HYDRODYNAMIC THEORY OF PLASMA OSCILLATIONS IN THE PRESENCE OF ELECTRON DENSITY CORRELATIONS

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Hydrodynamics of electronic plasma at presence of correlation of charge density is considered. Equations of ideal hydrodynamics with such condition for the case of immobile ions are received. It is shown that a new variable — second moment of charge density is included in Euler equation, for which temporal equation is received on the basis of equation of continuity in approximation of large radius of equilibrium correlations. Langmuir oscillations in this system are studied, an addition term to frequency of oscillations is found, arising up due to the account of correlation of charge density.

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1. INTRODUCTION

We will consider high-frequency oscillations of nonrelativistic coulomb plasma taking into account correlation of charge. We will carry out description of electronic components by the local values of the first and centered second moments of charge density of electronic component ρ, pressures P and velocity $\vec{v}$, on a background of immobile ionic components (model of jelly [1]). Except for these clean hydrodynamic values usually the state of environment is characterized by strength $\vec{E}$ of the electric field, which has the zero value in the equilibrium state. We will show that an term quadratic on the density of charge is included in standard Euler equation, for which we will build temporal equation in the hydrodynamic approximation.

2. ELECTRONIC PLASMA

We will write out standard equations of ideal hydrodynamics, ignoring all dissipative effects (viscidity, heat conductivity and electric resistance) [2]. Thus effects from the presence of electric charge show up most brightly. Continuity equation $\partial_t \sigma + \mathrm{div}\sigma\vec{v} = 0$ within a multiplier e/m_e gives the law of charge conservation

$$\partial_t \rho + \mathrm{div}\rho\vec{v} = 0, \tag{1}$$

Euler equation is

$$d\sigma\vec{v}/dt = -\nabla P - \rho\vec{E}. \tag{2}$$

In addition, we will write down the Poisson equation for the potential electric field

$$\mathrm{div}\vec{E} = 4\pi\left(\rho_0 - \rho\right), \tag{3}$$

where ρ_0 is charge density of compensating ionic background, $\rho > 0$ is charge density of electronic components. Using the condition of potentiality of the field $rot\vec{E} = 0$, coming from the theory of potential [3], nonaveraging strength is

$$\vec{E}\left(x\right) = \nabla \int d^3x' \frac{\rho_0 - \rho\left(x'\right)}{|x - x'|}. \tag{4}$$

Then, ignoring ionic correlations, we have after averaging for pondermotive force

$$\left\langle \rho\vec{E}\right\rangle_0 = -\int d^3x' \langle \rho\left(x\right)\rho\left(x'\right)\rangle_0 \nabla \frac{1}{|x - x'|}. \tag{5}$$

In an equilibrium there is no selected direction and this term is $\left\langle \rho\vec{E}\right\rangle_0 = 0$.

3. SMALL OSCILLATIONS

We will be interested only in small oscillations in this system. It allows to produce linearization on small amplitude of deviations from the equilibrium values. We will ignore thermal fluctuations. Putting (4) in (2) we will obtain

$$\begin{aligned}\left\langle \rho\vec{E}\right\rangle &= \int d^3x' \left(-\left\langle \rho^2\left(x - x', x'\right)\right\rangle\right) \nabla \frac{1}{|x - x'|} \\ &= \int d^3x' \left(-\left\langle \rho\left(x\right)\right\rangle \left\langle \rho\left(x'\right)\right\rangle - \left\langle \delta\rho^2\left(x - x', x'\right)\right\rangle\right) \\ &\qquad \times \nabla \frac{1}{|x - x'|},\end{aligned} \tag{6}$$

where for the centered correlation of density denotation $\left\langle \delta\rho^2\left(x - x', x'\right)\right\rangle$ is used. The first term will give standard plasma frequency [1]. For a variable $\left\langle \rho^2\left(x - x', x'\right)\right\rangle$ we will write equation, starting from the law of charge conservation (1), multiplying by a density in the proper point and linearizing

$$\begin{aligned}\partial_t \left\langle \rho^2\left(x - x', x'\right)\right\rangle + \left\langle \rho^2\left(x - x', x'\right)\right\rangle_0 \left(\partial_i v_i + \partial'_i v_i'\right) \\ + \left(v_i \partial_i + {v_i}' \partial'_i\right) \left\langle \rho^2\left(x - x', x'\right)\right\rangle_0 = 0.\end{aligned} \tag{7}$$

