SYMMETRY, PHASE STATES AND DYNAMICS OF MAGNETS WITH SPIN s=1

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The results of investigations of magnets with spin $s = 1$ are presented. The analysis of the possible symmetry of exchange interactions and its relationship with the magnetic degrees of freedom was done. We formulate the dynamics of normal non-equilibrium states. The generalization of the Bloch equations is obtained and the effect of magnetic field on the spectral characteristics is considered. The influence of dissipative processes is investigated and the relaxation fluxes corresponding to the exchange symmetry of the magnetic Hamiltonian are obtained.

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1. INTRODUCTION

The Landau-Lifshitz equation [1] defines the evolution of magnets in terms of the spin vector. This equation is well justified for the spin s=1/2 and used for studying the static and dynamic properties of magnetic insulators [2]. Discovery of quadrupole states and synthesizing of high-spin molecules have required clarifying the ideology of the macroscopic description of magnets with a spin $s > 1/2$ [3]. An additional stimulus came from the investigating of Bose-Einstein condensates of neutral atoms with a nonzero spin [4, 5]. For them the realization the magnetic states with higher symmetries of ordering, as compared with SO(3) symmetry, is possible. Using technology of optical lattices it will be possible, in principle, to construct magnetic materials with new physical properties at low temperatures.

Two points must be kept in mind regarding the development of notions pertaining to a abridge description of nonequilibrium magnetic states. The first is the need to extend the degrees of freedom in magnetic systems with spin $s > 1/2$. For pure quantum states these degrees of freedom are associated with the number of parameters characterizing the one-particle spin states. The normalization condition and the freedom to choose the wave-function phase lead to $N_{pure}(s) = 4s$ independent parameters for spin s. In the case of mixed quantum states because of the hermiticity and the normalization condition for density matrix, the number of such parameters is $N_{mix}(s) = 4s(s+1)$. The other important point in generalizing the macroscopic description of magnets is related to the notion of normal and degenerate equilibrium states for quantum objects. Normal states correspond to a paramagnetic state. The other states of magnets are states with spontaneously broken symmetry. Depending on the pattern of symmetry breaking due to the nature of the order parameter, adequately treating the system also requires extending the set of macroscopic parameters. Several options of dynamical behavior with different full sets of parameters of the abridge description can be realized in the $s = 1$ magnets. The set of these parameters essentially depends on the symmetry of the Hamiltonian and the symmetry of equilibrium states, which may not coincide in general.

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2. SYMMETRY OF THE EXCHANGE HAMILTONIAN AND EQUILIBRIUM STATES

The symmetry of the exchange Hamiltonian and the equilibrium states allows one to find a set of thermodynamic parameters describing the macroscopic magnetic states. To formulate these symmetry properties, we use the construction of the Gibbs statistical operator, which is not part of Hamiltonian mechanics. We give the necessary mathematical formulation and physical clarifications regarding the use of the terms "normal" and "degenerate" equilibrium states below, using the language of quantum mechanics. In the case SO(3) symmetry the exchange interaction Hamiltonian and normal equilibriums described by the Gibbs statistical operator $\hat{w}(Y) = \exp(\Omega - Y_a \hat{\gamma}_a)$ satisfy the equalities

$$\left[\hat{H}, \hat{S}_\alpha\right] = 0, \quad \left[\hat{w}, \hat{\Sigma}_\alpha(\mathbf{Y})\right] = 0. \qquad (1)$$

The generalized operator of spin moment is introduced here as

$$\hat{\Sigma}_\alpha(\mathbf{Y}) \equiv \hat{S}_\alpha + S_\alpha^{\mathbf{Y}}, \quad S_\alpha^{\mathbf{Y}} \equiv -i\varepsilon_{\alpha\beta\gamma} Y_\beta \frac{\partial}{\partial Y_\gamma}. \qquad (2)$$

