COMPLEX APPROACH OF BEAM DYNAMIC INVESTIGATION IN SC LINAC

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Beam dynamic investigation is difficult for superconducting linac consisting from periodic sequences of independently phased accelerating cavities and focusing solenoids. The matrix calculation was preferably used for previous estimate of accelerating structure parameters. The matrix calculation does not allow properly investigate the longitudinal motion. The smooth approximation can be used to investigate the nonlinear ion beam dynamics in such accelerating structure and to calculate the longitudinal and transverse acceptances. The potential function and equation of motion in the Hamiltonian form are devised by the smooth approximation. The advantages and disadvantages of each method will describe, the results of investigation will compare. Application package for ion beam dynamic analysis will create. A numerical simulation of beam dynamics in the full field will carry out for the different variants of the accelerator structure based on analytically obtained results.

PACS: 29.27.-A, 29.27.Bd

INTRODUCTION

High-current accelerators have great perspectives for problems of thermonuclear fusion, safe nuclear reactors, transmutation of radioactive wastes and free electron lasers. A large number of low energy particle accelerators are applied in micro- and nanoelectronics, material science, including the study of new construction materials for nuclear industry, in medical physics, in particular for cancer treatment by using of the accelerators of protons and light ions, in radiation technology. It is proposed to use one universal accelerator, consisting of independently phased cavities and solenoids sequence to solve these problems.

An ion superconducting linac is usually based on the superconducting (SC) independently phased cavities. This linac consists of the niobium cavities which can provide typically 1 MV of accelerating potential per cavity. Such structures can be used for ion acceleration with different charge-to-mass ratio in the low energy region [1] and for proton linac in the high-energy region (SNS, JHF, ESS project). It is desirable to have a constant geometry of the accelerating cavity in order to simplify manufacturing and to decrease the linac cost. Such geometry leads to a non-synchronism but a stable longitudinal particle motion can be provided by proper RF cavities phasing. The beam can be both longitudinally stable and accelerated in the whole system by control of the accelerating structure driven phase and the distance between the cavities. In this paper two methods of the beam dynamics investigation are compared for low ion velocities and for the charge-to-mass ratio Z/A = 1/66. This comparison can be demonstrated with an example a post-accelerator of radioactive ion beams (FRIB) linac, where beam velocity increases from $\beta = 0.01$ to $\beta = 0.06$ [1].

Beam focusing can be provided with the help of SC solenoid lenses, following each cavity and with the help of special RF fields. As was shown early the beam focusing can be realized for the solenoid field near $B \sim 20$ T. The value of magnetic field B can be reduced by using of addition APF. The smooth approximation has been applied to study the alternating phase focusing (APF) in RIB linac. By adjusting the drive phase (φ_1 and φ_2) of the two cavities, we can achieve the acceleration and the focusing by less magnitude of magnetic field B [2]. Adding a focusing solenoid into focusing period will also allow separate control of the transverse and longitudinal beam dynamics. A schematic plot of one period of the accelerator structure is shown in Fig.1. The low-charge-state low velocity beams require stronger transverse focusing than one is used in existing SC ion linac. Early investigation of beam dynamics shows that for the initial normalized transverse emittance $\varepsilon_T = 0.1$ π·mm·mrad and the longitudinal emittance $\varepsilon_V = 0.3$π·keV/u·nsec the connection between the longitudinal and transverse motion can be neglected if maximum beam envelop $X_m < 3…4$ mm and inner radius of drift tubes $a = 15$ mm.

Beam dynamics in such systems cannot be studied by means of analytical methods only. The initial setup of the system consisting of different types superconducting resonators and focusing solenoids or quadrupoles, can be performed using the transfer matrix calculation and the method of smooth approximation, and then refined the beam dynamics simulation in polyharmonic field.

