ELECTRON TEMPERATURE EFFECTS IN LINEAR COUPLING OF ELECTRON-CYCLOTRON WAVES NEAR THE CUT-OFF LAYERS IN FUSION PLASMAS

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Ordinary and extraordinary wave coupling in the electron-cyclotron frequency range in non-one-dimensionally inhomogeneous magnetized plasmas in a vicinity of the plasma cut-off surface is studied with taking into account electron thermal motion and tokamak magnetic field topology. Previously developed theory of the ultra-high-frequency O-X mode coupling in a toroidal plasma has been generalized. Reduced wave equations that describe the normal wave interaction in the considered case are found and solved analytically. Thermal effects essential for the rf heating of overdense plasma in large scale experiment (ITER like) are analyzed.

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INTRODUCTION

In this paper we study the conversion of ordinary (O) waves to extraordinary (X) plasma waves near the plasma cut-offs in the electron cyclotron resonance frequency range in a toroidal geometry. This process plays an important role in excitation of the electron Bernstein waves, which in turn provide an effective way for high-frequency heating and diagnostics of overdense plasma in spherical tokamaks and optimized stellarators [1].

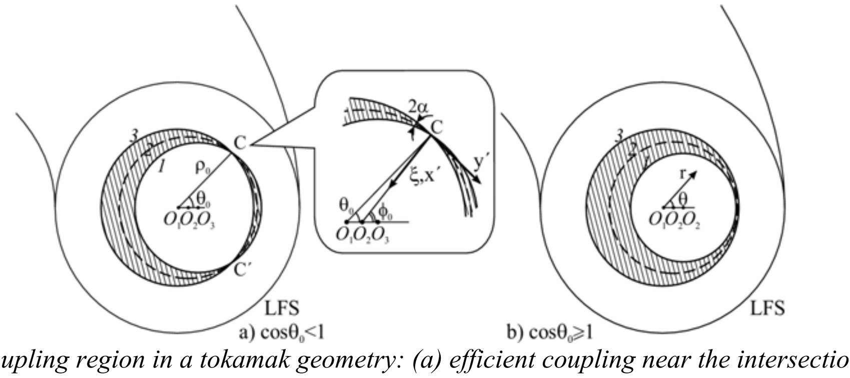
The O-X mode-conversion occurs when turning points of both modes becomes close to each other; for plane waves in cold plasma this is possible in the mode-coupling region defined by the following conditions [2]:

$$N_\perp << N_\parallel, \ |\varepsilon_\parallel| << 1, \ |\varepsilon_+ - N_\parallel^2| << 1, \qquad (1)$$

where $N_\perp = ck_\perp / \omega$ and $N_\parallel = ck_\parallel / \omega$ are wave refractive indexes transverse and parallel to the magnetic field, $\varepsilon_\parallel$ and ε_+ are components of the dielectric tensor in Stix representation [3],

$$\varepsilon_\parallel = 1 - X, \varepsilon_+ = 1 - X/(1+Y), X = \omega_{pe}^2/\omega^2, Y = \omega_{ce}/\omega,$$

ω, ω_{ce} and ω_{pe} are, correspondingly, the wave, electron cyclotron and plasma frequencies. With these conditions one can get an approximate solution to the dispersion relation for the left-polarized waves in a vicinity of the mode-region in the following form: $N_\perp^2 \approx 2\varepsilon_\parallel(\varepsilon_+ - N_\parallel^2)/\varepsilon_+$ .According to usual notation, this solution describes the O wave for $\varepsilon_\parallel > 0$ and $\varepsilon_+ > N_\parallel^2$, and the X wave for $\varepsilon_\parallel < 0$ and $\varepsilon_+ < N_\parallel^2$. While $N_\parallel$ is considered being constant due to the toroidal symmetry (neglecting the poloidal field), propagation regions for the O and X waves are separated by the evanescent region defined as $\varepsilon_\parallel(\varepsilon_+ - N_\parallel^2) < 0$. Typical view of this region in a toroidal device is shown in Fig. 1. The evanescent region is formed by two cut-off surfaces, $\varepsilon_\parallel = 0$ and $\varepsilon_+ - N_\parallel^2 = 0$, each of them may be the locus of turning points of either the O-mode or the X-mode depending on which surface corresponds to more dense plasma. In a toroidal geometry plasma density and ambient magnetic field vary along different directions resulting in misalignment of the cut-off surfaces. The first surface, $\varepsilon_\parallel = 0$, is a flux surface with the critical plasma density, $\omega_{pe} = \omega$; however due to variation of the magnetic field, the second surface, $\varepsilon_+ - N_\parallel^2 = 0$, does not correspond to a flux surface. For a certain range of $N_\parallel$ these surfaces may cross (see Fig.1, left), then the mode conversion actually occurs as a tunneling of the electromagnetic radiation through the non-slab evanescent region in the vicinity of the intersection line. After toroidal curvature is neglected, propagation of the O and X waves may be treated as a two-dimensionally inhomogeneous problem [4, 5]. In this paper we concentrate on the effects of the poloidal curvature and finite electron temperature.



