UNUSUAL RESONANCES IN SUPERFLUID ^{4}He – METAL DOUBLE-LAYER SYSTEM

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We consider the model of temperature and heat flow evolution in a double-layer system. The first layer is superfluid ^{4}He and the second layer is a metallic plate. At the left end of the first layer the oscillating heat source is situated. Temperature at the right end of the second layer is constant. It was found that temperature and heat flow resonances of unusual form appeared. Analytical expressions for width and amplitudes of these resonances are obtained.

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1. INTRODUCTION

The existence of second sound is a unique feature of superfluid ^{4}He. It is known that temperature and heat flow resonances of second sound appear in case of second sound excitation with an oscillating heat flow. Resonances appear both in even and odd harmonics. This phenomenon was described in [1, 2].

In [3–5] we demonstrated the possibility of unusual behavior of resonances when they appear only in odd harmonics. We attributed this feature in the resonance behavior to the specifics of the boundary conditions at the right end of the vessel. In [1, 2] the right end was heat insulated and in our case the right end was at the constant temperature.

In the present work we continue studying resonances in the standing second sound wave under boundary conditions as in [3–5], i.e. with oscillating heat flow at the left end and with constant temperature at the right end. Unlike [3–5] we considered superfluid ^{4}He as a first layer and a metallic wall as the second layer (in [3–5] heater and helium were considered as the double-layer system).

2. CALCULATIONS

So, we consider the following double-layer system — the first layer is superfluid helium of width l and the second layer is the right wall of width h. The heater is situated at the left end of the vessel and its width is neglected. Heat transfer in helium is described with the system of hydrodynamic equations for superfluids. For simplicity we consider temperature conductivity of helium equal to zero:

$$\frac{\partial v_n}{\partial t} + u_2^2 \frac{\partial T}{\partial x} = 0,$$
$$\frac{\partial T}{\partial t} + \frac{\partial v_n}{\partial x} = 0. \tag{1}$$

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We describe heat transfer in the second layer with the usual thermal conductivity equation. For our system we have the following boundary conditions:

$$Q(x=0,t) = Q_0 \cos(\omega t),$$
$$T(h,t) = 0. \tag{2}$$

where Q_0 is external heat sources amplitude, ω is frequency of external heat source.

Making temperatures and heat flows equal at the boundary of the layers we obtain the following expression for temperature in superfluid helium:

$$T(x,t) = \frac{Q_0(C_2(x)\cos(\omega t) + C_1(x)\sin(\omega t))}{\alpha u_2 Z}, \tag{3}$$

where functions $C_1(x)$ and $C_2(x)$ have the form:

$$\begin{aligned} C_1(x) &= 4C_v^2\omega\chi\cos(kl)\sin(k(l+x))b_1 \\ &\quad - 4\alpha^2 u_2^2 \sin(kl)\cos(k(l+x))b_2 \\ &\quad - C_v\alpha u_2\sqrt{2\omega\chi}\cos(k(2l+x))b_3, \\ C_2(x) &= -C_v\alpha u_2\sqrt{2\omega\chi}\cos(kx)b_4, \\ Z &= 4C_v^2\omega\chi\cos^2(kl)b_1 + 4\alpha^2u_2^2\sin^2(kl)b_2 \\ &\quad + C_v\sqrt{2\omega\chi}\sin(2kl)b_3. \end{aligned}$$

Then the expression for the heat flow in helium is:

$$Q(x,t) = \frac{Q_0(C_4(x)\cos(\omega t) + C_3(x)\sin(\omega t))}{Z}, \tag{4}$$

with functions $C_3(x)$ and $C_4(x)$ of the form:

$$\begin{aligned} C_3(x) &= 4C_v^2\omega\chi\cos(kl)\cos(k(l+x))b_1 \\ &\quad - 4\alpha^2u_2^2\sin(kl)\sin(k(l+x))b_2 \\ &\quad - C_v\alpha u_2\sqrt{2\omega\chi}\sin(k(2l+x))b_3, \\ C_4(x) &= C_v\alpha u_2\sqrt{2\omega\chi}\sin(kx)b_4. \end{aligned}$$

Hereinabove we used the following notations:

$$b_1 = \cos^2(\lambda h) + \sinh^2(\lambda h),$$
$$b_2 = \sin^2(\lambda h) + \sinh^2(\lambda h),$$
$$b_3 = \sin(2\lambda h) + \sinh(2\lambda h),$$
$$b_4 = \sin(2\lambda h) - \sinh(2\lambda h),$$
$$\alpha = T_0 C_{VHe}.$$

Here $\lambda = \sqrt{\omega/(2\chi)}$ is the inverse thermal wavelength in the right wall of the vessel, χ is temperature conductivity of it, $k = \omega/u_2$ defines the inverse wavelength of second sound, C_v and C_{vHe} are thermal capacity of the right wall and helium respectively, T_0 is equilibrium temperature of helium.

