# DRIFT MOTION OF CHARGED PARTICLE IN WAVE FIELD OF MAGNETIC PUMPING UNDER CHERENKOV AND CYCLOTRON RESONANCE CONDITIONS

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The charged particle motion problem in electromagnetic field of magnetic pumping under Chrenkov and cyclotron resonance conditions is solved in drift approximation. The wave field is produced by alternating surface azimuthal current, modeling the current of solenoidal antenna, which use is considered within the frames of a developed ICR-method of isotope separation. The drift motion equations are derived and their three first integrals are found at arbitrary values of Larmor radius. It is shown that the increasing of a particle Larmor radius involves the increasing of radius of the Larmor center, i.e. involves drift of heated particles to plasma edge. During Larmor gyration these ions transit near to a system axis.

PACS: 52.40.Fd; 52.50.Qt

Interaction of particles with a wave is a basis for calculation of effect of a selective heating of ions in a developed ICR-method of isotope separation [1]. In this paper the solution of a problem about a charged particle motion in the homogeneous magnetic field and in the vortex electromagnetic field of a wave of magnetic pumping of small amplitude under Cherenkov and cyclotron resonance conditions is presented in the drift approximation.

## 1. WAVE OF MAGNETIC PUMPING

The wave is produced by the azimuthal surface current $j_\varphi = j_0\delta(r-a)\cos(k_z z-\omega t)$ modeling the current in solenoidal antenna. Its usage is considered within the method of ICR-separation [1]. The wave field inside the solenoid ($r<a$) has components $E_\varphi, H_r, H_z$ [2]:

$$\begin{cases} E_\varphi = -CI_1(\Lambda r)\sin(k_z z-\omega t), \\ H_z = C\dfrac{k_z c}{\omega}\dfrac{\Lambda}{k_z}I_0(\Lambda r)\cos(k_z z-\omega t), \\ H_r = C\dfrac{k_z c}{\omega}I_1(\Lambda r)\sin(k_z z-\omega t)\ . \end{cases} \quad (1)$$

Here $C=\dfrac{E_0}{I_1(\Lambda a)}$, $E_0=\dfrac{4\pi\omega}{c^2}j_0 a K_1(\Lambda a)I_1(\Lambda a)$ – is the amplitude of azimuthal electric field near the solenoid ($r=a$), $\Lambda^2=k_z^2-\dfrac{\omega^2}{c^2}>0$, $N_{\parallel}=\dfrac{k_z c}{\omega}>>1$.

## 2. DERIVATION OF DRIFT EQUATIONS

We derive the equations of particle drift motion in the field (1) using the method [3]. Introducing a complex variable $u=x+iy=r\exp(i\varphi)$ we write equations of motion in this variable:

$$\begin{cases} \ddot u + i\omega_{ci}\dot u = \\ = i\left(\dfrac{e}{M}E_\varphi\cdot\exp(i\varphi)-\omega_{ci}\dfrac{H_z}{B_0}\dot u+\omega_{ci}\dfrac{H_r}{B_0}\dot z\cdot\exp(i\varphi)\right), \\ \ddot z = -\dfrac{e}{M}\dfrac{v_\varphi}{c}H_r\,. \end{cases} \quad (2)$$

Here $\omega_{ci}=eB_0/Mc>0$ – is the cyclotron frequency of a particle. We search he solution of equations (2) in the form

$$u = r\exp(i\varphi) = R\cdot\exp(i\theta)+\rho\cdot\exp(i\vartheta) \quad (3)$$

Particle motion is described by cylindrical coordinates of the Larmor center $R$, $\theta$, coordinates of a particle on the Larmor circle $\rho$, $\vartheta=\vartheta_0-\omega_c t$ ($\vartheta_0$ - initial phase of Larmor rotation) and longitudinal variables $z=z_0+\int_0^t v_z dt$, $v_z$.

According to (3) vectors $r\exp(i\varphi)$, $R\cdot exp(i\theta)$, $\rho\cdot\exp[i(\vartheta_0-\omega_{ci}t)]$ form a triangle. Taking this fact into account we apply the Graf summation theorem for Bessel functions [4], having in our case the form

$$I_1(\Lambda r)\exp(i\varphi) = \\ = \sum_{p=-\infty}^{p=+\infty} I_{1+p}(\Lambda R)I_p(\Lambda\rho)\exp\{i[\theta-p(\vartheta_0-\theta)+p\omega_{ci}t]\}, \quad (4)$$

to the terms in the right-hand sides of equations (2) and express them in variables $R,\theta,\rho,\vartheta_0,z,v_z$ .

