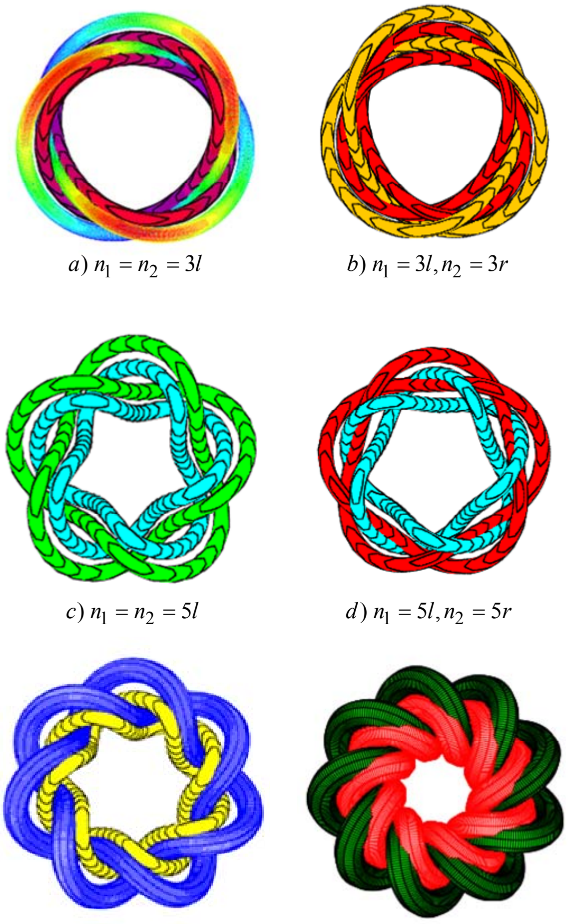
However spirality could be created at the expense of linking of various tubes of magnetic strings as well (in this case the spirality is called mutual). For example, in the case of two narrow linked current tubes the axis of which are closed curves C_1 and C_2 (Hopf linking [10]), magnetic field spirality is calculated by the formula

$$\kappa(\vec{B}) = 2\,\mathrm{lk}(C_1, C_2)\cdot Q_1 Q_2, \tag{8}$$

where $\mathrm{lk}(C_1, C_2)$ – a linking coefficient of the curves C_1 and C_2, Q_1 and Q_2 – magnetic strings in correspondent current tubes. Evidently, if the topology of tubes linking will be complicated (a current tube with writhings, self-linking, etc.), the spirality will be calculated not by the formulas (6) and (8), but by means of more complicated expression.

3. THE CONSTRUCTION OF THE REACTORS OF LINKING TORICAL KNOTS TYPE

Proceeding from mentioned above the given paper proposes new constructions of thermonuclear reactors where magnetic surfaces are presented in the form of linking torical knots (of the same- and different types n-foils [10]). For example, in Fig. 1,a the linking of two magnetic surfaces being left-hand trefoils is represented, in Fig. 1,b – a linking of left-and right-hand trefoils, in Fig. 1,c, d an analogous picture is done for five-foils, in Fig. 1,e – for left-hand seven-foils and at last in Fig. 1,f – for left-hand nine-foils.



a) $n_1 = n_2 = 3l$ *b)* $n_1 = 3l, n_2 = 3r$

c) $n_1 = n_2 = 5l$ *d)* $n_1 = 5l, n_2 = 5r$

e) $n_1 = n_2 = 7l$ *f)* $n_1 = n_2 = 9l$

Fig. 1. Schematic representation of knotted magnetic surfaces linking

One may propose just more complicated constructions of thermonuclear reactors in which magnetic surfaces could be presented in the form of multiple knots different linkings. Evidently, in such constructions the spirality should be just larger than in the above constructions, and in the long run it will contribute to the raise of thermonuclear reactor stability. May be such complicated magnetic fields with anomalously large spiralities are originated in nature spontaneously in ball lightnings.

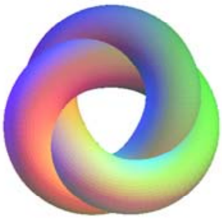


Fig. 2. A schematic representation of a thermonuclear reactor having a trefoil type cavity

CONCLUSIONS

Thus, the given paper deals with the additional arguments in favour of plasma confinement in multiple knotted volumes to the grounds presented in [1], since their complicated topology by itself leads to the origination of self-confined strong magnetic fields which when all is said and done may promote thermonuclear synthesis reaction origin.

And at last, on the basis of the phenomenological investigation carried out in this paper and in [1], the authors propose the following technical project: to construct in a laboratory a thermonuclear reactor with a cavity like the simplest tore knot – a trefoil (Fig. 2).

Preliminary evaluations show that the time of plasma confinement in such a construction just should have approximately an order increase.

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ТОПОЛОГИЧЕСКОЕ ОБОСНОВАНИЕ ВОЗНИКНОВЕНИЯ БЫСТРОГО КИНЕМАТИЧЕСКОГО ДИНАМО В ЗАУЗЛЕННОМ ТЕРМОЯДЕРНОМ РЕАКТОРЕ

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Проводится топологическое обоснование возможности возникновения эффективного быстрого кинематического динамо в так называемых заузленных термоядерных реакторах. На основе использования топологического инварианта спиральности приводятся дополнительные аргументы в пользу повышения устойчивости плазменных потоков в таких реакторах. Высказываются предположения, что для реакторов с полостями в виде зацепленных торических заузленностей характеристики удержания плазмы могут резко улучшиться, что позволило бы осуществить термоядерный синтез.

ТОПОЛОГІЧНЕ ОБҐРУНТУВАННЯ ВИНИКНЕННЯ ШВИДКОГО КІНЕМАТИЧНОГО ДИНАМО В ЗАВУЗЛЕНОМУ ТЕРМОЯДЕРНОМУ РЕАКТОРІ

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Проводиться топологічне обгрунтування можливості виникнення ефективного швидкого кінематичного динамо в так званих завузлених термоядерних реакторах. На основі використання топологічного інваріанта спіральності приводяться додаткові аргументи на користь підвищення стійкості плазмових потоків у таких реакторах. Висловлюються припущення, що для реакторів з порожнинами у вигляді зачеплених торічних завузленостей характеристики утримання плазми можуть різко покращитися, що дозволило б здійснити термоядерний синтез.