a) $\cos\theta_0 < 1$ b) $\cos\theta_0 \geq 1$

Fig.1. O-X mode-coupling region in a tokamak geometry: (a) efficient coupling near the intersection points C *and* C' *of the cutoff surfaces , (b) inefficient coupling for non-intersecting cutoff surfaces. The evanescent region for the left-polarized waves is shown in dashing*

1. EFFECTS OF POLOIDAL CURVATURE

Let us assume the simplest model for a toroidal magnetic configuration – the surfaces of constant pressure are formed by concentric circular tori, and the magnetic field strength is inversely proportional to a distance from the axis of symmetry, $B = B_0[1+(r/L_B)\cos\theta]^{-1}$, where θ is a poloidal angle counted from the equatorial plan and from the low-field-side. The considered geometry in a more symmetrical way, it is convenient to introduce new polar coordinates (ρ,ϕ) with the origin O_2 exactly between the centers O_1 and O_3 of two cut-off surfaces shown in Fig. 1. Also it is convenient to shift the coordinate $\widetilde{\rho} = \rho - \rho_0 + \frac{1}{2}\Delta\rho\cos\theta_0$ such that $\widetilde{\rho}$ is small inside the O-X coupling region, ρ_0 is the radius of a flux surface with critical density $X = 1$. Finally one obtains

$$\varepsilon_{\parallel} \approx g_{\parallel}\,[\widetilde{\rho} + \Delta\rho(\cos\phi - \cos\phi_0)/2],$$

$$\varepsilon_{+} - N_{\parallel}^2 \approx g_{+}\,[\widetilde{\rho} - \Delta\rho(\cos\phi - \cos\phi_0)/2]$$

where ϕ_0 is the poloidal coordinate of the intersection of the cut-off surfaces $g_{\parallel} = |n_e'(\rho_0)/n_e(\rho_0)| = 1/L_n$, $g_{+} = g_{\parallel}/(1+Y)$, $\Delta\rho = \rho_0 Y(L_n/L_B)/(1+Y)$. Parameters $\Delta\rho$ and ϕ_0 fully characterize the geometry of the mode-coupling region. Fortunately, such spatial variation of dielectric tensor allows analytical solution of the reference wave equations in the mode-coupling region [6]. Wave field distribution in the beam propagating towards the plasma center may be found as

$$A^{+}(\xi,\phi) = \sum_{n=0}^{\infty} A_n \Phi_n(\phi) D_{i\nu_n}\left(\sqrt{2}e^{i\pi/4}\xi\right), \qquad (2)$$

where $\xi = -\widetilde{\rho}/L_{\nabla}$, $L_{\nabla}^2 = N_{\parallel}/k_0\sqrt{2g_{+}g_{\parallel}}$, $D_{i\nu_n}$ are the parabolic cylinder functions, Φ_n are poloidal modes found the following equation

$$\left(-d^2/d\phi^2 + \delta^2(\cos\phi - \cos\phi_0)^2 - \delta\sin\phi\right)\Phi_n = \lambda_n\Phi_n,$$

$$\delta = \rho_0\Delta\rho/2L_{\nabla}^2,\ \ \lambda_n = 2\rho_0^2\nu_n/2L_{\nabla}^2$$

with boundary condition $\Phi_n(0) = -\Phi_n(2\pi)$. The obtained solutions in the cylindrical geometry suggest the "natural" coordinates – in the WKB region each poloidal mode propagates in radial direction and preserves its poloidal structure on surfaces $\rho = \mathrm{const}$. Indeed, as it follows from properties of parabolic cylinder functions in the limit $|\xi| >> 1$, Eq. (2) may be rewritten as

