VECTOR TO PSEUDOSCALAR MESON RADIATIVE TRANSITIONS IN CHIRAL THEORY WITH RESONANCES

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The form factor of the vector meson V conversion transition into the pseudoscalar meson $\mathcal{P}$ and the lepton pair l^+l^- is presented for $V \equiv \rho^0$, ω, ϕ and $\mathcal{P} \equiv \pi^0$, η. Our approach is based on the chiral effective field theory with resonances. The normalized form factor for $\omega \to \pi\gamma^*$ transition was fitted to the recent NA 60 data by varying the only free parameter σ_V. The results are compared to the available data and to the predictions of other models.

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1. INTRODUCTION

In current paper we study the conversion transition of the vector meson V into the pseudoscalar meson $\mathcal{P}$ and the lepton pair l^+l^-. The lepton pair is produced by the virtual photon γ^*:

$$V \quad \to \quad \gamma^*\mathcal{P} \to l^+l^-\mathcal{P} . \tag{1}$$

Experimentally, only the processes $\omega \to l^+l^-\pi^0$ and $\phi \to l^+l^-\eta$ have been studied out of the whole set of possible combinations of V and $\mathcal{P}$ ($V \equiv \rho^0$, ω, ϕ and $\mathcal{P} \equiv \pi^0$, η, η'). The most recent information on the former process comes from the CERN SPS experiment NA 60 [1–3]. The knowledge of the latter is less precise and given by Novosibirsk experiment SND [4]. The measured quantity is the transition form factor $\mathcal{F}_{V \to \gamma^*\mathcal{P}}(Q^2)$ as a function of the lepton-pair invariant mass $\sqrt{Q^2} \equiv M_{l+l-} \equiv M_{\gamma^*}$.

It is known from Refs. [1–3] that there is no good theoretical description of the recent data for $\omega \to l^+l^-\pi^0$. The most advanced model was presented recently in Ref. [5], but still it is not able to accommodate well the full range of M_{γ^*}. We would like to stress that a poor knowledge of $V \to \gamma^*\mathcal{P}$ transition can be one of the obstacles in the study of light scalar mesons: it was shown in [6] that the differential cross section of the e^+e^- annihilation to $\pi^0\pi^0\gamma$ and $\pi^0\eta\gamma$ for $\sqrt{s} = M_\phi$ are strongly affected by the contributions of the type $V \to \gamma^*\mathcal{P}$. The other important observation is that the $V \to \gamma^*\mathcal{P}$ transition and scalar meson decay contributions "contaminate" [7] the pion form factor precision measurement with the radiative return method [8]. The above issues make the study of the process (1) interesting.

The first aim of this paper is to present an effective field theory description of the processes (1). We apply the Lagrangian of the chiral perturbation theory with resonances [9–11]. This effective field theory is a universal tool for low-energy particle phenomenology and allows for a consistent description of the interactions of light pseudoscalar mesons (π, K, η, ...) with vector (ρ, ω, ϕ, ...), axial-vector (a_1, ...) and scalar (a_0, f_0, ...) resonances. The Lagrangian is organized as series in the masses of light quarks and derivatives acting on the pseudoscalar fields, which are considered as the Nambu-Goldstone fields of the spontaneous chiral symmetry breaking. The expansion coefficients are called the low energy constants (LECs). The state of the art is that these LECs are obtained from the fits to low-energy experimental data. It is known that this effective field theory is the correct limit of the Quantum Chromodynamics (QCD) at low energy. It naturally has the momentum-dependent vertices and exhibits the decoupling in the chiral limit. The energy scale of the applicability of the chiral effective theory with resonances is about 1 GeV.

In chiral theory with resonances, the strength of vector-vector-pseudoscalar meson transition ($VV\mathcal{P}$) is governed by the effective coupling σ_V which value cannot be theoretically calculated. This coupling appears in model description of various processes (see, for example, [6,12]) and, therefore, it is important to estimate its value from experiment. However, a direct measurement of this coupling is impossible.

A term proportional to σ_V is present in a model description of the vector-pseudoscalar radiative transition form factor for virtual photons [6]. The formulae are given in Section 2. The form factors of interest were obtained in the paper [6] only as a by-product and were not compared to data. In Section 3 we demonstrate that the value of σ_V coupling can be estimated from fitting the $\omega \to \pi\gamma^*$ form factor, recently measured in the NA 60 experiment [1,2]. The conclusions are drawn in Section 4.