3. UNUSUAL RESONANCES

Lets consider the limiting cases of relation the right wall width to the thermal wavelength in the right wall (i. e. limiting cases of relation h to $1/\lambda$).

When $h\lambda << 1$ odd harmonics resonances of the heat flow and temperature appear. For the heat flow we obtain the expression for resonance of the form:

$$Q(\omega) = \frac{Q_0 u_2}{l\sqrt{(\omega-\omega_0)^2+\gamma^2}}, \tag{5}$$

where $\omega_0 = \pi/2(2k-1)u_2/l$, $k = 1, 2, \ldots$ and

$$\gamma = \frac{u_2^2}{C_v \chi l} h. \tag{6}$$

present respectively the frequency and peak width of the resonance. So heat flow resonances have usual form. Unlike heat flow resonances, the temperature resonances are of unusual form (Fig. 1).

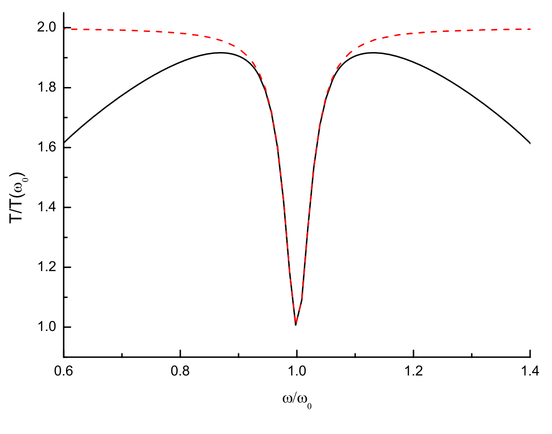

Fig. 1. *An example of temperature resonance of unusual form. Solid line corresponds to exact formulae (3). Dashed line corresponds to approximate expression (7)*

The expression for temperature resonances is:

$$T(\omega) = \frac{Q_0}{u_2}\sqrt{\frac{4(\omega-\omega_0)^2+\gamma^2}{(\omega-\omega_0)^2+\gamma^2}}. \tag{7}$$

It should be noted that resonance width depends on h (as we can see from (6)).

When $h\lambda >> 1$ even harmonics resonances of the heat flow and temperature appear. So changing relation h/λ one can switch the resonance harmonics from odd to even or vice versa. For temperature resonance we obtain the following expression:

$$T(\omega) = \frac{Q_0}{l\sqrt{(\omega-\omega_0+\gamma)^2+\gamma^2}}. \tag{8}$$

Here $\omega_0 = \pi k u_2/l$, $k = 1, 2, \ldots$ is a resonance frequency and

$$\gamma = \frac{C_v\sqrt{\omega\chi}}{\alpha\sqrt{2}l} \tag{9}$$

is the width of the resonance peak. So temperature resonances have usual form but their frequency is shifted on γ. Heat flow resonances are of unusual form (Fig. 2) and expression for them can be written in the form:

$$Q(\omega) = Q_0\frac{\sqrt{4(1+\varpi)^4 - 4\varpi(0.75+\varpi)^2 - 1.75\varpi}}{(1+\varpi)^2+1}, \tag{10}$$

where $\varpi = (\omega-\omega_0)/\gamma$ is normalized frequency. As we can see from (10) amplitude of heat flow resonance depends only on Q_0 and ϖ . So maximum and minimum value of resonance depends only on Q_0 and does not change with resonance width (Fig. 3). We have calculated that $Q_{max} \approx 2.29\,Q_0$ and $Q_{min} \approx 0.87\,Q_0$. The values of ω_{max} and ω_{min} can be found from the trivial relations $\omega_{max} = \omega_0 + \varpi_{max}\gamma, \omega_{min} = \omega_0 + \varpi_{min}\gamma$, where $\varpi_{max} = -0.316, \varpi_{min} = -2.618$. Such behavior near a resonance frequency is very much unexpected. The nature of this phenomenon is connected with the unusual type of the considered system. It consists of two layers which have different type of heat transfer — acoustic and dissipative. This frequency dependence could be observed in future experiments. All the conditions under which such resonances can be observed are presented in this paper.

