INTEGRABLE WZNW MODELS AND STRING MODELS OF WZNW MODEL TYPE WITH CONSTANT SU(2) TORSION

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The integrability of WZNW model and string model of WZNW model type with constant $SU(2)$ torsion is investigated. The closed boson string model in the background gravity and antisymmetric B-field is considered as integrable system in terms of initial chiral currents. The model is considered under assumption that internal torsion related with metric of Riemann-Cartan space and external torsion related with antisymmetric B-field are (anti)coincident. New equation of motion and exact solution of this equation was obtained for string model with constant $SU(2)$ torsion. New equations of motion and new Poisson brackets (PB) for infinite dimensional hydrodynamic chains was obtained for string model with constant $SU(n)$, $SO(n)$, $SP(n)$ torsion for $n \to \infty$.

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1. INTRODUCTION

1.1 String Model

A closed string model in the background gravity $g_{ab}(\phi)$ and antisymmetric fields $B_{ab}(\phi)$ in the conformal $g_{\alpha\beta} = e^{\phi}\eta_{\alpha\beta}$ and light-cone gauge is described by the Lagrangian:

$$L = \frac{1}{2}\int_0^{2\pi}\left[\sqrt{-g}g^{\alpha\beta}g_{ab}(\phi)\frac{\partial\phi^a}{\partial x^\alpha}\frac{\partial\phi^b}{\partial x^\beta} + \epsilon^{\alpha\beta}B_{ab}\frac{\partial\phi^a}{\partial x^\alpha}\frac{\partial\phi^b}{\partial x^\beta}\right]dx.$$

Here $g_{ab}(\phi(t,x))$ is the metric tensor of curve n-dimensional space $\phi^a(x+2\pi) = \phi^a(x)$, $(a,b = 1,2,\ldots,n)$:

$$g_{ab}(\phi) = g_{ba}(\phi), B_{ab}(\phi) = -B_{ba}(\phi).$$

$g^{\mu\nu}$ is the metric tensor of flat space, tangent space to curve space in point $\phi(t,x)$ and $\mu,\nu = 1,2,...,n$. Both metric can have the arbitrary signature.
$g_{\alpha\beta}(t,x)$ is the metric tensor of curve 2-d space, $(\alpha,\beta = 0,1)$. In the repers formalism

$$g_{ab}(\phi) = e^\mu_a(\phi)e^\nu_b(\phi)g_{\mu\nu}.$$

The repers e^μ_a satisfy to condition $g^{\mu\nu} = e^\mu_a(\phi)e^\nu_b(\phi)g^{ab}(\phi)$. In the conformal gauge

$$g_{\alpha\beta} = e^{\theta(t,x)}\eta_{\alpha\beta}$$

Lagrangian does not depend on field $\theta(t,x)$.

Equation of motion. The equation is:

$$g_{ab}(\phi)\eta^{\alpha\beta}\frac{\partial^2\phi^a}{\partial x^\alpha\partial x^\beta} + \Gamma_{abc}\eta^{\alpha\beta}\frac{\partial\phi^b}{\partial x^\alpha}\frac{\partial\phi^c}{\partial x^\beta} + H_{abc}\epsilon^{\alpha\beta}\frac{\partial\phi^b}{\partial x^\alpha}\frac{\partial\phi^c}{\partial x^\beta} = 0, \quad (1)$$

$$\Gamma_{abc} = \frac{1}{2}(\frac{\partial g_{ab}}{\partial\phi^c} + \frac{\partial g_{ac}}{\partial\phi^b} - \frac{\partial g_{bc}}{\partial\phi^a}),$$

$$H_{abc} = \frac{\partial B_{ab}}{\partial\phi^c} + \frac{\partial B_{ca}}{\partial\phi^b} + \frac{\partial B_{bc}}{\partial\phi^a}.$$

In the terms of repers the connection

$$\Gamma^c_{ab}(\phi) = \frac{e^c_\mu}{2}[\frac{\partial e^\mu_a}{\partial\phi^b} + \frac{\partial e^\mu_b}{\partial\phi^a}]$$

is symmetric on a, b. The function H_{abc} is a total antisymmetric function on a, b, c.

