EXPLOSIVE INSTABILITY IN THE PLASMA-BEAM SYSTEMS

V.A. Buts, I.K. Koval'chuk
National Science Center "Kharkov Institute of Physics and Technology", Kharkov, Ukraine
E-mail: vbuts@kipt.kharkov.ua

The results of investigation of explosive instability in the plasma-beam systems are presented. It has been shown that in the general case, the dynamics of this instability can be essentially changed, right up to its full suppression, if one takes into account a fast beam wave (wave with positive energy). The parameters when explosive instability is realized and when it is suppressed are defined. It was shown that in the case of such four wave interaction there are regimes with attributes of irregularity.

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INTRODUCTION

Processes of nonlinear wave interaction belong to the key processes of the plasma theory and are well investigated. It is possible to mark out two subsections of the theory of wave interaction: (i) coherent nonlinear wave interaction, and (ii) interaction of waves in the random phase approximation. Here we will consider the coherent interaction only. The process of three wave interaction is the main element of such interactions. Similar interaction can be useful, for example, at the wave excitation in the scheme of acceleration in beat-wave. This interaction may essentially restrict the level of exciting oscillations in the plasma-beam generators (e.g., [1–5]).

Among three wave interaction the decay processes are essentially interest. Especially, when decaying wave has negative energy. The explosive instability arises in this case. Usually stabilization of this instability occurs due to cubic nonlinearities, and therefore the level of exciting oscillations can be high enough. Often, when considering such processes, the authors do restrict themselves by a three wave interaction.

However in many cases besides decaying wave with negative energy there are waves with positive energy. If the characteristic of such waves is near the characteristics of wave with negative energy, such waves can take part in the nonlinear interaction and significantly change the character of such interaction. Especially this concerns the beam systems. In reality, both the fast and slow waves exist in such systems. The slow wave has a negative energy and fast one has a positive energy. The wave characteristics of these waves are always close. So, in the general case it is naturally and necessary to take into account both these waves in the nonlinear dynamics.

This article is devoted just to this problem. We show that decay processes of the beam waves, when taking into account the fast and slow waves, can significantly differ from the processes when only one of these waves takes part in the process.

1. BASIC EQUATIONS

Let us consider electrodynamic system through which electron beam moves. There are two beam waves in this system, the dispersion of which may be presented as follows:

$$\omega_{11,12} = k_1 V \pm \delta\omega \, , \qquad (1)$$

where $\omega_{11,12}$ – are frequencies of beam waves; k_1 – projection of wave vector on the direction of beam propagation; V – its velocity, $\delta\omega \sim \omega_b$, ω_b – is the plasma frequency of beam. The second of these two waves is the wave with negative energy. Besides, it is supposed that in this electrodynamics system there are two natural waves with frequencies less than $\omega_{11,12}$. We will consider interaction of two beam modes and natural waves of system which is effectively realized when synchronism conditions are satisfied (conservation laws), Fig 1:

$$\omega_{12} = \omega_2 + \omega_3 , \; \vec{k}_1 = \vec{k}_2 + \vec{k}_3 , \qquad (2)$$

where $\omega_{2,3}$ – frequencies of third and fourth waves (system of eigen waves) taking part in interaction, $k_{2,3}$ – are their wave numbers.

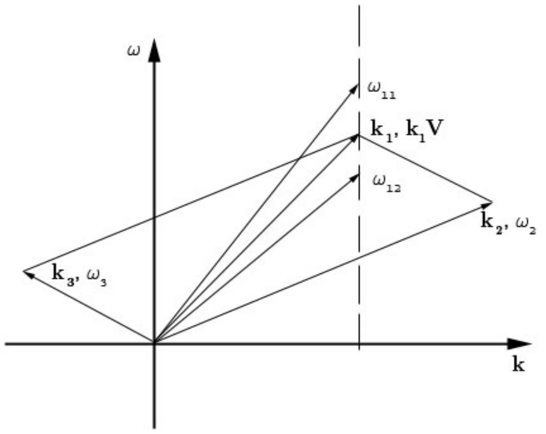

Fig. 1. The possible scheme of wave interaction

Such nonlinear interaction, in which slow beam wave (wave with negative energy) takes part, ordinarily results in rising an explosive instability. If in expressions (2) to change index 12 to 11, this scheme will correspond to a decay process. Thus in the beam system there are two close waves participating in the fully different processes with two other modes of electrodynamics system.