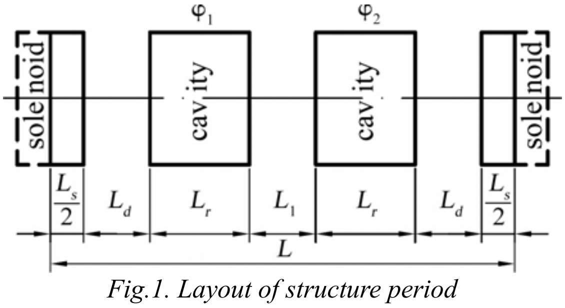

Fig.1. Layout of structure period

Code BEAMDULAC-SCL developed in the laboratory DINUS allows comprehensive research ion beam dynamics in a different structures that satisfy the acceleration of many methods. To evaluate the accelerator parameters implemented transfer matrix calculation method. With a smooth averaging can determine the stability region and to calculate the dynamics of a single particle and beam. And for completion, to verify selections, we perform the calculation of beam dynamics in polyharmonic field.

1. MATRIX CALCULATIONS

The conditions of longitudinal and transverse beam stabilities for the structure consisting from the periodic sequence of the cavities and solenoids were studied early using transfer matrix calculation [2]. The code window with the plots of dependencies dimensionless parameter α, phase advances per period (Floke parameters), magnetic field which need for beam stable motion with envelope value Xm = 3 mm is shown in the Fig.2.

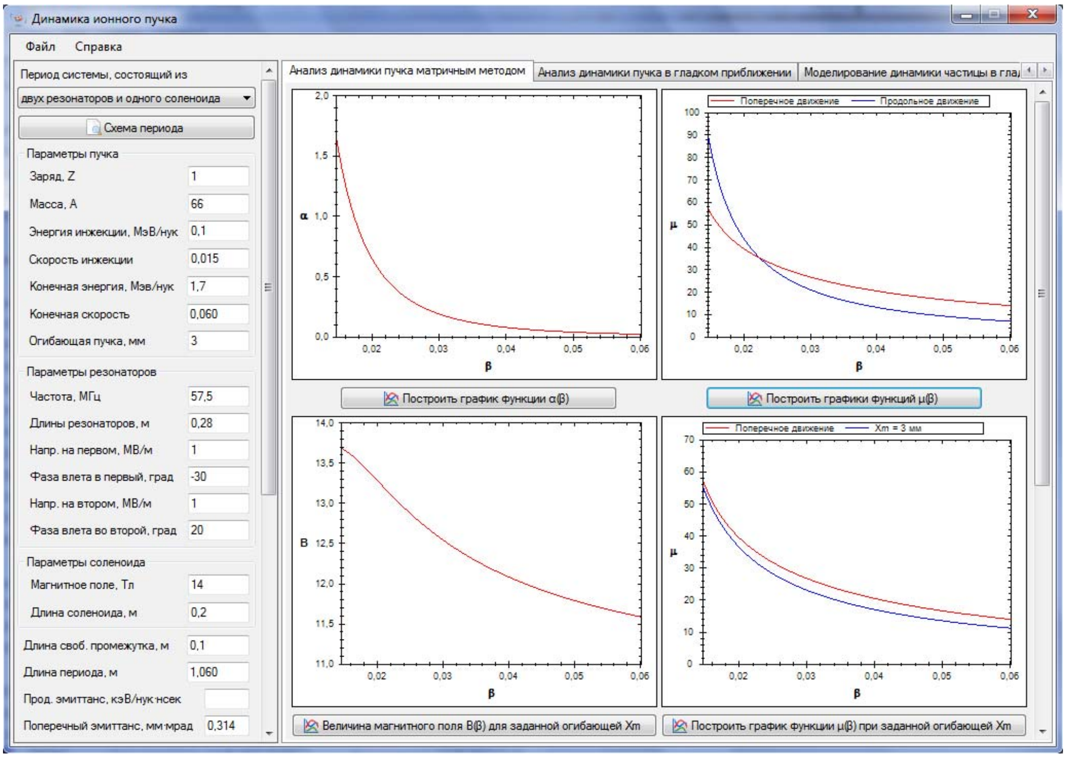

Fig.2. Beam dynamic transfer matrix calculation

2. BEAM DYNAMICS IN SMOOTH APPROXIMATION

The general axisymmetric equations of motion for ion moving inside an accelerator can be written as

$$\frac{d}{dt}\left(\gamma\frac{dz}{dt}\right)=\frac{eZ}{Am}E_z(\vec{r},t)-\frac{e^2Z^2}{2A^2m^2\gamma}\frac{\partial}{\partial z}A_\varphi^2,$$
$$\frac{d}{dt}\left(\gamma\frac{dr}{dt}\right)=\frac{eZ}{Am}E_r(\vec{r},t)(1-\beta\beta_G)-\frac{e^2Z^2}{2A^2m^2\gamma}\frac{\partial}{\partial r}A_\varphi^2. \qquad (1)$$