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2. FORMALISM

We use the chiral Lagrangian in the vector formulation for spin-1 fields, following the papers [9–11]. At the moment we neglect the G-parity breaking and OZI breaking effects and use the $SU(3)$ flavor symmetry relations for the couplings. We also omit the η' meson terms for a while. The Lagrangian terms relevant for the calculation of $\mathcal{F}_{V \to \gamma^* \mathcal{P}}(Q^2)$ read

$$\mathcal{L}_{\gamma V} = -ef_V \partial^\mu B^\nu \left(\tilde{\rho}^0_{\mu\nu} + \frac{1}{3}\tilde{\omega}_{\mu\nu} - \frac{\sqrt{2}}{3}\tilde{\phi}_{\mu\nu} \right) , \quad (2)$$

where $\tilde{V}_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu$;

$$\begin{aligned}
\mathcal{L}_{V\gamma P} &= -\frac{4\sqrt{2}eh_V}{3f_\pi}\epsilon_{\mu\nu\alpha\beta}\partial^\alpha B^\beta \left[\left[\rho^{0\mu} + 3\omega^\mu \right] \partial^\nu \pi^0 \right. \\
&\quad \left. + \left[(3\rho^{0\mu} + \omega^\mu)C_q + 2\phi^\mu C_s \right] \partial^\nu \eta \right] ; \quad (3)
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{VVP} &= -\frac{4\sigma_V}{f_\pi}\epsilon_{\mu\nu\alpha\beta} \left[\pi^0 \partial^\mu \omega^\nu \partial^\alpha \rho^{0\beta} \right. \\
&\quad + \eta \left[(\partial^\mu \rho^{0\nu}\partial^\alpha\rho^{0\beta} + \partial^\mu\omega^\nu\partial^\alpha\omega^\beta)\frac{1}{2}C_q \right. \\
&\quad \left. \left. - \partial^\mu\phi^\nu\partial^\alpha\phi^\beta\frac{1}{\sqrt{2}}C_s \right] \right] . \quad (4)
\end{aligned}$$

The $\epsilon_{\mu\nu\alpha\beta}$ is the totally antisymmetric Levi-Civita tensor. The pion decay constant is $f_\pi = 92.4$ MeV. The coupling constants f_V, h_V and θ_V are model parameters: $f_V = 0.20173(86)$ from $\Gamma(\rho^0 \to e^+e^-) = \frac{e^4 M_\rho f_V^2}{12\pi}$; $h_V = 0.041(3)$ from $\Gamma(\rho \to \pi\gamma) = \frac{4\alpha M_\rho^3 h_V^2}{27f_\pi^2}\left(1 - \frac{m_\pi^2}{M_\rho^2}\right)^3$ according to the PDG values for the widths [13]. The coefficients $C_q \approx 0.720$, $C_s \approx 0.471$ account for the η mixing.

According to the above Lagrangian terms, at the tree level, the form factors $\mathcal{F}_{V \to \gamma^* \mathcal{P}}(Q^2)$ read [6]

$$\begin{aligned}
\mathcal{F}_{\rho\pi\gamma^*}(Q^2) &= \frac{4}{3f_\pi}\left[\sqrt{2}h_V - \sigma_V f_V Q^2 D_\omega(Q^2) \right] \\
&= \frac{1}{C_q}\mathcal{F}_{\omega\eta\gamma^*}(Q^2) , \quad (5)
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_{\omega\pi\gamma^*}(Q^2) &= \frac{4}{f_\pi}\left[\sqrt{2}h_V - \sigma_V f_V Q^2 D_\rho(Q^2) \right] \\
&= \frac{1}{C_q}\mathcal{F}_{\rho\eta\gamma^*}(Q^2) , \quad (6)
\end{aligned}$$

$$\mathcal{F}_{\phi\eta\gamma^*}(Q^2) = \frac{8C_s}{3f_\pi}\left[\sqrt{2}h_V - \sigma_V f_V Q^2 D_\phi(Q^2) \right], (7)$$

where the vector meson propagator is

$$D_V(Q^2) = [Q^2 - M_V^2 + i\sqrt{Q^2}\Gamma_{tot,V}(Q^2)]^{-1} \quad (8)$$

with $M_V = M_\rho$, M_ω, M_ϕ.