Canonical currents. Let us introduce new variables to obtain first order equation instead of second one:

$$J^\mu_0(\phi) = e^a_\mu(\phi)[p_a - B_{ab}(\phi)\phi'^b]$$
$$J^\mu_1(\phi) = e^\mu_a\phi'^a,$$

Canonical momentum

$$p_a(t,x) = g_{ab}(\phi)\dot\phi^b + B_{ab}\phi'^b,\ \dot\phi^a = \frac{\partial\phi^a}{\partial t},\ \phi'^a = \frac{\partial\phi^a}{\partial x}.$$

Equations of motion in new variables are:

$$\begin{aligned}\partial_0 J^\mu_1 - \partial_1 J^\mu_0 &= C^\mu_{\nu\lambda}(\phi)J^\nu_0 J^\lambda_1,\\ \partial_0 J^\mu_0 - \partial_1 J^\mu_1 &= -H^\mu_{\nu\lambda}(\phi)J^\nu_0 J^\lambda_1.\end{aligned} \quad (2)$$

Here $C^{\mu\nu\lambda}$ is the torsion:

$$C^\mu_{\nu\lambda} = \frac{\partial e^\mu_a}{\partial x^b}(e^b_\nu e^a_\lambda - e^a_\nu e^b_\lambda) = (\frac{\partial e^\mu_a}{\partial x^b} - \frac{\partial e^\mu_b}{\partial x^a})e^b_\nu e^a_\lambda. \quad (3)$$

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Algebra of PBs. Let us consider commutation relation function J^{μ}_{α}, $\alpha = 0,1$ in the phase space on the PB:

$$\{J^{\mu}_0(x), J^{\nu}_0(y)\} = C^{\mu\nu}_{\lambda}(\phi)J^{\lambda}_0(x)\delta(x-y)$$
$$+H^{\mu\nu}_{\lambda}(\phi)J^{\lambda}_1(x)\delta(x-y),$$
$$\{J^{\mu}_0(x), J^{\nu}_1(y)\} = C^{\mu\nu}_{\lambda}(\phi)J^{\lambda}_1(x)\delta(x-y)$$
$$+g^{\mu\nu}\frac{\partial}{\partial x}\delta(x-y),$$
$$\{J^{\mu}_1(x), J^{\nu}_1(y)\} = 0.$$

Let us introduce chiral variables:

$$U^{\mu} = \delta^{\mu\nu}J_{0\nu} + J^{\mu}_1,\ V^{\mu} = \delta^{\mu\nu}J_{0\nu} - J^{\mu}_1.$$

The chiral variables satisfy the following relations under PB [1]:

$$\{U^{\mu}(x), U^{\nu}(y)\} = \frac{1}{2}[(3C^{\mu\nu}_{\lambda} + H^{\mu\nu}_{\lambda})U^{\lambda} -$$
$$(C^{\mu\nu}_{\lambda} + H^{\mu\nu}_{\lambda})V^{\lambda}]\delta(x-y) + \delta^{\mu\nu}\frac{\partial}{\partial x}\delta(x-y),$$
$$\{V^{\mu}(x), V^{\nu}(y)\} = \frac{1}{2}[(3C^{\mu\nu}_{\lambda} - H^{\mu\nu}_{\lambda})V^{\lambda} -$$
$$(C^{\mu\nu}_{\lambda} - H^{\mu\nu}_{\lambda})U^{\lambda}]\delta(x-y) - \delta^{\mu\nu}\frac{\partial}{\partial x}\delta(x-y),$$
$$\{U^{\mu}(x), V^{\nu}(y)\} = \frac{1}{2}[(C^{\mu\nu}_{\lambda} + H^{\mu\nu}_{\lambda})U^{\lambda} +$$
$$+(C^{\mu\nu}_{\lambda} - H^{\mu\nu}_{\lambda})V^{\lambda}]\delta(x-y).$$

Here function $H_{\mu\nu\lambda}(\phi)$ is an additional external torsion. These PB's form algebra if:

$$1) C^{\mu\nu}_{\lambda} = 0, H^{\mu\nu}_{\lambda} = 0$$

and functions $U^{\mu}(x)$ are abelian currents;

$$2) C^{\mu\nu}_{\lambda},\ H^{\mu\nu}_{\lambda}$$

are structure constants $f^{\mu\nu}_{\lambda}$ of Lie algebra, and the functions $U^{\mu}(x)$ are chiral currents. Here are two possibility to simplify this algebra:

$$1)\ H^{\mu\nu}_{\lambda} = -C^{\mu\nu}_{\lambda},$$
$$\{U^{\mu}(x), U^{\nu}(y)\} = C^{\mu\nu}_{\lambda}U^{\lambda}\delta(x-y) + \delta^{\mu\nu}\frac{\partial}{\partial x}\delta(x-y), \tag{4}$$
$$2)\ H^{\mu\nu}_{\lambda} = C^{\mu\nu}_{\lambda},$$
$$\{V^{\mu}(x), V^{\nu}(y)\} = C^{\mu\nu}_{\lambda}V^{\lambda} - \delta^{\mu\nu}\frac{\partial}{\partial x}\delta(x-y).$$