$$A^{+}(\rho,\phi) = A_{\phi}^{+}(\phi)\exp\left(-i(\rho-\rho_0)^2/2L_{\nabla}^2\right),$$

i.e. the modulation over ϕ is conserved. Conservation of a poloidal structure results in modulation over the Cartesian coordinate y' across the beam propagation direction in the poloidal plane. However, just this coordinate seems to be most natural for specification of an incident beam. Let us consider for definiteness, an incident beam $A_y^{+}(y')$ specified on the plane tangent to the surface $\rho = \rho_i$ at point $\phi = \phi_i$:

$$A_{\phi}^{+}(\phi) = A_y^{+}\left((\phi-\phi_i)\rho_0\right)e^{i\chi(\phi-\phi_i)^2/2 + i\chi^2(\phi-\phi_i)^4/8\xi_i^2}$$

with $\chi = \xi_i\rho_0/L_{\nabla}$ and $\xi_i = (\rho_i - \rho_0)/L_{\nabla}$. One can see an additional poloidal phase modulation over ϕ due to not conserving beam structure over y' in the WKB region. For example, for the Gaussian incident beam with a local poloidal width a_{ϕ} and a flat wave front in a vicinity of the surface $\rho = \rho_i$ we retain the quadratic phase modulation resulted from the different curvature of the wave front and the cut-off surfaces. Note, that in tokamak conditions $\chi >> 1$, so the curvature may be essential even for narrow beams with a small local width compared to the curvature itself. Strong phase-modulation in the poloidal direction results in strong degradation of the O-X coupling efficiency for wave beams with a flat wave front. For example, in case of a new ECRH launching system at the FTU tokamak [7] the maximum coupling efficiency is about 40% for a standard wave beam without phase-front tailoring [6]. Similar effective phase modulation corresponds to toroidal curvature, this effect is characterized by $\chi_{tor} \approx \chi R/\rho_0$, where R is major radius.

2. EFFECTS OF THERMAL MOTION

In a next generation fusion experiment the electron temperature may achieve a value at which thermal electron motion results in non-neglidgible contribution to the O-X mode-coupling. This is possible when relativistic factor of thermal electrons, $\beta_e^2 = T_e/m_ec^2$, becomes comparable to the small parameter of the geometric-optics, $1/k_0L_n$. The thermal effects may be taken into account by retaining only the first-order terms over β_e^2 in the plasma dielectric tensor [8], and assuming condition $N_{\perp} << N_{\parallel}$ usual for the O-X mode-coupling. In this approximation, the dielectric tensor remains diagonal in the Stix representation with the following diagonal components:

$$\varepsilon_{\pm} = 1 - \frac{X}{1 \pm Y}\left(1 + \frac{N_{\parallel}^2\beta_e^2}{(1 \pm Y)^2}\right), \quad \varepsilon_{\parallel} = 1 - X\,(1 + 3N_{\parallel}^2\beta_e^2).$$

One can see that small perturbation of the diagonal components results only to shifts of the cut-off surfaces $\varepsilon_{\parallel} = 0$ and $\varepsilon_{+} = N_{\parallel}^2$.

Thermal shift of the cut-off layers results in modification of the optimal launching conditions for the wave beams. For example in the slab geometry with $Y = Y_0 = \mathrm{const}$ and $X = 1 + x/L_n$, the optimal propagation direction with taken into account the finite temperature shift may be defined as

$$N_{\parallel}^2 = \frac{Y_0}{1+Y_0}\left(1 + \frac{3(1+Y_0)^2 - 1}{(1+Y_0)^2}\beta_e^2\right).$$

One can see that the thermal corrections may be easily compensated by tuning $N_{\parallel}$-spectrum, but can lead to essential degradation of the O-X mode-coupling efficiency if this tuning is omitted. Level of this degradation may be estimated as a coupling coefficient calculated with thermal effects being taken into account but for the propagation direction optimal for the cold plasma:

$$T = \exp(-\delta),\ \ \delta = AL_nk_0\beta_T^4\ ,$$

$$A = \pi\sqrt{2Y_0}[3(1+Y_0)^2 - 1]^2(1+Y_0)^{-6}.$$

Here coefficient δ is calculated for the case of the slab geometry, however in a more complex tokamak geometry it differs only by a multiplier of the order of unity. For present-day devices this effect is not very essential. But for the case of ITER $T = 10^{-1}\ldots10^{-4}$ ($\beta_e^2 = 1/50$, $\lambda = 1.7\text{mm}$, L_n = 200…1000 mm depending on a scenario), i.e. not-compensated thermal degradation of the coupling efficiency may be very essential.