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We will assume that the equilibrium centered correlation of charges poorly depends on distance between points (differences of coordinates) and it is possible to neglect this dependence $\left\langle \delta\rho^2\left(x-x',x'\right)\right\rangle_0 \approx \left\langle \delta\rho^2\right\rangle =$ const. It can be named the smoothed correlation (homogeneity of equilibrium is implied). Then instead of (7) we have

$$\partial_t \left\langle \rho^2\left(x-x',x'\right)\right\rangle + \left\langle \rho^2\right\rangle_0 \left(\partial_i v_i + \partial'_i v_i'\right) = 0. \quad (8)$$

For simplicity we will ignore thermal effects $P = 0$. And differentiating linearized averaged equation (2) at times, we will obtain equation

$$\sigma_0 \partial_t^2 \partial_i v_i = \left\langle \rho^2\right\rangle_0 \int d^3x' \left(\partial_i v_i + \partial'_i v_i'\right) \nabla \frac{1}{|x-x'|}. \quad (9)$$

The first term applies in a zero by standard appearance on the Ostrogradsky-Gauss theorem as integral on an infinitely remote surface [4]. We will take divergence from linearized equation (9)

$$\sigma_0 \partial_t^2 \nabla \vec{v} = -\nabla \left\langle \rho \vec{E}\right\rangle \quad (10)$$

$$= \left\langle \rho^2\right\rangle_0 \int d^3x' \left(\nabla' \vec{v}'\right) \Delta \frac{1}{|x-x'|} = -4\pi \left\langle \rho^2\right\rangle_0 \nabla \vec{v}.$$

From where we find frequency of longitudinal plasma oscillations

$$\omega^2 = 4\pi \left(\rho_0^2 + \left\langle \delta\rho^2\right\rangle_0\right)/\sigma_0 = \Omega_e^2 + \left\langle \delta\Omega^2\right\rangle_e, \quad (11)$$

where standard electronic plasma frequency Ω_e [1, 5] and correlation addition $\left\langle \delta\Omega^2\right\rangle_e = 4\pi \left\langle \delta\rho^2\right\rangle_0/\sigma_0$ are distinguished.

4. CONCLUSIONS

Thus, evolution of electronic coulomb plasma in approximation of ideal hydrodynamics is studied. The research made in the present work allows to extract the following results:

- The linear system of temporal equations for velocity and second moment of charge density is obtained.
- In approximation of weak dependence of the centered correlation function of charge density from coordinates, correction of frequency of plasma oscillations is found, arising up due to the presence of equilibrium correlation with the indicated descriptions.
- When equilibrium value of the centered second moment terns to zero then frequency of oscillations passes to ordinary Langmuir frequency in electronic plasma.

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ГИДРОДИНАМИЧЕСКАЯ ТЕОРИЯ ПЛАЗМЕННЫХ КОЛЕБАНИЙ ПРИ НАЛИЧИИ КОРРЕЛЯЦИИ ПЛОТНОСТИ ЭЛЕКТРОНОВ

А.А. Ступка

Рассмотрена гидродинамика электронной плазмы при наличии корреляции плотности заряда. Получены уравнения идеальной гидродинамики с таким условием для случая неподвижных ионов. Показано, что в уравнение Эйлера входит новая переменная — второй момент плотности заряда, для которого получено временное уравнение на основе уравнения непрерывности в приближении большого радиуса равновесных корреляций. Изучены ленгмюровские колебания в этой системе, найдено значение добавки к частоте колебаний, возникающее благодаря учёту корреляции плотности заряда.

ГІДРОДИНАМІЧНА ТЕОРІЯ ПЛАЗМОВИХ КОЛИВАНЬ ЗА НАЯВНОСТІ КОРЕЛЯЦІЇ ГУСТИНИ ЕЛЕКТРОНІВ

А.А. Ступка

Розглянуто гідродинаміку електронної плазми за наявності кореляції густини заряду. Отримані рівняння ідеальної гідродинаміки з такою умовою для випадку нерухомих іонів. Показано, що в рівняння Ейлера входить нова змінна — другий момент густини заряду, для якого отримано часове рівняння на основі рівняння неперервності в наближенні великого радіусу рівноважних кореляцій. Вивчено ленгмюрівські коливання в цій системі, знайдено значення добавки до частоти коливань, що виникає завдяки врахуванню кореляції густини заряду.