

# STOCHASTIC HEATING OF ELECTRONS IN CAPACITIVE RF DISCHARGES BY PLASMA OSCILLATIONS

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Stochastic heating of electrons by plasma oscillations excited in the capacitive discharge plasma is investigated theoretically. We have obtained criteria, when the stochastic heating take place, and demonstrated numerically that this heating mechanism can be sufficiently effective.

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## INTRODUCTION

It was demonstrated experimentally in [1] and confirmed numerically in [2-6] that electron energy distribution function (EEDF) in an argon capacitively coupled discharge (CCD) has a large number of low-energy electrons. The measurements [1] done in the discharge driven at  $f_i = 13.56$  MHz (for more high frequency drive see, for example, [4-6]) in argon at pressure 0.1 Torr. It was found that the EEDF can be represented as a sum of two Maxwellian distributions with two electron temperatures:  $T_l = 0.3$  eV and  $T_h = 3.1$  eV, and plasma densities  $n_l = 1.3 \cdot 10^{10}$  cm<sup>-3</sup> and  $n_h = 1.3 \cdot 10^9$  cm<sup>-3</sup>. The low-energy electron group, with its temperature close to the energy of the Ramsauer minimum, has an extremely low electron-neutral (e-n) collision cross section corresponding to a low e-n collision frequency  $\nu_{enl} = 10^7$  s<sup>-1</sup>. These electrons oscillates collisionlessly ( $\nu_{enl}^2 \ll \Omega^2$ ) in the weak bulk RF field, and unable to gain energy either from the RF field due to Ohmic heating or from the oscillating RF sheath due to stochastic heating. The high-energy electrons have a large e-n collision frequency  $\nu_{enh} = 5 \cdot 10^8$  s<sup>-1</sup> ( $\nu_{enh}^2 \gg \Omega^2$ ), and effectively interact with argon atoms in elastic, excitation and ionization collisions and serve as the source of the low-energy electrons for the most part due to ionization. These electrons compensate energy losses mainly through stochastic heating on the oscillating plasma sheath boundaries. In order to increase plasma density in CCDs one needs to increase the high-energy electrons population. This can be done by heating of low-energy electrons in the plasma bulk by (i) the collisionless electron bounce resonance heating [7-10], and by (ii) the stochastic electron heating due to plasma oscillations. It is necessary to emphasize, that the stochastic heating in the bulk plasma is usually more effective than the stochastic heating due to interaction with oscillating boundaries [11,12]. Stochastic heating of electrons by plasma oscillations excited in the CCD plasma is investigated theoretically and numerically. We have obtained the criteria, when the stochastic heating of low-energy electrons by plasma oscillations take place. PIC-MCC simulation shows that this heating mechanism can be sufficiently effective to increase plasma density and to control EEDF in CCDs.

## 1. RESULTS AND DISCUSSION

To find out conditions, when stochastic heating take place, let us consider the motion of charged particle in the field of a plasma wave under the action of low-frequency oscillation. In dimensionless variables  $\tau = \omega_p t$ ,  $\xi = k_p x$  the equations of particle motion have the form:

$$\begin{aligned} \frac{d\xi}{d\tau} &= v, \\ \frac{dv}{d\tau} &= -\varepsilon_p \cos(\tau - \xi) - \varepsilon_o \sin(\omega\tau) - \gamma, \end{aligned} \quad (1)$$

where  $\omega = \Omega / \omega_p$ ,  $\varepsilon_p = |e| E_p k_p / m \omega_p^2 \ll 1$ ,  $\varepsilon_o = |e| E_o k_p / m \omega_p^2 \ll 1$ ,  $\gamma = \nu_m / \omega_p \ll 1$ ,  $E_p$  is the plasma wave amplitude,  $E_o$  is the amplitude of low-frequency drive,  $k_p$  is the plasma wave number,  $\omega_p$  is the plasma frequency,  $\Omega = 2\pi f_i$ ,  $\nu_m$  is collision frequency. The equations (1) can be presented as

$$\frac{d^2 \tilde{\xi}}{d\tau^2} = -\varepsilon_p \sum_{n=-\infty}^{+\infty} J_n \left( \frac{\varepsilon_o}{\omega^2} \right) \cos((1-n\omega)\tau - \tilde{\xi}) \quad (2)$$

where the dissipation term is neglected ( $\nu_{enl}^2 \ll \Omega^2 \ll \omega_p^2$ ) and the following representation for Bessel function is used:

$$\cos(\theta - \beta \sin \varphi) = \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\theta - n\varphi),$$