It acts in the Hilbert space and in the space of thermodynamic forces $Y_a = (Y_0, Y_\alpha)$. The thermodynamic forces determine the temperature $Y_0^{-1} \equiv T$ and the internal magnetic field $-Y_\alpha/Y_0 \equiv h_\alpha$ and conjugate with the motion integrals $\hat{\gamma}_a \equiv \hat{H}, \hat{S}_\alpha$. The thermodynamic potential is can be determined from the condition of normalization $\mathrm{Sp}\hat{w} = 1$. Relations (1) mean that the Hamiltonian and the equilibrium are invariant under unitary transformations of homogeneous spin rotation $\hat{U} = \exp i\theta_\alpha \hat{\Sigma}_\alpha(\mathbf{Y})$, whose generator is the operator (2). Degenerate equilibriums have a symmetry lower than that of the Hamiltonian, with $\left[\hat{w}, \hat{\Sigma}_\alpha(\mathbf{Y})\right] \neq 0$. In the case of degenerate equilibrium states their description requires to use the quasiaverages conception [6, 7]. It hence follows that the equilibrium states depend on the unitary transformation parameters: $\hat{w} = \hat{w}(Y, \theta_\alpha)$. Table 1 shows the relationship of the symmetry properties of the Hamiltonian and the equilibrium states with the number of magnetic degrees of freedom for magnets with spin $s = 1/2$.

Now we consider the magnets with spin $s = 1$. This case, as seen from Table 2, is much more complicated. In addition to the well-known SO(3) symmetry, for spin s=1 it is possible to implement the SU(3) symmetry. Along with a vector order parameter a tensor order parameter may exist and also more diverse ways of symmetry breaking of the equilibrium state may be realized. A number of magnetic degrees of freedom increases. The statistical operator of normal equilibriums of magnets with the SU(3) symmetry has a similar form. In addition to the Hamiltonian, a set of additive integrals of motion $\hat{\gamma}_a \equiv \left(\hat{H}, \hat{G}_{\alpha\beta}\right)$ contain the matrix operator $\hat{G}_{\alpha\beta} = \int d^3x \hat{g}_{\alpha\beta}(\mathbf{x})$. Here, following [8], we introduce the tensor density operator $\hat{g}_{\alpha\beta}(\mathbf{x}) \equiv \hat{\psi}^+_\alpha(\mathbf{x})\hat{\psi}_\beta(\mathbf{x}) - \delta_{\alpha\beta}\hat{\psi}^+_\gamma(\mathbf{x})\hat{\psi}_\gamma(\mathbf{x})/3$ in terms of the Bose field creation and annihilation operators $\hat{\psi}^+_\alpha(\mathbf{x}), \hat{\psi}_\alpha(\mathbf{x})$. The SU(3) symmetry of normal equilibriums is formulated similarly according to relations (1) and (2). For this, we introduce the operator

$$\hat{G}_{\alpha\beta}(\mathbf{Y}) \equiv \hat{G}_{\alpha\beta} + G^{\mathbf{Y}}_{\alpha\beta},$$

$$G^{\mathbf{Y}}_{\alpha\beta} \equiv Y_{\alpha\lambda}\frac{\partial}{\partial Y_{\beta\lambda}} - Y_{\lambda\beta}\frac{\partial}{\partial Y_{\lambda\alpha}}. \quad (3)$$

Table 1. *Normal and degenerate equilibrium states of magnets with spin s = 1/2*

The symmetry of Hamiltonian	The symmetry of the equilibrium state	The symmetry group	Magnetic degrees of freedom	The number of magnetic degrees of freedom	Order parameter
$\left[\hat{H}, \hat{S}_\alpha\right] = 0$	$\left[\hat{w}, \hat{\Sigma}_\alpha(\mathbf{Y})\right] = 0$	SO(3)	s_α	3	-
$\left[\hat{H}, \hat{S}_\alpha\right] = 0$	$\left[\hat{w}, \hat{\Sigma}_\alpha(\mathbf{Y})\right] \neq 0$	SO(3) broken	s_α $R_{\alpha\beta}$	6	vector