We find the approximate solution of equations (2) supposing that a wave amplitude is small ($eE_0/(M\omega_{ci}r)<<\omega_{ci}$) and a particle moves under Cherenkov or cyclotron resonance conditions with a pumping wave

$$\omega = +k_z v_z + n\omega_{ci}+\Delta\omega,\ n=\pm1,\pm2,\pm3,...,\ \ |\Delta\omega|<<\omega_{ci}. \quad (5)$$

In the absence of wave the coordinates $R,\theta,\rho,\vartheta_0,z_0,v_z$ are the integrals of motion. In the presence of wave of small amplitude the right-hand sides of equations (2) contain small fast and slow oscillating summands. Neglecting the fast oscillating summands and taking into account only the slow oscillating ones (i.e. using the method of averaging), we obtain the equations of particle drift motion:

$$
\begin{cases}
\dot{R} = -\dfrac{e}{M\omega_{ci}} C \dfrac{\rho}{R} \dfrac{n\omega_{ci}}{\omega} I_n(\Lambda R) \dfrac{dI_n(\Lambda\rho)}{d(\Lambda\rho)} \sin\Psi_n, \\
R\dot{\theta} = \dfrac{e}{M\omega_{ci}} C \dfrac{\omega_{ci}}{\omega} \Lambda\rho \dfrac{dI_n(\Lambda R)}{d(\Lambda R)} \dfrac{dI_n(\Lambda\rho)}{d(\Lambda\rho)} \cos\Psi_n, \\
\dot{\rho} = -\dfrac{e}{M\omega_{ci}} C \dfrac{n\omega_{ci}}{\omega} I_n(\Lambda R) \dfrac{dI_n(\Lambda\rho)}{d(\Lambda\rho)} \sin\Psi_n, \\
\rho\dot{\vartheta}_0 = \\
-\dfrac{e}{M\omega_{ci}} C \dfrac{\omega_{ci}}{\omega} I_n(\Lambda R) \dfrac{d}{d(\Lambda\rho)} \left[ (\Lambda\rho) \dfrac{dI_n(\Lambda\rho)}{d(\Lambda\rho)} \right] \cos\Psi_n, \\
\dot{z} = v_z, \\
\dot{v}_z = -\dfrac{e}{Mc} C \dfrac{k_z c}{\omega} \omega_{ci} \rho I_n(\Lambda R) \dfrac{dI_n(\Lambda\rho)}{d(\Lambda\rho)} \sin\Psi_n.
\end{cases}
\qquad (6)
$$

Here $\Psi_n \equiv n(\vartheta_0 - \theta) + (\omega - n\omega_{ci})t - k_z z$ – is a resonance phase slowly changing under conditions (5). Using the equations for $\theta$ and $\vartheta$ in (6) we obtain the equation for $\Psi_n$ in the form

$$
\begin{aligned}
\dot{\Psi}_n &= (\omega - n\omega_{ci} - k_z v_z) - \\
&- \frac{e}{M\omega_{ci}} C \frac{n\omega_{ci}}{\omega} \Lambda \left\{ I_n(\Lambda R) \frac{1}{(\Lambda\rho)} \frac{d}{d(\Lambda\rho)} \left[ (\Lambda\rho) \frac{dI_n(\Lambda\rho)}{d(\Lambda\rho)} \right] + \right. \\
&\left. + \frac{\rho}{R} \frac{dI_n(\Lambda R)}{d(\Lambda R)} \frac{dI_n(\Lambda\rho)}{d(\Lambda\rho)} \right\} \cos\Psi_n .
\end{aligned}
\qquad (7)
$$

The equations for $R$, $\rho$, $\Psi_n$, $v_z$ (6), (7) form the closed set of equations. The phases $\theta$ and $\vartheta$ are determined by the solution of this closed set.

## 3. INTEGRALS OF DRIFT MOTION

Combining the equations for $\dot{\rho}$ and $\dot{R}$ we find one first integral of the equations set (6):

$$
R^2 - \rho^2 = C_1 = \frac{2P_\varphi}{M\omega_{ci}} . \qquad (8)
$$

It determines the form of drift trajectories in $R\rho$ plane.