One can expect the Q^2-dependent total width of the vector meson $\Gamma_{tot,V}(Q^2)$ to be important only for the ρ meson within the scope of current research, because we are interested in the time-like region of momenta $0 < \sqrt{Q^2} < 0.7$ GeV, which partly overlaps with the broad ρ resonance. A possible choice for $\Gamma_{tot,\rho}(Q^2)$ is (see, for example, [6])

$$\begin{aligned}
\Gamma_{tot,\rho}(Q^2) &= \frac{G_V^2 M_\rho^2}{48\pi f_\pi^4 Q^2}\left[(Q^2 - 4m_\pi^2)^{3/2}\theta(Q^2 - 4m_\pi^2) \right. \\
&\quad \left. + \frac{1}{2}(Q^2 - 4m_K^2)^{3/2}\theta(Q^2 - 4m_K^2) \right], \quad (9)
\end{aligned}$$

where the coupling constant G_V can be determined from the decay width $\Gamma(\rho^0 \to \pi^+\pi^-)$ via

$$\Gamma(\rho^0 \to \pi\pi) = \frac{G_V^2}{48\pi f_\pi^4}(M_\rho^2 - 4m_\pi^2)^{3/2}. \quad (10)$$

Using the PDG [13] value for the width, one obtains $G_V = 65.18(15)$ MeV.

For the ω and ϕ mesons below we use the constant width approximation with the width values according to PDG [13].

3. RESULTS

From Section 2 we see that the only ambiguous parameter in the model is σ_V. Experimentally, only the normalized form factors are known

$$F_{V \to \gamma^* \mathcal{P}}(Q^2) \equiv \frac{\mathcal{F}_{V \to \gamma^* \mathcal{P}}(Q^2)}{\mathcal{F}_{V \to \gamma^* \mathcal{P}}(0)} . \quad (11)$$

We perform the least-square fit of the recent data from CERN SPS NA 60 experiment ($\omega \to \pi\mu^+\mu^-$) [1, 2] by varying the value of σ_V. The best χ^2 value is obtained for $\sigma_V \approx 0.58$. The $\omega \to \gamma^*\pi^0$ form factor (normalized) is shown in Fig. 1. We observe an unavoidable big discrepancy in the region of $M_{\gamma^*} > 0.6$ GeV, however in the rest of the range the theory is fairly consistent with the data.

In Fig. 1 we can notice that our model agrees with the data slightly better than the model of Ref. [5], but for both models the problematic region starts at $M_{\gamma^*} \approx 0.6$ GeV. In order to investigate the impact of the ρ meson line-shape on the form factor at $M_{\gamma^*} > 0.6$ GeV we perform a complimentary calculation neglecting the resonance width and make a new fit. It is found that the effects of zero-width approximation are almost negligible below 0.6 GeV and are much smaller than the data/model discrepancy above 0.6 GeV. A similar conclusion was found also in Ref. [5]. We conclude that the line-shape of the ρ meson alone can not be responsible for the discrepancy between the model and the data.

It is also interesting to demonstrate that a simple Vector Meson Dominance ansatz (VMD)

$$F_{V \to \gamma^* \mathcal{P}}(Q^2) = \frac{M_V^2}{M_V^2 - Q^2} \quad (12)$$

utterly fails to describe the data already as low as at $M_{\gamma^*} \approx 0.45$ GeV (see Fig. 1). This fact by itself makes the problem of $F_{V \to \gamma^* \mathcal{P}}$ modeling very important.

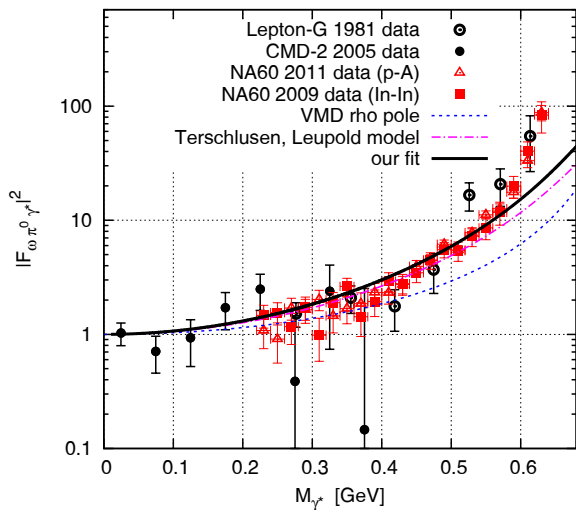**Fig. 1.** *The* $\omega \to \gamma^* \pi^0$ *form factor (normalized to unity). Lepton-G data is given in Ref. [14]; CMD-2 data — [15]; NA 60 data — (p-A) [2], (In-In) [1]. The model of Terschlüsen and Leupold is given in Ref. [5]. VMD rho pole formula — eq. (12) with* $M_V = M_\rho$