Table 2. *Normal and degenerate equilibrium states of magnets with spin s = 1*

The symmetry of Hamiltonian	The symmetry of the equilibrium state	The symmetry group	Magnetic degrees of freedom	The number of magnetic degrees of freedom	Order parameter
$\left[\hat{H}, \hat{S}_\alpha\right] = 0$ $\left[\hat{H}, \hat{Q}_{\alpha\beta}\right] \neq 0$	$\left[\hat{w}, \hat{\Sigma}_\alpha(\mathbf{Y})\right] = 0$	SO(3)	s_α	3	—
$\left[\hat{H}, \hat{S}_\alpha\right] = 0$ $\left[\hat{H}, \hat{Q}_{\alpha\beta}\right] \neq 0$	$\left[\hat{w}, \hat{\Sigma}_\alpha(\mathbf{Y})\right] \neq 0$	SO(3) broken	s_α $R_{\alpha\beta}$	6	vector
$\left[\hat{H}, \hat{S}_\alpha\right] = 0$ $\left[\hat{H}, \hat{Q}_{\alpha\beta}\right] \neq 0$	$\left[\hat{w}, \hat{\Sigma}_\alpha(\mathbf{Y})\right] \neq 0$	SO(3) broken	s_α $q_{\alpha\beta}$	8	tensor
$\left[\hat{H}, \hat{G}_{\alpha\beta}\right] = 0$	$\left[\hat{w}, \hat{G}_{\alpha\beta}(\mathbf{Y})\right] = 0$	SU(3)	$g_{\alpha\beta}$	8	—
$\left[\hat{H}, \hat{G}_{\alpha\beta}\right] = 0$	$\left[\hat{w}, \hat{G}_{\alpha\beta}(\mathbf{Y})\right] \neq 0$	SU(3) broken	$g_{\alpha\beta}$ $R_{\alpha\beta}$	11	vector
$\left[\hat{H}, \hat{G}_{\alpha\beta}\right] = 0$	$\left[\hat{w}, \hat{G}_{\alpha\beta}(\mathbf{Y})\right] \neq 0$	SU(3) broken	$g_{\alpha\beta}$ $\Delta_{\alpha\beta}$	16	tensor

This operator acts in a Hilbert space and in the space of thermodynamic parameters. Using this generator, we can write property of SU(3) symmetry for Hamiltonian and equilibrium states. Operator $\hat{G}_{\alpha\beta}(\mathbf{Y})$ satisfies the relations $\left[\hat{G}_{\alpha\beta}(\mathbf{Y}), \hat{G}_{\mu\nu}(\mathbf{Y})\right] = \hat{G}_{\alpha\nu}(\mathbf{Y})\delta_{\beta\mu} - \hat{G}_{\mu\beta}(\mathbf{Y})\delta_{\alpha\nu}$. The SU(3) symmetry conditions for the Hamiltonian and normal equilibriums then become $\left[\hat{H}, \hat{G}_{\alpha\beta}\right] = 0$, $\left[\hat{w}, \hat{G}_{\alpha\beta}(\mathbf{Y})\right] = 0$. These formulas mean that the Hamiltonian and the equilibrium are invariant under homogeneous linear transformation $\hat{U} = \exp i\theta_{\alpha\beta}\hat{G}_{\beta\alpha}(\mathbf{Y})$, whose generator is the operator in (3). In the case of spontaneous symmetry breaking (degenerate states), $\left[\hat{w}, \hat{G}_{\alpha\beta}(\mathbf{Y})\right] \neq 0$, which results in an additional dependence of the equilibrium on the parameters of the unitary transformation $\hat{w} = \hat{w}(Y, \theta_{\alpha\beta})$. Table 2 shows the relationship of the symmetry properties of the Hamiltonian and the equilibrium states with the number of magnetic degrees of freedom for magnets with spin $s = 1$.

3. NONEQUILIBRIUM PROCESSES

In [9] the Poisson brackets for the Hermitian matrix $\hat{g}(\mathbf{x})$ were obtained:

$$i\{g_{\alpha\beta}(\mathbf{x}), g_{\gamma\rho}(\mathbf{x}')\} = \left(-g_{\alpha\rho}(\mathbf{x})\,\delta_{\gamma\beta} + g_{\gamma\beta}(\mathbf{x})\,\delta_{\alpha\rho}\right)\delta(\mathbf{x}-\mathbf{x}'). \tag{4}$$