In every cavity the acceleration RF field of periodic H-cavity can be represented as an expansion in spatial harmonics

$$\left.\begin{aligned}E_z&=E_0\sum \mathrm{I}_0(h_n r)\cos(h_n(z-z_i))\cos(\omega t),\\ E_r&=E_0\sum \mathrm{I}_1(h_n r)\sin(h_n(z-z_i))\cos(\omega t),\end{aligned}\right\} \qquad (2)$$

where E_0 is the amplitude of RF field at the axis ($E_0 \neq 0$ if $-L_r/2 < z-z_i < L_r/2$), $h_n = \pi/D + 2\pi n/D$, $n = 0, 1, 2, \ldots$, $D_i = \beta_G\lambda/2$ is the cavity accelerating structure period length, L_r is the cavity length, z_i is the coordinate of the i-th cavity center, I_0, I_1 are the modified Bessel functions. Let we call particle which accelerating on axis and does not have slow phase and transverse oscillation term as reference. In our case the reference particle velocity β_c and the geometrical velocity β_G are closely for each class of the identical cavities. Retaining in (2) only zeroth RF field harmonic we can use the traveling wave system. In this system ωt can be replaced by $h_0(z-z_i) + \varphi_{0i}$, where φ_{0i} is the RF phase when the reference particle traverses the cavity center. In equation (1) the value A_φ is the azimuthal vector-potential of the magnetic field in solenoid (***B*** = rot***A***).

In SC linac design, it is very important to know the bucket size since it relates to the longitudinal RF focusing. But the linac longitudinal acceptance cannot be obtained by matrix method because of the assumption that the particles have small longitudinal oscillation amplitude. In order to investigate the nonlinear ion beam dynamics in such accelerated structure and to calculate the longitudinal and transverse acceptances it can be used smooth approximation [3,4]. In this paper, equation of motion for ion beam in the Hamiltonian form is derived in the smooth approximation for superconducting linac.

Let us consider the particle motion in the polyharmonic fields of the cavities and solenoids. The ion dynamics in such periodic structure is complicated. The particles trajectories can be presented as a sum of the slowly term and a fast oscillation term with a period L. The normalized particle velocity deviation with respect to the reference particle velocity, $\Delta\beta$, can be represented as a sum of a slow motion term and a fast oscillation term also.

Following Ref. [5] one can apply an averaging over the fast oscillations and obtain the phase (ψ) and radial ($\rho = h_0 r$) motion equations in smooth approximation:

$$\frac{d^2\psi}{d\xi^2}+3\left[\frac{d}{d\xi}(\ln\beta\gamma)\right]\frac{d\psi}{d\xi}=-\frac{\partial\overline{U}_{eff}}{\partial\psi},$$
$$\frac{d^2\rho}{d\xi^2}+\left[\frac{d}{d\xi}(\ln\beta\gamma)\right]\frac{d\rho}{d\xi}=-\frac{\partial\overline{U}_{eff}}{\partial\rho}, \qquad (4)$$

where $U_{eff} = U_0 + U_1 + U_2$ is effective potential function. We use the following designations:

$$U_0 = -4\alpha\left[\psi\{\cos\varphi_1 + \cos\varphi_2\} - I_0(\rho)\{\sin(\varphi_1+\psi) + \sin(\varphi_2+\psi)\}\right],$$