This effect is illustrated in Fig 2. Here the O-X coupling coefficients as dependent on a launching angle are plotted for cold plasma and electron temperatures typical of FTU and ITER conditions.

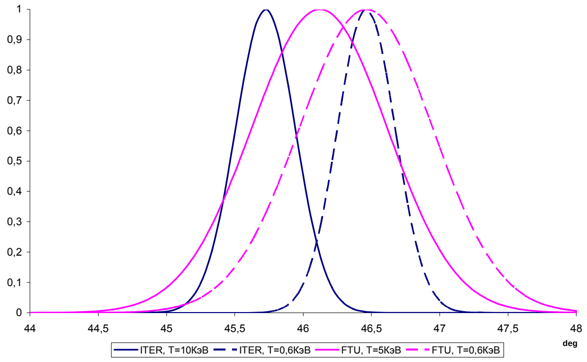


Fig. 2. O-X mode coupling coefficiens for FTU tokamak (wide) and for ITER (narrow), solid lines – cold plasma, dashing – hot plasma (0.6 keV for FTU and 10 keV for ITER)

CONCLUSIONS

In the present paper two effects are considered that may result in essential degradation of the O-X coupling efficiency. These are the influence of the magnetic flux surface on the curvature of the phase fronts in optimal beams and the shift of the cut-off layers due to spatial dispersion induced by thermal electron motion. It is shown that each of these effects must be taken into account while planning future O-X-B heating experiments with overdense plasma, from the other side these effects may be responsible (along with plasma fluctuations [9] and parametric decay instabilities [10]) for low O-X-B heating efficiency in some modern experiment.

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ЭФФЕКТЫ ТЕПЛОВОГО ДВИЖЕНИЯ ЭЛЕКТРОНОВ ПРИ ЛИНЕЙНОМ ВЗАИМОДЕЙСТВИИ ВОЛН ЭЛЕКТРОННОГО ЦИКЛОТРОННОГО ДИАПАЗОНА ЧАСТОТ ВБЛИЗИ ПОВЕРХНОСТЕЙ ОТСЕЧКИ В ТЕРМОЯДЕРНОЙ ПЛАЗМЕ

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Взаимодействие обыкновенной и необыкновенной волн электронного циклотронного диапазона частот в неодномерно-неоднородной магнитоактивной плазме рассмотрено с учетом теплового движения электронов и реалистичной топологии магнитного поля в тороидальных установках. Получено обобщение ранее разработанной теории взаимодействия сверхвысокочастотных волн в тороидальной плазме, в частности, аналитически решено укороченное волновое уравнение, описывающее распределение поля в области взаимодействия. Продемонстрировано, что эффекты теплового движения частиц могут быть значимы в экспериментах по нагреву закритической плазмы в крупномасштабных установках следующего поколения.

ЕФЕКТИ ТЕПЛОВОГО РУХУ ЕЛЕКТРОНІВ ПРИ ЛІНІЙНІЙ ВЗАЄМОДІЇ ХВИЛЬ ЕЛЕКТРОННОГО ЦИКЛОТРОННОГО ДІАПАЗОНУ ЧАСТОТ ПОБЛИЗУ ПОВЕРХОНЬ ВІДСІЧКИ В ТЕРМОЯДЕРНІЙ ПЛАЗМІ

Є.Д. Господчиков, Т.А. Хусаінов, А.Г. Шалашов

Взаємодія звичайної і незвичайної хвиль електронного циклотронного діапазону частот у неодномірно-неоднорідній магнітоактивній плазмі розглянуто з урахуванням теплового руху електронів і реалістичної топології магнітного поля в тороїдальних установках. Отримано узагальнення раніше розробленої теорії взаємодії надвисокочастотних хвиль у тороїдальній плазмі, зокрема, аналітично вирішено вкорочене хвильове рівняння, що описує розподіл поля в області взаємодії. Продемонстровано, що ефекти теплового руху частинок можуть бути значущими в експериментах з нагріву закритичної плазми у великомасштабних установках наступного покоління.