$\tilde{\xi} = \xi - \frac{\varepsilon_o}{\omega^2} \sin(\omega\tau)$ . Equation (2) has the “angular standard form”:

$$\begin{aligned} \frac{d\theta_n}{d\tau} &= 1 - n\omega - I, \\ \frac{dI}{d\tau} &= -\varepsilon_p \sum_{n=-\infty}^{+\infty} J_n \left( \frac{\varepsilon_o}{\omega^2} \right) \cos(\theta_n), \end{aligned} \quad (3)$$

where  $\theta_n(\tau) = (1-n\omega)\tau - \tilde{\xi}$ ,  $I(\tau) = \dot{\tilde{\xi}}$ . Taking into account that the efficient wave-particle interaction takes place under the resonance conditions

$$I_n \stackrel{def}{=} \dot{\tilde{\xi}}_n = 1 - n\omega, \quad (4)$$

by averaging (3), one can obtain the equations for particle motion in the isolated resonance (4), and find out the half-width of the nonlinear resonance:

$$\Delta_{1/2}^n(\xi) = 2\sqrt{\varepsilon_p J_n\left(\frac{\varepsilon_o}{\omega^2}\right)}. \quad (5)$$

The distance between resonances with numbers  $n$  and  $n+1$  can be obtained from (4):  $\delta_{n+1}^n = \omega$ , and the criterion of the dynamical chaos emergence and stochastic heating of electrons can be presented as:

$$\mu_{n+1}^n = \frac{\Delta_{1/2}^n + \Delta_{1/2}^{n+1}}{\delta_{n+1}^n} - 1 > 0, \quad (6)$$

or, in dimensions variables:

$$2\sqrt{\frac{eE_p k_p}{m\Omega^2}} \left[ \sqrt{J_n\left(\frac{eE_o k_p}{m\Omega^2}\right)} + \sqrt{J_{n+1}\left(\frac{eE_o k_p}{m\Omega^2}\right)} \right] - 1 > 0.$$

Conditions (6) are satisfied due to small distance between resonances  $\omega$  and possibility to excite plasma oscillations of sufficiently large amplitude. Fig. 1 and 2 show the contour plot of the overlapping criterion (6) as function of  $\varepsilon_p = |e| E_p k_p / m\Omega^2$  and  $\varepsilon_o = |e| E_o k_p / m\Omega^2$  for  $n=0$  (Fig. 1) and  $n=1$  (Fig. 2).

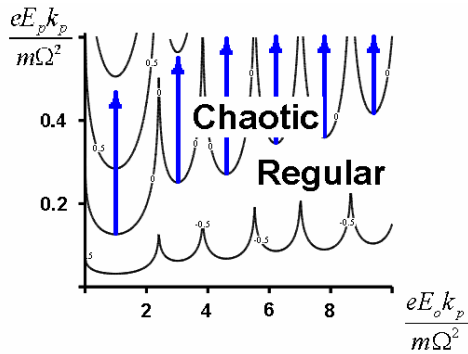


Fig. 1. The overlapping criterion (6) for  $n=0$ .

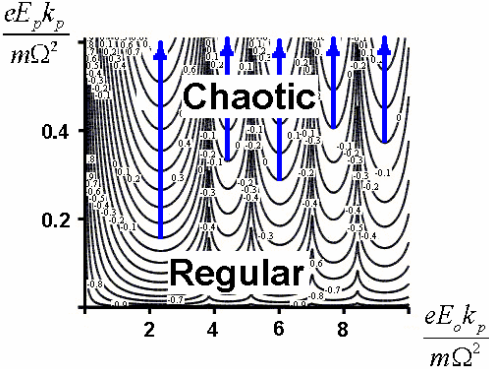


Fig. 2. The overlapping criterion (6) for  $n=1$ .

Let's take into account backward plasma wave in (1):

$$\begin{aligned} \frac{d\xi}{d\tau} &= v, \\ \frac{dv}{d\tau} &= -\varepsilon_p^f \cos(\tau - \xi) - \varepsilon_p^b \cos(\tau + \xi) - \\ &\quad - \varepsilon_o \sin(\omega\tau) - \gamma. \end{aligned} \quad (7)$$

Equation (7) can be treated as the set (1). The backward plasma wave leads to an additional set of resonances in comparison with (1):  $\tilde{\xi}_k = -1 - k\omega$ , where  $k$  is integer.

The half-width of these resonances is

$$\Delta_{1/2}^k(\xi) = 2\sqrt{\varepsilon_p^b J_k\left(\frac{\varepsilon_o}{\omega^2}\right)}. \quad (8)$$

These resonances can be overlapped with resonances, which correspond to interaction of a particle with a forward plasma wave:

$$\mu_k^n = \frac{\Delta_{1/2}^n + \Delta_{1/2}^k}{\delta_k^n} > 1, \quad \text{where } \delta_k^n = |2 + (k - n)\omega|.$$