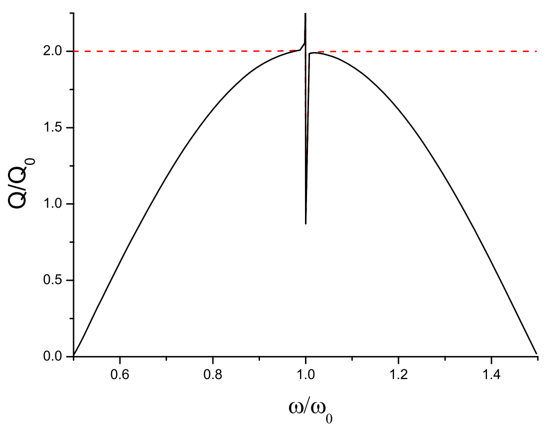

Fig. 2. *An example of heat flow resonance of unusual form. Solid line corresponds to exact formulae (4). Dashed line corresponds to approximate expression (10)*

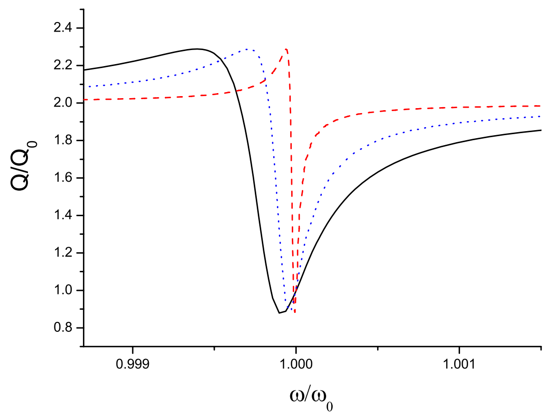

Fig. 3. *The examples of heat flow resonances of unusual form with different γ. Dashed, solid and dotted lines correspond to the least, the greatest and intermediate value of γ, respectively. The width changes but amplitude does not change*

4. CONCLUSIONS

We developed the double-layer model (with superfluid ^{4}He as a first layer and right wall as a second layer) of heat emission and propagation. Using this model we study influence of the right wall of the vessel on resonance characteristics. In particular we found that the resonance width in helium can be determined not by dissipative properties of helium, but by the thermodynamic parameters of the right wall of the vessel.

We obtained accurate expressions for temporal and spatial dependencies of temperature (3) and heat flow (4).

Unusual resonances of heat flow and temperature were found out and experimental conditions under which such phenomenon may be observed were defined.

It was established that heat flow resonances amplitudes depend only on external heat sources amplitude when the right wall width is much larger than the thermal wavelength in it and does not change with resonance width.

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НЕОБЫЧНЫЕ РЕЗОНАНСЫ В ДВУХСЛОЙНОЙ СИСТЕМЕ СВЕРХТЕКУЧИЙ ^{4}He – МЕТАЛЛ

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Рассматривается модель распространения температуры и теплового потока в двухслойной системе. Первый слой — сверхтекучий ^{4}He, а второй слой — металлическая пластина. На левом конце первого слоя находится осциллирующий источник тепла. На правом конце второго слоя поддерживается постоянная температура. Обнаружено возникновение резонансов температуры и теплового потока необычной формы. Получены аналитические выражения для амплитуд и ширин резонансных пиков.

НЕЗВИЧАЙНІ РЕЗОНАНСИ В ДВОШАРОВІЙ СИСТЕМІ НАДПЛИННИЙ ^{4}He – МЕТАЛ

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Розглядається модель поширення температури та теплового потоку в двошаровій системі. Перший шар — надплинний ^{4}He, а другий шар — металічна пластина. На лівому кінці першого шару знаходиться осцилюючe джерело тепла. На правому кінці другого шару підтримується постійна температура. Виявлено виникнення резонансів температури та теплового потоку незвичайної форми. Отримано аналітичні вирази для амплітуд та ширин резонансних піків.