$$\begin{aligned} U_1 = \chi_1\alpha^2 \{ & 2(\cos\varphi_1 + \cos\varphi_2)(\sin\varphi_1 + \sin\varphi_2)\psi + \\ & + I_0^2(\rho)[\cos(\varphi_1+\psi) + \cos(\varphi_2+\psi)]^2 + \\ & + I_1^2(\rho)[\sin(\varphi_1+\psi) + \sin(\varphi_2+\psi)]^2 \} + \\ & + \chi_2\alpha^2\{2(\cos\varphi_1 - \cos\varphi_2)(\sin\varphi_1 - \sin\varphi_2)\psi + \\ & + I_0^2(\rho)[\cos(\varphi_1+\psi) - \cos(\varphi_2+\psi)]^2 + \\ & + I_1^2(\rho)[\sin(\varphi_1+\psi) - \sin(\varphi_2+\psi)]^2 \}, \end{aligned} \quad (4)$$

$$\begin{aligned} U_2 = & -\frac{\chi_3}{2}\alpha\tilde{B}\rho I_1(\rho)\frac{L_{sol}}{L}[\sin(\varphi_1+\psi) + \sin(\varphi_2+\psi)] + \\ & \chi_4\tilde{B}^2\frac{L_{sol}^2}{L^2}\rho^2 + \tilde{B}\frac{L_{sol}}{L}\frac{\rho^2}{2}. \end{aligned}$$

Here $\alpha = \pi eZUL/2A\lambda mc^2\beta_g^3\gamma_g^3$ is the interaction parameter, $\tilde{B} = (eZBL/2Amc\beta_c\gamma_c)^2$ is the focusing parameter.

In this expression for U_{eff} we take into account the coherent oscillations of bunches and the effective potential function describe slowly oscillations in the reference particle frame. Earlier, in [5] the effective potential function was found in the frame where averaged velocity of the reference particle, $\bar{\beta}_c = 0$.

The analysis of the effective potential function makes it possible to study the condition at which the phase and radial stability of the beam is achieved and to calculate the longitudinal acceptance.

$$U_{eff} = U_{eff}(0,0) + \frac{1}{2}\Omega_z^2\psi^2 + \frac{1}{2}\Omega_r^2\rho^2 + \ldots \quad (5)$$

The code window with the frequencies of longitudinal and transverse oscillations, the cross section of the potential function and change the size of the separatrix is shown in Fig.3.