This matrix is related to the quadrupole matrix $q_{\alpha\beta}(\mathbf{x})$ and spin density $s_\alpha(\mathbf{x})$ by relation: $g_{\alpha\beta}(\mathbf{x}) \equiv q_{\alpha\beta}(\mathbf{x}) - i\varepsilon_{\alpha\beta\gamma}s_\gamma(\mathbf{x})/2$. Dynamics of normal nonequilibrium states with spin 1 is described by the Hamiltonian, which is a functional of matrix $\hat{g}(\mathbf{x})$: $H = H(\hat{g}(\mathbf{x}))$. Using standard Hamiltonian formalism, we obtain the equations of nonlinear dynamics for the matrix

$$\dot{\hat{g}}(\mathbf{x}) = i\left[\hat{g}(\mathbf{x}), \frac{\delta\hat{H}(g)}{\delta g(\mathbf{x})}\right], \tag{5}$$

which generalizes the Landau-Lifshitz equation for the considered magnets. In the case of SU(3) symmetry of the Hamiltonian a set of integrals of motion consists of the exchange Hamiltonian and the matrix $G_{\alpha\beta}$: $\gamma_a \equiv (H, G_{\alpha\beta}) = \int d^3x\zeta_a(\mathbf{x})$, $\{H, \gamma_a\} = 0$. Here $\zeta_a(\mathbf{x}) = \varepsilon(\mathbf{x}), g_{\alpha\beta}(\mathbf{x})$ are densities of the additive integrals of motion ($a = 0, \alpha\beta$). Using the representation of the flux densities of additive integrals of motion [10], we obtain the dynamic equations, reflecting the conservation laws in the differential form

$$\begin{aligned} \dot{\varepsilon}(\mathbf{x}) &= -\nabla_k q_k(\mathbf{x}), \\ \dot{\hat{g}}(\mathbf{x}) &= -\nabla_k \hat{j}_k(\mathbf{x}), \end{aligned} \tag{6}$$

$$\begin{aligned} q_k(\mathbf{x}) &\equiv \varsigma_{0k}^{(0)}(\mathbf{x}) = \\ &= \frac{1}{2}\int d^3x' x'_k \int_0^1 d\lambda\{\varepsilon(\mathbf{x}+\lambda\mathbf{x}'), \varepsilon(\mathbf{x}-(1-\lambda)\mathbf{x}')\}, \\ \hat{j}_k(\mathbf{x}) &\equiv \hat{\varsigma}_k^{(0)}(\mathbf{x}) = \\ &= \int d^3x' x'_k \int_0^1 d\lambda\{\hat{g}(\mathbf{x}+\lambda\mathbf{x}'), \varepsilon(\mathbf{x}-(1-\lambda)\mathbf{x}')\}, \end{aligned}$$

where $q_k(\mathbf{x})$ is the energy flux density and $\hat{j}_k(\mathbf{x})$ is the flux density corresponding to the conserved quantity $\hat{G}$. Taking into account the equality (4), from (5), (6) we obtain expressions for the flux densities of the additive integrals of motion

$$\hat{j}_k = i\left[\hat{g}, \frac{\partial\hat{\varepsilon}}{\partial\nabla_k g}\right], \quad q_k = Sp\frac{\delta\hat{H}}{\delta g}\hat{j}_k. \tag{7}$$

Consider the homogeneous dynamics of the magnetic medium in an external constant magnetic field. Hamiltonian $V(\mathbf{h})$, describing such an interacting medium, in its simplest form can be written as $V(\mathbf{h}) \equiv Sp\hat{\mathbf{G}}\hat{\mathbf{h}} = V_1(\mathbf{h}) + V_2(\mathbf{h})$, where the term linear in magnetic field

$$V_1(\mathbf{h}) = -ih_\alpha\varepsilon_{\alpha\beta\gamma}G_{\beta\gamma} = -h_\alpha S_\alpha \tag{8}$$

is the Zeeman interaction. Term

$$V_2(\mathbf{h}) = Q_{\alpha\beta}h_{\beta\alpha}, \; h_{\alpha\beta} \equiv h_\alpha h_\beta - \delta_{\alpha\beta}h^2/3 + i\varepsilon_{\alpha\beta\gamma}h_\gamma \tag{9}$$

is squared in the magnetic field. In the absence of spatial inhomogeneities and of term $V_2(\mathbf{h})$ for matrix $g_{\alpha\beta}$, according (4), (5), (8) we obtain the dynamic equation $\dot{g}_{\alpha\beta} = h_\sigma(g_{\alpha\rho}\varepsilon_{\sigma\beta\rho} - \varepsilon_{\sigma\rho\alpha}g_{\rho\beta})$. Hence, separating the symmetric and antisymmetric parts, we obtain the equations

$$\dot{s}_\alpha = \varepsilon_{\alpha\beta\gamma}s_\beta h_\gamma, \quad \dot{q}_{\alpha\beta} = h_\sigma(q_{\alpha\rho}\varepsilon_{\sigma\beta\rho} - \varepsilon_{\sigma\rho\alpha}q_{\rho\beta}). \tag{10}$$