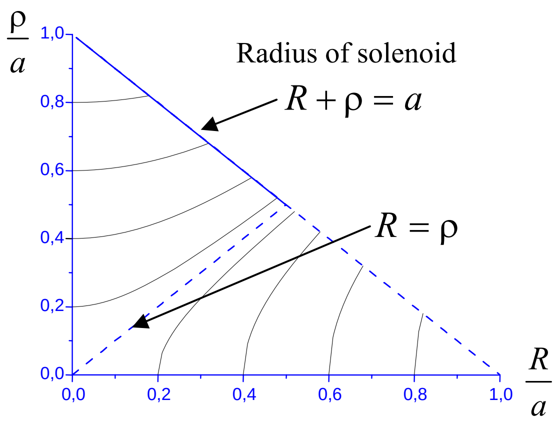


*Fig. 1. Drift trajectories in $R\rho$ plane. Line $R+\rho=a$ corresponds to a radius of solenoid, line $R=\rho$ is the asymplote of hyperbolas*

The trajectories have the form of hyperbola (Fig. 1) along which the ions drift under action of the wave field of magnetic pumping (1) under cyclotron resonance conditions (5). As it is seen from integral (8), the form of drift trajectories does not depend on the number of cyclotron harmonic $n$ on which the resonance is realized. This is a consequence of axial symmetry of a wave field (1) and conservation of the generalized angular momentum $P_\varphi$ .

If the values of radius of Larmor center $R$ and Larmor radius $\rho$ satisfy the condition $R+\rho<a$, then the particle, moving on Larmor trajectory, remains inside the cylinder $r=a$, on which the solenoidal antenna is placed, and interacts with a wave of magnetic pumping. If coordinates of a particle satisfy the condition $R+\rho \geq a$, then particle falls on the antenna during Larmor rotation and stop the interaction with a wave.

Combining the equations for $\dot{R}$ and $\dot{v}_z$ we find one more first integral of the equations set (6), determining drift trajectories in $Rv_z$ plane:

$$
v_z - (\omega_c k_z / 2n) R^2 = C_2 . \qquad (9)
$$

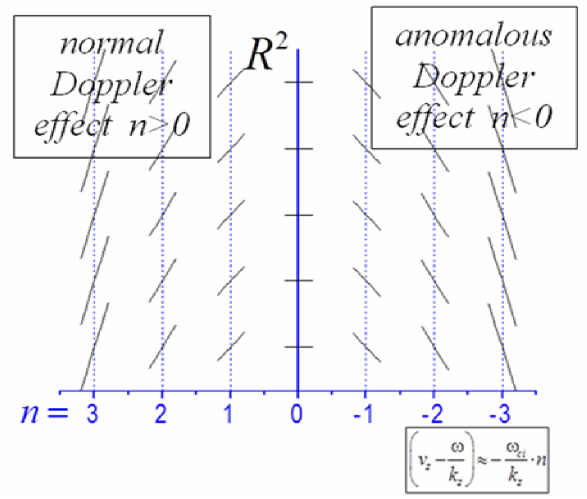


*Fig. 2. Drift trajectories in $R^2 v_z$ plane. Along $y$ axis the value $R^2$ varies. Along $x$ axis the longitudinal velocity varies in frame of reference, moving with a wave $(v_z - \omega/k_z)$. The cyclotron resonances (5) take place only if the approximate equality is fulfilled $(v_z - \omega/k_z) \approx -n \cdot \omega_{ci}/k_z$ . Integer numbers $n$ along $x$ axis specify the number of resonance cyclotron harmonic. Positive $n$ correspond to a normal Doppler effect, negative $n$ – to anomalous Doppler effect*

At $k_z \neq 0$ the trajectory has the form of parabola in $Rv_z$ plane. More accurately, they are the segments of parabola placed near velocity values, where the resonance condition (5) is fulfilled. In Fig. 2 the trajectories are presented in axes $R^2 v_z$ . In these axes the trajectories are the straight line segments.

At $k_z = 0$ the trajectories degenerate into the lines $v_z = const$ . In this case drift equations for $R$, $\rho$ and $\Psi_n$ form the closed set.