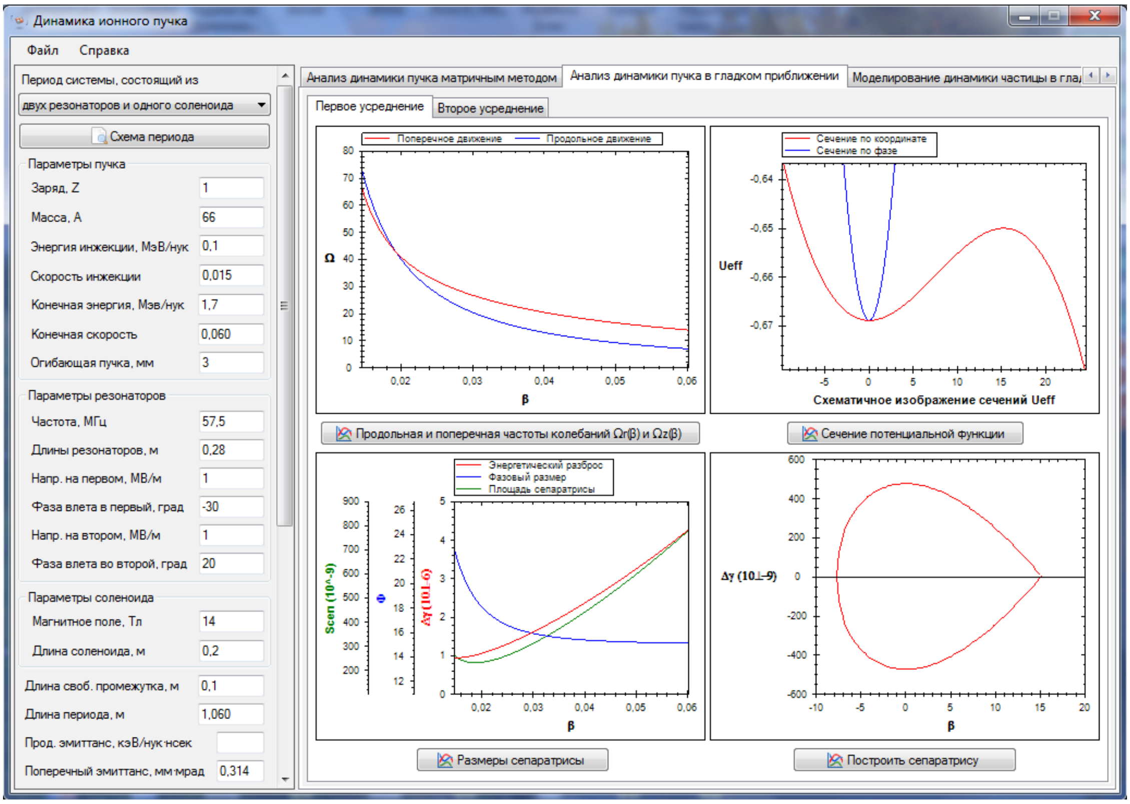

Fig.3. The frequencies of longitudinal and transverse oscillations, potential function cross-section and separatrixes for different approximation cases

3. NUMERICAL SIMULATION OF ION BEAM DYNAMICS

3.1. SINGLE PARTICLE MOTION

For the analyses of longitudinal and transverse motions the beam dynamic was studied in averaged on fast oscillations field. Field's components can be obtained from the effective potential function in smooth approximation. The solution can be obtained only by numerical simulation because the field components are nonlinear functions. The greatest interest represents modeling of the dynamics of the full field. The results of ion beam numerical simulations in polyharmonic field with Z/A = 1/66 are shown in Fig.4.