Let's neglect last two terms in the second equation of system (7). Assuming that the Cherenkov's resonance condition ( $v_o = \xi = \pm 1$ , where  $v_o$  is the initial particle velocity) are not satisfied, the solutions can be presented as  $\xi = v_o \tau + \hat{\xi} + \tilde{\xi}$ , where  $\hat{\xi}$  is slow,  $\tilde{\xi}$  is fast variables. The Bogolyubov-Miropolskiy averaging method gives for  $\hat{\xi}$  following equation:

$$\ddot{\xi} = -\left\langle \sum_{k,p} W_{k,p} \cos\left[G_{k,p} \tau + (k+p)\hat{\xi} + (k+p-1)\frac{\pi}{2}\right] \right\rangle,$$

where

$$W_{k,p} = \varepsilon_p^f J_{k-1}\left(\frac{\varepsilon_p^f}{\Omega_-^2}\right) J_p\left(\frac{\varepsilon_p^b}{\Omega_+^2}\right) + \varepsilon_p^f J_k\left(\frac{\varepsilon_p^f}{\Omega_-^2}\right) J_{p-1}\left(\frac{\varepsilon_p^b}{\Omega_+^2}\right),$$

$\Omega_{\pm} = v_o \pm 1$ ,  $G_{k,p} = k\Omega_- + p\Omega_+$ . Under the resonance conditions  $G_{k,p} = 0$ ,  $k \neq 0$ ,  $p \neq 0$  one can find out the the half-width for resonance  $\{k, p\}$ :

$$\Xi_{k,p} = 2\sqrt{W_{k,p}} / (k+p). \quad (8)$$

The distance between resonances is  $\delta_{k',p'}^{k,p} = \left| \frac{k-p}{k+p} - \frac{k'-p'}{k'+p'} \right|$ . The resonances are overlapped if

$$\Xi_{k,p} + \Xi_{k',p'} > \delta_{k',p'}^{k,p}. \quad (8)$$

The Poincare maps of equations (7) are presented in Figs. 3-4. In Fig. 3, orbits are regular practically in all phase space, except for separatrix region.

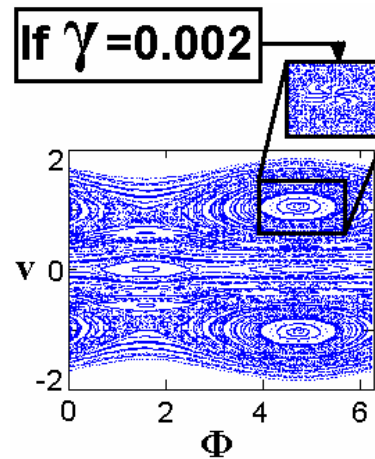


Fig. 3. Poincare map.  $\varepsilon_p^f = 0.09$ ,  $\varepsilon_p^b = 0.09$ ,

$$\varepsilon_o = 0.0, \quad \omega = 0.0, \quad \gamma = 0.0$$

It is easy to see (Fig. 4), that the weak low-frequency signal results in chaotic dynamics of particles, however Cherenkov's resonances do not collapse. Weak dissipation essentially does not reduce the max

imal energy of particles at stochastic heating, however this case requires additional investigations.

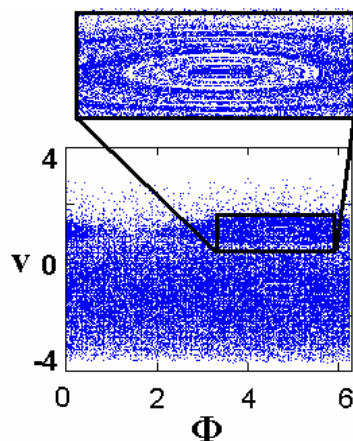


Fig. 4. Poincaré map.  $\varepsilon_p^f = 0.09$ ,  $\varepsilon_p^b = 0.09$ ,

$$\varepsilon_o = 0.01, \omega = 0.01, \gamma = 0.0$$

Particle-in-Cell Monte Carlo simulations of CCD (for details see, for example, [2-6]) for 40 mTorr and 100V/27MHz drive show that the high-energy population of electrons in EEDF and averaged plasma density are increased in the case of stochastic heating by plasma oscillations.

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### СТОХАСТИЧЕСКИЙ НАГРЕВ ЭЛЕКТРОНОВ В ЕМКОСТНЫХ РАДИОЧАСТОТНЫХ РАЗРЯДАХ ПЛАЗМЕННЫМИ КОЛЕБАНИЯМИ

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Изучен стохастический нагрев электронов плазменными колебаниями, возбужденными в плазме емкостного разряда. Получены условия, при которых стохастический нагрев имеет место и показано численно, что такой механизм нагрева может быть достаточно эффективным.

### СТОХАСТИЧНЕ НАГРІВАННЯ ЕЛЕКТРОНІВ У ЄМКІСНИХ РАДІОЧАСТОТНИХ РОЗРЯДАХ ПЛАЗМОВИМИ КОЛИВАННЯМИ

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Досліджено стохастичне нагрівання електронів плазмовими коливаннями, які збуджено в плазмі ємкісного розряду. Одержано умови, при яких стохастичне нагрівання має місце і показано чисельно, що зазначений механізм нагріву може бути досить ефективним.