We do not write down the remaining commutation relations. The chiral currents U^{μ} in first case and V^{μ} in second case form Kac-Moody algebras. Equations of motion in light-cone coordinates

$$x^{\pm} = \frac{1}{2}(t \pm x), \frac{\partial}{\partial x^{\pm}} = \frac{\partial}{\partial t} \pm \frac{\partial}{\partial x}$$

have the following form:

$$H^{\mu}_{\nu\lambda} = -C^{\mu}_{\nu\lambda},\ \partial_{-}U^{\mu} = 0,\ \partial_{+}V^{\mu} = 2C^{\mu\nu\lambda}U^{\nu}V^{\lambda},$$

$$H^{\mu}_{\nu\lambda} = C^{\mu}_{\nu\lambda},\ \partial_{+}V^{\mu} = 0,\ \partial_{-}U^{\mu} = -2C^{\mu\nu\lambda}U^{\nu}V^{\lambda}.$$

The chiral currents $U^{\mu}(x)$ are generators of translation on the curved space

$$\delta\phi^a(x) = \{\phi^a(x), c^{\mu}U_{\mu}(x)\} = c^{\mu}e^a_{\mu}(\phi) = c^a(\phi).$$

Simultaneously, they are generators of group transformations with structure constant $C^{\mu\nu}_{\lambda}$ in the tangent space.

1.2 Integrable string model with null torsion $C^{\mu\nu}_{\lambda} = 0$

To construct integrable dynamical system we must have hierarchy of PB brackets and find hierarchy of Hamiltonians through bi-Hamiltonity condition. We have used the hydrodynamic approach of Dubrovin, Novikov [2,3] to integrable systems and Dubrovin [4] solutions of WDVV associativity equation to construct new integrable string equations of hydrodynamic type in the torsionless Riemann space of chiral currents [5–7]. The basic idea of hydrodynamic approach is construction of compatible PB of abelian currents from PB on the flat space of currents and from PB on the curved space of currents. The Jacobi identity for compatible PB led to WDVV associativity equation for metric tensor of curved space. Also, we can construct recursion operator from Hamiltonian operators of flat PB and of compatible PB. The degrees of recursion operator generate new PBs and new Hamiltonians through bi-Hamiltonity condition.

2. INTEGRABLE STRING MODEL WITH CONSTANT TORSION

Another way to construct integrable dynamical system is the following. We must have the hierarchy of Hamiltonians and find the hierarchy of PB brackets. This way is more simple if the dynamical system have some group structure. Let torsion $C^a_{bc}(\phi)) \neq 0$ and C_{abc} are structure constants of a Lie algebra. In bi-Hamiltonian approach to integrable string models with constant torsion we have considered the conserved primitive chiral currents $C_n(U(x))$ as local fields of the Riemann manifold. The primitive and non-primitive local charges of invariant chiral currents form the hierarchy of new Hamiltonians. The primitive invariant currents are densities of Casimir operators. The non-primitive currents are functions of primitive currents. Commutation relations (4) show that currents U^{μ} form closed algebra. Therefore, we will consider PB of right chiral currents U^{μ} and Hamiltonians constructed only from right currents. The constant torsion does not contribute to equation of motion, but it gives possibility to introduce group structure and to introduce symmetric structure constant.

Let t_μ are the generators $SU(n)$, $SO(n)$, $SP(n)$ of Lie algebras:

$$[t_\mu, t_\nu] = 2if_{\mu\nu\lambda}t_\lambda. \quad (5)$$

There is additional relation for generators Lie algebra in the definition of matrix representation. There is following relation for symmetric double product generators of $SU(n)$ algebra:

$$\{t_\mu, t_\nu\} = \frac{4}{n}\delta_{\mu\nu} + 2d_{\mu\nu\lambda}t_\lambda,\ \mu = 1, ..., n^2 - 1. \quad (6)$$