The first of them is the Bloch equation, which describes the spin dynamics. The second one is the equation of motion for the quadrupole matrix. Obviously, in equilibrium, the spin is directed along the magnetic field $s_\alpha || h_\alpha$, quadrupole matrix is uniaxial and has the form $q_{\alpha\beta} = q(n_\alpha n_\beta - \delta_{\alpha\beta}/3)$, where unit vector $n_\alpha \equiv h_\alpha/h$. The solution of the first equation in (10) leads to two spin-wave spectra $\omega = 0$ and $\omega = h$. The solution of the second equation in (10) leads to three quadrupole spectra of waves: $\omega = 0$, $\omega = h$, $\omega = 2h$. Consider now the effect of interaction $V_2(\mathbf{h})$ on dynamics of the system. The corresponding equation for the matrix $g_{\alpha\beta}$ has the form $\dot{\hat{g}} = i\left[\hat{g}, \hat{h}\right]$. This implies the following dynamical equations for the density matrix of spin and quadrupole:

$$\dot{s}_\alpha = 2\varepsilon_{\alpha\beta\rho}q_{\gamma\rho}h_{\beta\gamma},$$

$$\dot{q}_{\alpha\beta} = s_\sigma h_\rho(h_\beta\varepsilon_{\alpha\rho\sigma} + h_\alpha\varepsilon_{\beta\rho\sigma}). \tag{11}$$

Stationary solutions of these equations lead to the condition of collinearity of the vectors $s_\alpha || h_\alpha$. The quadrupole matrix still has the form $q_{\alpha\beta} = q\left(n_\alpha n_\beta - \delta_{\alpha\beta}/3\right)$, where there are two possible solutions for the unit vector $n_\alpha = h_\alpha / h$ and $n_\alpha \perp h_\alpha$. The solution of (11) leads to the two spectra of collective excitations: $\omega = 0$, $\omega = h^2$.

Let us consider the relaxation processes in magnetic materials with spin s=1. For this we use the approach of [11], where the dissipative Poisson brackets were introduced and the relaxation equations for the dynamics of condensed matter were obtained. The equations of motion for the densities of additive integrals of motion can be written as

$$\dot{\zeta}_a\left(\mathbf{x}\right) \equiv \{\zeta_a\left(\mathbf{x}\right), H\} - T_0 \{\zeta_a\left(\mathbf{x}\right), \Sigma\}_D. \qquad (12)$$

Here $\Sigma = \int d^3x s\left(\mathbf{x}\right)$ is the entropy and T_0 is a constant having the dimensionality of temperature. Reactive Poisson bracket describes the dynamics of the system in the adiabatic approximation, while the dissipative bracket – the relaxation processes. Dissipative brackets are symmetric and satisfy the Leibnitz identity

$$\begin{array}{l} \{A, B\}_D = \{B, A\}_D, \\ \{A, BC\}_D = \{A, B\}_D C + B\{A, C\}_D. \end{array}$$

For the densities of additive integrals of motion, using (4) and (6), (7), we obtain the Poisson brackets:

$$\begin{array}{l} \{\zeta_a\left(\mathbf{x}\right), \zeta_b\left(\mathbf{x}'\right)\} = -i\delta_{a,\alpha\beta}\delta_{b,\gamma\rho}\delta\left(\mathbf{x}-\mathbf{x}'\right) \times \\ \times \left(g_{\gamma\beta}\left(\mathbf{x}\right)\delta_{\alpha\rho} - g_{\alpha\rho}\left(\mathbf{x}\right)\delta_{\gamma\beta}\right) + \\ + \left[\delta_{a0}\zeta_{bk}^{(0)}\left(\mathbf{x}\right) + \delta_{b0}\zeta_{ak}^{(0)}\left(\mathbf{x}'\right)\right]\nabla'_k \delta\left(\mathbf{x}-\mathbf{x}'\right) \end{array} \qquad (13)$$