Usually in theoretical investigations only explosive instability is considered in which slow beam wave takes part but the influence of the fast one is not taken in account. It is interesting to define how accounting of fast wave influences the explosive instability.

The set of shortened equations for dimensionless slowly varying amplitudes of all four interacting modes was obtained in the standard way from Maxwell equations for electromagnetic fields and the hydrodynamical equation for particles. This set is as following:

$$\frac{dE_{11}}{d\tau} = -\mu E_2 E_3 \exp\left(i(\Delta+\delta\omega)\tau\right);$$
$$\frac{dE_{12}}{d\tau} = \mu E_2 E_3 \exp\left(i(\Delta-\delta\omega)\tau\right); \quad (3)$$
$$\frac{dE_2}{d\tau} = \left[E_{11}\exp\left(-i(\Delta+\delta\omega)\tau\right) + E_{12}\exp\left(-i(\Delta-\delta\omega)\tau\right)\right]E_3^*;$$
$$\frac{dE_3}{d\tau} = \left[E_{11}\exp\left(-i(\Delta+\delta\omega)\tau\right) + E_{12}\exp\left(-i(\Delta-\delta\omega)\tau\right)\right]E_2^*,$$

where E_{11} and E_{12} – ($E \to eE/(mc\omega)$, m – is electron mass; c – the velocity of light) – are dimensionless slowly varying complex amplitudes of HF and LF beam waves; E_2 and E_3 – dimensionless slowly varying complex amplitudes of any other natural waves of the electrodynamic system, which can take part in the investigated process; μ – dimensionless coefficient which is proportional to cube of beam density. Dimensionless time is measured in the periods of $k_1 V$. Dimensionless frequencies are normalized to $k_1 V$. Δ – characterizes synchronism conditions 2 and 3 waves with beam modes and is defined by means correlation:

$$\omega_2 + \omega_3 = 1 - \Delta . \quad (4)$$

When there is synchronism with slow beam wave the condition $\Delta = \delta\omega$ is satisfied. If there is synchronism with fast beam wave the condition $\Delta = -\delta\omega$ is satisfied.

2. RESULTS OF INVESTIGATION

The set of equations (3) has integral:

$$|E_2|^2 - |E_3|^2 = const . \quad (5)$$

The equations describing three wave explosive processes have similar integral. It follows from (5) that amplitudes of second and third waves can grow infinitely.

The set of equations (3) was investigated numerically for two cases: $\Delta = \delta\omega$ when second and third waves are in synchronism with slow beam mode (wave with negative energy), and $\Delta = -\delta\omega$ when these waves are in synchronism with fast beam mode.

When condition $\Delta = \delta\omega \to 0$ is satisfied the set of equations has the integral:

$$E_{11} + E_{12} = C_0 , \quad (6)$$

and for non beam modes the next expressions can be found:

$$E_2 = C_{21}\exp\left(|C_0|\tau\right) + C_{22}\exp\left(-|C_0|\tau\right),$$
$$E_3 = C_{31}\exp\left(|C_0|\tau\right) + C_{32}\exp\left(-|C_0|\tau\right), \quad (7)$$

what corresponds to infinite exponential growth. For amplitudes of beam modes similar expressions can be obtained but in an exponent the constant C_0 has to be changed to $2|C_0|$. This exponential growth is not connected with linear stage of beam instability, but is the result of nonlinear four wave interaction.

When there is synchronism of second and third waves with fast and slow beam waves, the following values of parameter $\delta\omega$ were selected: $\delta\omega = 1.0\times10^{-6}$, 0.001, 0.01, 0.1, 0.2. Besides, two regimes corresponding to different initial conditions were investigated. In the first case the initial dimensionless values of beam mode amplitudes were taken equal to 0.1 in dimensionless units, and initial values of two other modes of the investigated electrodynamic system were much less. In the second regime the initial value of slow beam mode having negative energy was chosen equal to zero. The characteristic temporal dependence of slow wave amplitude in the logarithm scale for case $\Delta = \delta\omega \neq 0$ is shown in Fig. 2.