Here $d_{\mu\nu\lambda}$ is total symmetric structure constant tensor. The similar relation for total symmetric triple product of $SO(n)$ and $SP(n)$ algebras has form:

$$t_{(\mu}t_\nu t_{\lambda)} = v^\rho_{\mu\nu\lambda}\, t_\rho. \quad (7)$$

Here $v_{\rho\mu\nu\lambda}$ is total symmetric structure constant tensor. The invariant chiral currents can be constructed as product of invariant symmetric tensors:

$$d_{(\mu_1...\mu_n)} = d^{k_1}_{(\mu_1\mu_2}d^{k_2}_{\mu_3k_1}...d^{k_{n-3}}_{\mu_{m-1}\mu_M)},$$

$$d_{\mu_1\mu_2} = \delta_{\mu_1\mu_2}$$

for $SU(n)$ group and initial chiral currents U^μ:

$$C_n(U(x)) = d_{(\mu_1...\mu_n)}U_{\mu_1}U_{\mu_2}...U_{\mu_n},\ C_2 = \delta_{\mu\nu}U^\mu U^\nu. \quad (8)$$

Any of this currents satisfy to equation of motion $\partial_- C(n)(U(x)) = 0$. A similar construction can be used for $SO(n)$, $SP(n)$ groups. The invariant chiral currents can be constructed as product of invariant symmetric constant tensor

$$v_{(\mu_1...\mu_{2n})} = v^{\nu_1}_{(\mu_1\mu_2\mu_3}v^{\nu_1\nu_2}_{\mu_4\mu_5}...v^{\nu_{2n-3}}_{\mu_{2n-2}\mu_{2n-1}\mu_{2n})},$$

$$v_{\mu_1\mu_2} = \delta_{\mu_1\mu_2}.$$

and initial chiral currents U^μ:

$$C_{2n} = v_{\mu 1...\mu_{2n}}U^{\mu_1}...U^{\mu_{2n}},\ C_2 = \delta_{\mu_1\mu_2}U^{\mu_1}U^{\mu_2}. \quad (9)$$

The invariant chiral currents for $SU(2)$, $SO(3)$, $SP(2)$ have form:

$$C_{2n} = (C_2)^n. \quad (10)$$

Another family of invariant symmetric currents J_n is based on the invariant symmetric chiral currents on simple Lie groups, realized as symmetric trace of n product chiral currents $U(x) = t_\mu U^\mu$, $\mu = 1, \ldots, n^2 - 1$:

$$J_n = \mathrm{SymTr}(U...U). \quad (11)$$

These invariant currents are polynomials of product of basic chiral currents C_k, $k = 2, 3, ..., k$. The commutation relations for chiral curents have form:

$$\{C_m(x), C_n(y)\} = W_{mn}(y)\frac{\partial}{\partial y}\delta(y - x)$$

$$-W_{nm}(x)\frac{\partial}{\partial x}\delta(x - y).$$

Hamiltonian function $W_{mn}(x)$ for finite dimensional $SU(n)$, $SO(n)$, $SP(n)$ group has form:

$$W_{mn}(x) = \frac{n-1}{m+n-2}\sum_k a_k C_{m+n-2,k}(x), \quad (12)$$

$$\sum_{k=0} a_k = mn.$$

Here the invariant total symmetric currents $C_{n,k}$, $k = 1, 2...$ are new currents, which are polynomials of product of basic invariant currents $C_{n_1}C_{n_2}...C_{n_n}$, $n_1 + ... + n_n = n$. They can be obtained during calculation of total symmetric invariant currents J_n by different replacements of double product using (6) for $SU(n)$ group and triple product using (7) for $SO(n)$ group. This PB can be rewritten as PB of hydrodynamic type

$$\{C_m(x), C_n(y)\} = -\frac{n-1}{m+n-2} \times \frac{\partial}{\partial x}C_{m+n-2,k}(x)\delta(x-y) - C_{m+n-2,k}(x)\frac{\partial}{\partial x}\delta(x-y). \quad (13)$$

Here are only $l = n-1$ primitive invariant tensors for $SU(n)$ algebra, $l = \frac{n-1}{2}$ for $SO(n)$ algebra and $l = \frac{n}{2}$ for $SP(n)$ algebra. Higher invariant currents C_n for $n \geq l + 1$ are non-primitive currents and they are polynomials of primitive currents. The charges, corresponding to non-primitive chiral currents, are not Casimir operators. The expression for these polynomials are obtained from the generating function

$$\det(1 - \lambda t_\mu U^\mu) = e^{\mathrm{Tr}(ln(1-\lambda U))} = \exp\left(-\sum_{n=2}^{\infty}\frac{\lambda^n}{n}J_n\right).$$