The right side of the bracket is represented in terms of densities and the corresponding fluxes of the additive integrals of motion. The explicit form of the dissipative Poisson brackets can be expressed in terms of the dissipation function, which under consideration for magnets has the form

$$\begin{array}{l} R \equiv \dfrac{1}{2}\displaystyle\int d^3x \nabla_k Y_a\left(\mathbf{x}\right) I_{ak,bl}\left(\mathbf{x}\right) \nabla_l Y_b\left(\mathbf{x}\right) \\ \quad = \displaystyle\int d^3x r\left(\mathbf{x}\right). \end{array} \qquad (14)$$

Here $Y_a\left(\mathbf{x}\right) = \delta\Sigma / \delta\zeta_a\left(\mathbf{x}\right)$ are thermodynamic forces conjugate to the additive integrals of motion, $I_{ak;bl}$ are generalized kinetic coefficients, which satisfy the Onsager principle of the kinetic coefficients symmetry $I_{ak,bl} = I_{bl,ak}$. Since the matrix $\hat{g}$ is traceless, then we have the additional relations $I_{\alpha\alpha k,bl} = 0$, $I_{ak,\gamma\gamma l} = 0$. Taking into account [11] we obtain expression

$$\begin{array}{l} \{\zeta_a\left(\mathbf{x}\right), \zeta_b\left(\mathbf{x}'\right)\}_D \equiv -\delta^2 R / \delta Y_a\left(\mathbf{x}\right)\delta Y_b\left(\mathbf{x}'\right) \\ \quad = -\dfrac{1}{T_0}\nabla_k \nabla'_l \left(I_{ak,bl}\left(\mathbf{x}\right)\delta\left(\mathbf{x}-\mathbf{x}'\right)\right). \end{array} \qquad (15)$$

Accounting for the relaxation processes leads to the equations of dynamics for the densities of additive integrals of motion

$$\dot{\varsigma}_a\left(\mathbf{x}\right) = -\nabla_k \left(\zeta_{ak}^{(0)}\left(\mathbf{x}\right) + \zeta_{ak}^{(1)}\left(\mathbf{x}\right)\right) \equiv L_a^R\left(\mathbf{x}\right) + L_a^D\left(\mathbf{x}\right), \qquad (16)$$

where we obtain

$$L_a^D\left(\mathbf{x}\right) = -T_0 \int d^3x' \frac{\delta\Sigma}{\delta\zeta_b\left(\mathbf{x}'\right)} \{\zeta_a\left(\mathbf{x}\right), \zeta_b\left(\mathbf{x}'\right)\}_D.$$

The equations (13), (15), (16) yield the dynamic equation for the entropy density

$$\dot{s}\left(\mathbf{x}\right) = -\nabla_k j_{sk}^{(1)}\left(\mathbf{x}\right) + I\left(\mathbf{x}\right), \qquad (17)$$

where $j_{sk}^{(1)} = Y_a \zeta_{ak}^{(1)}$ is the flux density of entropy and $I = \zeta_{ak}^{(1)} \nabla_k Y_a$ is the entropy production. Taking into account formulas (14), (17), we see that the dissipation function is associated with the densities of the dissipative flow of the additive integrals of motion equation $L_a^D\left(\mathbf{x}\right) = -\nabla_k \zeta_{ak}^{(1)}\left(\mathbf{x}\right) = \delta R / \delta Y_a\left(\mathbf{x}\right)$. In the exchange approximation the tensor structure of the generalized transport coefficients is such that the spatial and spin indices are not mixed and there is no preferred direction in configuration space. Therefore, $I_{ak,bl} = \delta_{kl} I_{ab}$. In this case, for the dissipative flux densities of additive integrals of motion we obtain the expressions

$$\begin{array}{l} j_{\alpha\beta}^{(1)k} = -D_{\alpha\beta}\nabla_k T - \sigma_{\alpha\beta,\gamma\rho}\nabla_k h_{\rho\gamma}, \\ q_k^{(1)} = -\left(\kappa + h_{\beta\alpha}D_{\alpha\beta}\right)\nabla_k T - \\ -T D_{\alpha\beta}\nabla_k h_{\beta\alpha} - \sigma_{\alpha\beta,\gamma\rho} h_{\beta\alpha}\nabla_k h_{\rho\gamma}. \end{array} \qquad (18)$$