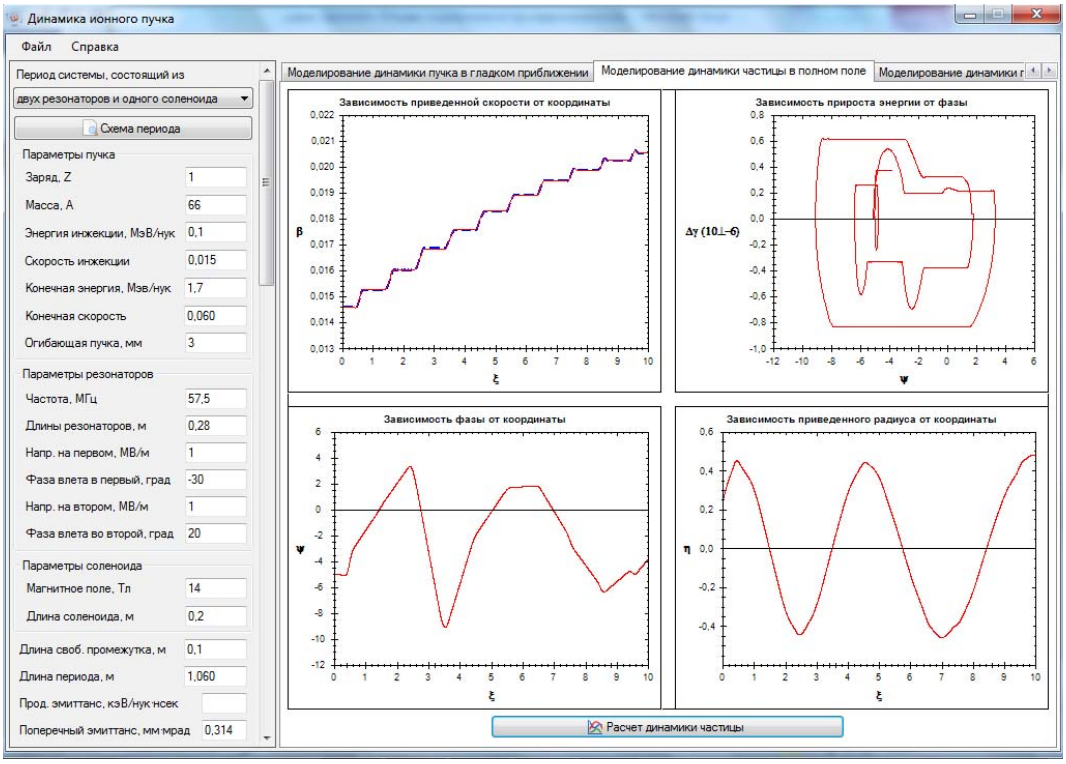

Fig.4. Particle trajectories during acceleration

3.2. BEAM DYNAMICS

The initial parameters of numerical simulation for this system are similar to the previous one. The initial particle phase in the cavity are $\varphi_1 = -30°$ and $\varphi_2 = 20°$, and magnetic field of solenoid is $B = 14$ T. The beam moves through 8 periods in this case. The numerical simulation in polyharmonic field was performed to verify of the result obtained bellow. Geometrical velocity β_G varies in each cavity. The results of ion beam numerical simulations in a polyharmonic field are close to results received in smooth approximation. The beam longitudinal volume increase is negligible and transverse emittance slowly vying. The results are shown in Fig.5 agree with previously calculation.

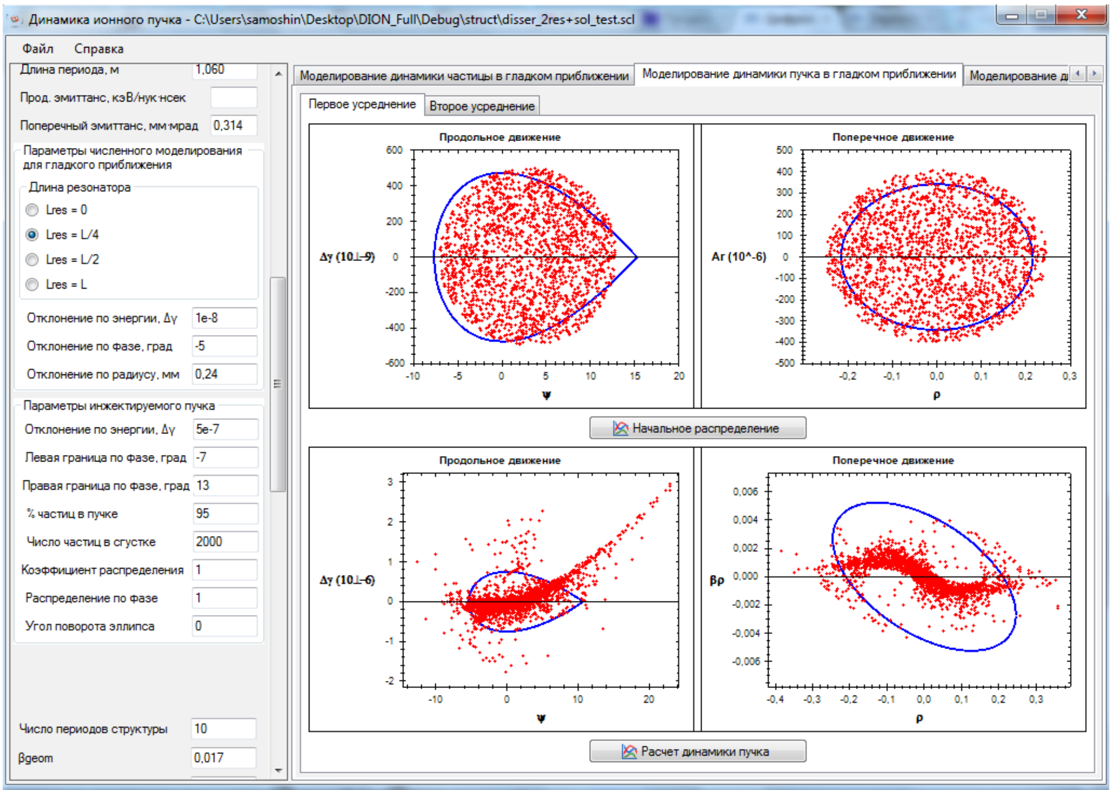

Fig.5. Beam dynamics in the polyharmonic field

4. NUMERICAL SIMULATION IN POLYHARMONIC FIELD

The numerical simulation in polyharmonic field was performed to verify of the result obtained bellow. The simulation was spent for the same focusing periods and the same initial parameters as in section 4 accordingly. Geometrical velocity β_G varies in each cavity too. The results of ion beam numerical simulations in a polyharmonic field are close to results received in smooth approximation. The beam longitudinal volume increase negligible and transverse emittance slowly vying.