Here exists only one primitive invariant tensor in $SU(2)$. The invariant non primitive tensors for $n \geq 2$ are functions of primitive tensor. Let us introduce the local chiral currents based on the invariant symmetric polynomials on $SU(2)$ Lie group:

$$C_2(U) = \delta_{ab}U^aU^b, C_n(U) = (\delta_{ab}U^aU^b)^n,$$

where $n = 1, 2, ...$

$$\{C_2(x), C_2(y)\} = 2C_2(y)\partial_y\delta(y-x) - 2C_2(x)\partial_x\delta(x-y).$$

$C_2(x)$ is local field on the Riemann space of chiral currents. As Hamiltonians we choose functions

$$H_n = \frac{1}{2(n+1)}\int_0^{2\pi} C_2^{n+1}(y)dy. \quad (14)$$

The equation of motion for density of first Casimir operator has form:

$$\frac{\partial C_2}{\partial t_n} + (2n+1)(C_2)^n\frac{C_2}{\partial x} = 0. \quad (15)$$

The equation for currents C_2^n is following:

$$\frac{\partial C_2^n}{\partial t_n} + (2n+1)(C_2)^n\frac{C_2^n}{\partial x} = 0.$$

This equation is inviscid Burgers equation. We will to find the solution in the form:

$$C_2^n(t_n,x) = \exp\left(a + i(x - t_n C_2^n(t_n,x))\right).$$

To obtain solution we rewrite equation of motion in following form: $Y_n = Z_n e^{Z_n}$, where $Y_n = it_n e^{(a+ix)}$, $Z_n = it_n C_2^n$. The inverse transformation $Z_n = Z_n(Y_n)$ is defined by Lambert function:

$$C_2^n = \frac{1}{i(2n+1)t_n} W(i(2n+1)t_n e^{a+ix}).$$

Consequently, solution for first Casimir operator is:

$$C_2(t_n,x) = \left(\frac{1}{i(2n+1)t_n} W(i(2n+1)t_n e^{a+ix})\right)^{\frac{1}{n}}. \tag{16}$$

The equation of motion for initial chiral current U^μ defined by PB (5) and Hamiltonian (14) is:

$$\frac{\partial U^\mu}{\partial t_n} = \partial_x [U^\mu (UU)^n],\ \mu = 1,2,3.$$

3. INFINITE DIMENSIONAL HYDRODYNAMIC CHAIN

In the case, if dimension of matrix representation n is not finite, all chiral currents are primitive currents. This easy to see from expression for new chiral currents $C_{m,k}$. For example:

$$C_{8,1} = C_8 + \frac{2}{n} C_3 C_5 - \frac{2}{n} C_2 C_6.$$

The algebra of PB for chiral currents has the form:

$$\{C_m(x), C_n(y)\} = W_{mn}(y) \frac{\partial}{\partial y} \delta(y-x) - $$
$$+ W_{nm}(x) \frac{\partial}{\partial x} \delta(x-y). \tag{17}$$

$$W_{mn}(x) = \frac{mn(n-1)}{m+n-2} C_{m+n-2}(x). \tag{18}$$

This PB satisfies to skew-symmetric condition $\{C_m(x), C_n(y)\} = -\{C_n(y), C_m(x)\}$. Jacobi identity imposes conditions on the Hamiltonian function $W_{mn}(x)$:

$$(W_{kp} + W_{pk}) \frac{\partial}{\partial C_k} W_{mn} = (W_{km} + W_{mk}) \frac{\partial}{\partial C_k} W_{pn}$$
$$\frac{\partial}{\partial x} W_{kp} \frac{\partial}{\partial C_k} W_{nm} = \frac{\partial}{\partial x} W_{km} \frac{\partial}{\partial C_k} W_{np}. \tag{19}$$

The Jacobi identity satisfies for metric tensor $W_{mn}(U)$ (18) for $m = p$ from compatibility condition of Kronekers $\delta_{m+n-2,k}$ and $\delta_{p+n-2,k}$. This PB can be rewritten as PB of hydrodynamic type and describes the hydrodynamic chain (see [8, 9] and references therein):

$$\{C_m(x), C_n(y)\} = -\frac{mn(n-1)}{m+n-2} \tag{20}$$
$$\times \frac{\partial}{\partial x} C_{m+n-2}(x) \delta(x-y)$$
$$- mn C_{m+n-2}(x) \frac{\partial}{\partial x} \delta(x-y).$$