The third first integral is a Hamiltonian of a particle in the magnetic field and in the field of a pumping wave. Its form can be determined by combining the equations (6) or by averaging the exact expression of Hamiltonian. Finally, we obtain its form:

$$H \equiv H_0 + H_1 = C_3 . \qquad (10)$$

Here

$$H_0 = \frac{M\omega_{ci}^2\rho^2}{2} + \frac{p_z^2}{2M} \qquad (11)$$

is the well-known unperturbed (without pumping wave) Hamiltonian of a particle and

$$H_1 = e\frac{\omega_{ci}}{\omega} C I_n(\Lambda R)\rho I_n'(\Lambda\rho)\cos\Psi_n \qquad (12)$$

is the addition describing the interaction of a particle with a pumping wave. It should be noted, that expressions (10) – (12) determine Hamiltonian function relative to canonically conjugated variables $(J_R, \theta)$, $(J_\rho, -\vartheta)$, $(p_z, z)$, where $J_R = (1/2)M\omega_{ci}R^2$, $J_\rho = (1/2)M\omega_{ci}\rho^2$ are the known adiabatic invariants of a particle in magnetic field.

## CONCLUSIONS

The drift equations are derived and its integrals are found for particle moving in a homogeneous magnetic field and in a vortex wave field of magnetic pumping under Cherenkov and cyclotron resonance conditions. The drift equations (6), (7) and integrals (8)-(12) are obtained at arbitrary value of particle Larmor radius $\rho$.

The found integrals (8)-(12) make it possible to integrate on time the equations of drift motion (6), (7), to build trajectories in a phase space $R, \rho, v_z, \Psi_n$ and thus to solve the motion problem completely.

As it is seen from integrals (8), (9) the form of drift trajectories does not depend on amplitude and the distribution of the wave field on radius. The drift velocities, of course, depend on these factors.

The integrals (8), (9) coincide with correspondent integrals of the drift motion of a particle in the field of a running along magnetic field potential wave having azimuthal number $m = 0$ [3].

As results from the integral (8), the increasing of a particle Larmor radius $\rho$ involves the increasing of radius of the Larmor centre $R$, i.e. pumping wave involves drift of heated particles outside, to plasma edge. This peculiarity explains the observed radial drift of particles interacting with a pumping wave in numerical calculations [4, 5].

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*Article received 10.10.12*

## ДРЕЙФОВОЕ ДВИЖЕНИЕ ЗАРЯЖЕННОЙ ЧАСТИЦЫ В ПОЛЕ ВОЛНЫ МАГНИТНОЙ НАКАЧКИ В УСЛОВИЯХ ЧЕРЕНКОВСКОГО И ЦИКЛОТРОННОГО РЕЗОНАНСОВ

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В дрейфовом приближении решена задача о движении заряженной частицы в поле волны магнитной накачки в условиях черенковского и циклотронного резонансов. Поле волны создается поверхностным переменным азимутальным током, моделирующим ток соленоидальной антенны, использование которой рассматривается в рамках разрабатываемого ИЦР-метода разделения элементов и изотопов. Выведены уравнения дрейфового движения, справедливые при произвольной величине ларморовского радиуса, и найдены три их первых интеграла. Показано, что увеличение ларморовского радиуса частицы под действием волны накачки сопровождается увеличением радиуса ее ларморовского центра, т.е. дрейфом нагретых частиц на периферию плазмы. При ларморовском вращении эти ионы проходят вблизи оси системы.

## ДРЕЙФОВИЙ РУХ ЗАРЯДЖЕНОЇ ЧАСТКИ В ПОЛІ ХВИЛІ МАГНІТНОГО НАКАЧУВАННЯ В УМОВАХ ЧЕРЕНКОВСЬКОГО Й ЦИКЛОТРОННОГО РЕЗОНАНСІВ

***Ю.М. Єлісеєв, К.М. Степанов***

У дрейфовому наближенні вирішено задачу про рух зарядженої частки в полі хвилі магнітного накачування в умовах черенковського й циклотронного резонансів. Поле хвилі створюється поверхневим змінним азимутальним струмом, що моделює струм соленоїдальної антени, використання якої розглядається в рамках розроблюваного ІЦР-методу розділення елементів і ізотопів. Виведені рівняння дрейфового руху справедливі при довільній величині ларморовського радіуса, і знайдено три їх перших інтеграла. Показано, що збільшення ларморовського радіуса частки під дією хвилі накачування супроводжується збільшенням радіуса її ларморовського центра, тобто дрейфом нагрітих часток на периферію плазми. При ларморовському обертанні ці іони проходять поблизу осі системи.