CONCLUSIONS

The methods of the beam focusing in SC linac analysis are compared for low ion velocities. By the smooth approximation it was studied more detailed nonlinear ion beam dynamics and founded the beam stability area. It was done the recommendation for choice of the reference particle phases and the value of solenoid magnetic field B. It was shown that the smooth approximation gives very good agreement with the simulation in polyharmonic field. By the smooth approximation it is studied nonlinear ion beam dynamics in linac with combined focusing.

REFERENCES

1. P.N. Ostroumov, et al. // *Proc. of PAC'*2001, p. 4080.
2. E.S. Masunov, D.A. Efimov, P.N. Ostroumov // *Proc. of EPAC'*2004, p. 1405-1407.
3. E.S. Masunov, N.E. Vinogradov // *Phys. Rev. ST-AB* 4, 2001, 070101.
4. Ji Qiang, R.W. Garnett // *Nucl. Instr. and Meth. A.* 2003, v.496, p.33.
5. E.S. Masunov, A.V. Samoshin // *Proc. of PAC'*2007, p.1568-1570.
6. E.S. Masunov, A.V. Samoshin // *Proc. of RuPAC'*2006, p.162-164.

Статья поступила в редакцию 23.09.2011 г.

КОМПЛЕКСНЫЙ ПОДХОД К АНАЛИЗУ ДИНАМИКИ ПУЧКА В СВЕРХПРОВОДЯЩЕМ ЛИНЕЙНОМ УСКОРИТЕЛЕ

А.В. Самошин

Анализ динамики ионного пучка в сверхпроводящем ускорителе, состоящем из последовательности независимо фазированных ускоряющих резонаторов и фокусирующих соленоидов, представляет сложную задачу. Для первоначальной оценки параметров ускоряющей структуры удобно использовать матричный метод расчета. Матричный метод не дает возможности корректно исследовать продольное движение. Для исследования нелинейной динамики в такой ускоряющей структуре и определения продольного и поперечного аксептанса может быть использовано гладкое приближение. В гладком приближении найдена потенциальная функция и уравнения движения в форме уравнения Гамильтона. Описываются достоинства и недостатки каждого метода, проводится сравнение результатов исследования. Создан пакет прикладных программ для анализа динамики пучка ионов. На основе полученных данных проведено численное моделирование динамики пучка в полном поле для различных вариантов ускорителя.

КОМПЛЕКСНИЙ ПІДХІД ДО АНАЛІЗУ ДИНАМІКИ ПУЧКА В НАДПРОВІДНОМУ ЛІНІЙНОМУ ПРИСКОРЮВАЧІ

О.В. Самошин

Аналіз динаміки іонного пучка в надпровідному прискорювачі, що складається з послідовності незалежно фазованих прискорюючих резонаторів і фокусуючих соленоїдів, представляє складну задачу. Для початкової оцінки параметрів прискорюючої структури зручно використовувати матричний метод розрахунку. Матричний метод не дає можливості коректно досліджувати поздовжній рух. Для дослідження нелінійної динаміки в такій прискорюючій структурі та визначення поздовжнього і поперечного аксептанса може бути використано гладке наближення. У гладкому наближенні знайдено потенційну функцію і рівняння руху в формі рівняння Гамільтона. Описуються переваги і недоліки кожного методу, проводиться порівняння результатів дослідження. Створено пакет прикладних програм для аналізу динаміки пучка іонів. На основі отриманих даних проведено чисельне моделювання динаміки пучка в повному полі для різних варіантів прискорювача.