The algebra of charges $\int_0^{2\pi} C_n(x)dx$ is abelian algebra. Let us choose as Hamiltonians the Casimir operators C_n:

$$H_n = \frac{1}{n} \int_0^{2\pi} C_n(x) dx,\ n = 2,3\dots. \tag{21}$$

The equations of motion for densities of Casimir operators are the following:

$$\frac{\partial C_m(x)}{\partial t_n} = \frac{1}{n} \int_0^{2\pi} W_{mn}(y) \partial_y \delta(y-x) dy - \tag{22}$$

$$-\frac{1}{n} \int_0^{2\pi} W_{nm}(x) \partial_x \delta(x-y) dy = \frac{m(n-1)}{m+n-2} \partial_x C_{m+n-2}.$$

We can construct equations of motion for initial chiral currents U^μ using flat PB (4) and Hamiltonians H_n (18), where $C_n(x)$ is defined by (8) for $SU(\infty)$ group:

$$\frac{\partial U^\mu(x)}{\partial t_n} = \frac{1}{n} \int_0^{2\pi} dy \{U^\mu(x), C_n(y)\}_0,$$

$$\frac{\partial U_{\underline{\mu}}(x)}{\partial t_n} = \partial_x (d^{k_1}_{\mu_1\mu_2} d^{k_2}_{k_1\mu_3} \dots d^{k_{n-3}}_{\mu_{n-1}\underline{\mu}} U^{\mu_1}(x) \dots U^{\mu_{n-1}}(x)). \tag{23}$$

As example we consider $n = 3$:

$$\frac{\partial U_\mu}{\partial t_3} = \partial_x (d_{\mu\nu\lambda} U^\nu U^\lambda).$$

By similar manner we can obtain equation of motion for chiral currents of $SO(n)$, $SP(n)$:

$$\frac{\partial U_{\underline{\mu}}(x)}{\partial t_n} = \partial_x (v^{k_1}_{\mu_1\mu_2\mu_3} \dots v^{k_{2n-3}}_{\mu_{2n-2}\mu_{2n-1}\underline{\mu}} U^{\mu_1} \dots U^{\mu_{2n-1}}). \tag{24}$$

As example we consider $n = 4$:

$$\frac{\partial U_\mu}{\partial t_4} = \partial_x (d_{\mu\nu\lambda\rho} U^\nu U^\lambda U^\rho).$$

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ИНТЕГРИРУЕМЫЕ ВЗНВ-МОДЕЛИ И СТРУННЫЕ МОДЕЛИ ВЗНВ-ТИПА С ПОСТОЯННЫМ SU(2)-КРУЧЕНИЕМ

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Исследована интегрируемость ВЗНВ-модели и струнной модели ВЗНВ-типа с постоянным $SU(2)$-кручением. Замкнутая бозонная струна во внешних гравитационном и антисимметричном полях рассмотрена как интегрируемая система в терминах начальных киральных токов. Модель рассмотрена в предположении, что внутреннее кручение, связанное с метрикой Римана-Картана пространства, и внешнее кручение, связанное с антисимметричным B-полем, (анти)совпадают. Для модели струны с постоянным $SU(2)$-кручением получено новое уравнения движения и найдено точное решение в виде функции Ламберта. Для модели струны с постоянным $SU(n)$-, $SO(n)$-, $SP(n)$-кручением для $n \to \infty$ получены новые уравнения движения для бесконечномерных гидродинамических цепочек и новые скобки Пуассона.

ИНТЕГРОВАНІ ВЗНВ-МОДЕЛІ ТА СТРУННІ МОДЕЛІ ВЗНВ-ТИПУ З ПОСТІЙНИМ $SU(2)$-СКРУТОМ

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Досліджена інтегрованість ВЗНВ-моделі та струнної моделі ВЗНВ-типу с постійним $SU(2)$-скрутом. Замкнута бозонна струна в зовнішніх гравітаційному и антисиметричному полях розглянута як інтегрована система у термінах первісних кіральних струмів. Модель розглянута за припущенням що внутрішній скрут, зв'язаний з метрикою Рімана-Картана простору, та зовнішній скрут, зв'язаний з антисиметричним B-полем, (анти)співпадають. Для моделі струни з постійним $SU(2)$-скрутом одержано нове рівняння руху та знайдені точні рішення у вигляді функції Ламберта. Для моделі струни з постійним $SU(n)$-, $SO(n)$-, $SP(n)$-скрутом з $n \to \infty$ одержані нові рівняння руху для нескінченновимірних гідродинамічних ланцюгів та нові дужки Пуассона.