Coefficients of thermal conductivity κ, magnetic thermodiffusion $D_{\alpha\beta}$ and magnetic diffusion $\sigma_{\alpha\beta,\gamma\rho}$ are associated with generalized kinetic coefficients by the relations: $I_{\alpha\beta,0} = T^2 D_{\alpha\beta} + T h_{\gamma\rho}\sigma_{\alpha\beta,\rho\gamma}$, $I_{\alpha\beta,\gamma\rho} = T\sigma_{\alpha\beta,\rho\gamma}$, $I_{0,0} = T^2\kappa + 2T^2 h_{\gamma\rho}D_{\rho\gamma} + T h_{\beta\alpha}h_{\gamma\rho}\sigma_{\alpha\beta,\rho\gamma}$. Account now for the specific structure of the transport coefficients for the paramagnetic state, where in equilibrium $g_{\alpha\beta} = 0$ and $h_{\alpha\beta} = 0$. The expressions for the tensor of kinetic coefficients become simplified and take the form

$$\begin{array}{l} \sigma_{\alpha\beta,\gamma\rho} = \sigma\left(\delta_{\alpha\rho}\delta_{\beta\gamma} - \delta_{\alpha\gamma}\delta_{\beta\rho}\right)/4 + \\ +\sigma'\left(\delta_{\alpha\gamma}\delta_{\beta\rho} + \delta_{\alpha\rho}\delta_{\beta\gamma} - \frac{2}{3}\delta_{\alpha\beta}\delta_{\gamma\rho}\right)/2 \end{array},$$

$$D_{\alpha\beta} = 0. \qquad (19)$$

Here σ, σ' are, respectively, the spin diffusion coefficient and diffusion of the quadrupole matrix. As a result, we obtain the flux density of the matrix $g_{\alpha\beta}$ and energy density

$$\begin{array}{l} j_{\alpha\beta}^{(1)k} = i\sigma\varepsilon_{\alpha\beta\gamma}\nabla_k h_\gamma / 2 - \sigma'\nabla_k h_{\alpha\beta}^s, \\ q_k^{(1)} = -\kappa\nabla_k T, \end{array} \qquad (20)$$

where $h_{\alpha\beta}^a \equiv -i\varepsilon_{\alpha\beta\gamma}h_\gamma$, $h_{\alpha\beta}^s \equiv \left(h_{\alpha\beta} + h_{\beta\alpha}\right)/2$. From (17) - (20) the expressions for the dissipative flux and entropy production follow:

$$\begin{array}{l} j_s^{(1)k} = -\frac{\kappa}{T}\nabla_k T, I = \frac{\kappa}{T^2}\left(\nabla_k T\right)^2 + \\ +\frac{\sigma}{T}\left(\nabla_k h_\alpha\right)^2 + \frac{\sigma'}{T}\left(\nabla_k h_{\alpha\beta}^s\right)^2 \geq 0. \end{array}$$

Positivity of entropy production is ensured by the inequalities $\kappa \geq 0$, $\sigma \geq 0$, $\sigma' \geq 0$.

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СИММЕТРИЯ, ФАЗОВЫЕ СОСТОЯНИЯ И ДИНАМИКА МАГНЕТИКОВ СО СПИНОМ s=1

М.Ю. Ковалевский, О.А. Котельникова

Представлены результаты исследований магнетиков со спином 1. Дан анализ возможной симметрии обменных взаимодействий и ее связь с магнитными степенями свободы. Сформулирована динамика нормальных неравновесных состояний. Получено обобщение уравнения Блоха и изучено влияние магнитного поля на спектральные характеристики. Рассмотрено влияние диссипативных процессов и найдены релаксационные потоки, обусловленные обменной симметрией магнитного гамильтониана.

СИМЕТРІЯ, ФАЗОВИЙ СТАН ТА ДИНАМІКА МАГНЕТИКІВ ЗІ СПІНОМ s=1

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Представлені результати досліджень магнетиків зі спіном 1. Дан аналіз можливої симетрії обмінних взаємодій та їх зв'язок з магнітними ступенями свободи. Сформульована динаміка нормальних нерівноважних станів. Отримано узагальнення рівняння Блоха і вивчено вплив магнітного поля на спектральні характеристики. Розглянуто вплив дисипативних процесів і знайдені релаксаційні потоки, зумовлені обмінної симетрією магнітного гамільтоніана.