The amplitudes of other waves have qualitatively similar temporal dependence. As follows from Fig. 2, the exponential growth is changed by explosive instability. The time of its rising decreases when parameter $\delta\omega$ increases. The threshold value of amplitudes corresponding to the beginning of the explosive instability decreases also. When there is synchronism of second and third waves with fast beam mode (the condition $\Delta = -\delta\omega$ is satisfied) for the values $\delta\omega = 0\ldots0.1$ the dynamics of the process is practically identical with the case $\Delta = \delta\omega$. But for $\delta\omega = 0.2$ it essentially changes.

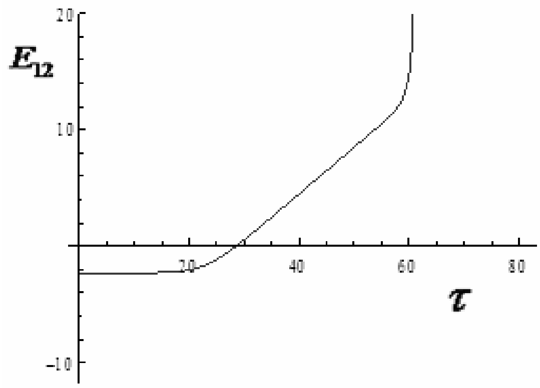

Fig.2. Temporal dependence of slow beam wave amplitude in the logarithmic scale

The explosive instability arises noticeably later than in the considered cases when characteristic time of instability growth was approximately 40…60 time units. Now this instability appears at more than 400 units. Up to this moment the well expressed exponential growth is absent and the dynamics has an oscillating character with an attribute of nonregularity. This is confirmed by investigations of frequency spectra and autocorrelation functions. The spectrum is wide and autocorrelation function decreases. The time of the explosive instability start is very sensitive to initial conditions in this case.

In the case when initial amplitude of slow beam wave equals zero for values $\delta\omega = 1.0\times10^{-6}$, 0.001, 0.01 the explosive instability arises. But when $\delta\omega = 0.1$ it is absent. If to approach to this value from left, the time of explosive instability start tends to infinity.

CONCLUSIONS

Thus, in the general way, for the beam and plasma-beam systems as well as for the systems where there are waves with positive and negative energy and closed wave characteristics, it is necessary to consider the dynamics of four waves instead of three. Such four wave dynamics can essentially differ from the three wave one. Most impotent distinctive features of four wave dynamics are:

1. At three wave interaction the decay of fast beam wave takes place. This decay is characterized by a periodical dynamics. If we take into consideration close waves with negative energy, the periodical dynamics does not occur, but the result will be an exponential growth of amplitudes of interacting waves.

2. If the beam wave with negative energy decays then the existence of fast beam wave can break the process of explosive instability.

3. Four wave processes have the interval of parameters with non regular dynamics.

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ВЗРЫВНАЯ НЕУСТОЙЧИВОСТЬ В ПЛАЗМЕННО-ПУЧКОВЫХ СИСТЕМАХ

В.А. Буц, И.К. Ковальчук

Представлены результаты исследования динамики взрывной неустойчивости в плазменно-пучковых системах. Показано, что в общем случае учет быстрой пучковой волны (волны с положительной энергией) может существенно менять динамику этой неустойчивости, вплоть до ее срыва. Определены параметры, при которых реализуется взрывная неустойчивость, и параметры, при которых происходит ее подавление. Показано, что при таком четырехволновом взаимодействии существуют режимы с признаками нерегулярности.

ВИБУХОВА НЕСТІЙКІСТЬ У ПЛАЗМОВО-ПУЧКОВИХ СИСТЕМАХ

В.А. Буц, І.К. Ковальчук

Представлені результати дослідження динаміки вибухової нестійкості в плазмово-пучкових системах. Показано, що в загальному випадку врахування швидкої пучкової хвилі (хвилі з позитивною енергією) може істотно змінити динаміку цієї нестійкості, навіть до зриву. Визначені параметри, при яких реалізується вибухова нестійкість, та параметри, при яких виникає її зрив. Показано, що при такій чотирьоххвилевій взаємодії існують режими з ознаками нерегулярності.