Equations (5)–(7) are inter-related via the $SU(3)$ flavor symmetry for the coupling constants and predict the transition form factors for the poorly measured process $\phi \to \eta\gamma^*$ and for the not measured processes $\rho \to \pi\gamma^*$, $\rho \to \eta\gamma^*$ and $\omega \to \eta\gamma^*$.

The model prediction for the $\phi \to \gamma^*\eta$ form factor (normalized) is shown in Fig. 2. Here we observe a full consistency with available data from Novosibirsk [4]. We would like to remark that new precise data from KLOE experiment will appear soon [16] and serve as an important test of the models. Obviously, the new data are required in order to cross-check the validity of VMD for the case of $\phi \to \gamma^*\eta$ transition.
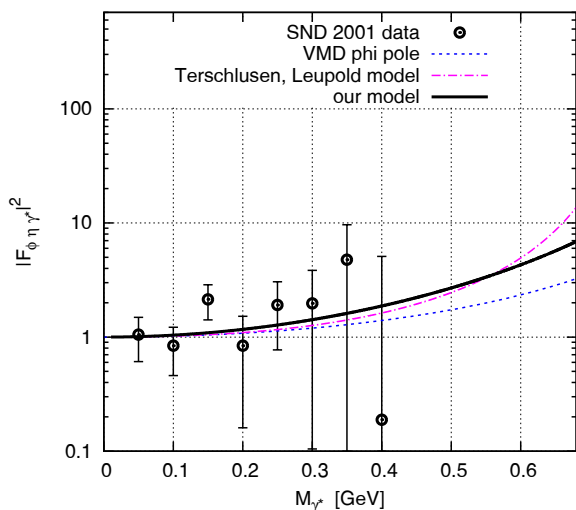

Fig. 2. *The* $\phi \to \gamma^*\eta$ *form factor (normalized to unity). SND data is given in Ref. [4]. The model of Terschlüsen and Leupold is given in Ref. [5]. VMD phi pole formula — eq. (12) with* $M_V = M_\phi$

4. CONCLUSIONS

Within the chiral perturbation theory with resonances, following the Lagranigian formalism given in Refs. [9–11], the description of the conversion transition of the vector meson V into the pseudoscalar meson $\mathcal{P}$ and the lepton pair l^+l^- is presented ($V \equiv \rho^0$, ω, ϕ and $\mathcal{P} \equiv \pi^0$, η, η'). The normalized form factor for $\omega \to \pi\gamma^*$ transition was fitted to the recent data of CERN SPS experiment NA 60 [1,2] and compared to the Lepton-G data [14] and CMD-2 data [15]. It was possible to accommodate the data, except for the very high M_{γ^*}, by adjusting only one model parameter — σ_V. The quality of the data description by our model is slightly better than that by the model of Ref. [5] and much better than that by the Vector Meson Dominance ansatz. A possible way to reduce a big data/model discrepancy in the region of $M_{\gamma^*} > 0.6$ GeV is to go beyond a tree level approximation. We leave this option for future investigations.

After the value of σ_V is fixed, the model gives the predictions for various $V \to \gamma^*\mathcal{P}$ transitions, among which there are the poorly measured process $\phi \to \eta\gamma^*$ [4] and the not measured processes $\rho \to \pi\gamma^*$, $\rho \to \eta\gamma^*$, $\omega \to \eta\gamma^*$. The new precise results on $\phi \to \eta\gamma^*$ from Frascati [16] could help to benchmark the models.

Acknowledgements

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РАДИАЦИОННЫЕ ПЕРЕХОДЫ ВЕКТОРНОГО МЕЗОНА В ПСЕВДОСКАЛЯРНЫЙ МЕЗОН В КИРАЛЬНОЙ ТЕОРИИ С РЕЗОНАНСАМИ

С.А. Ивашин

Формфакторы для конверсионного перехода векторного мезона V в псевдоскалярный мезон $\mathcal{P}$ и лептонную пару $l^+ l^-$ представлены для случаев $V \equiv \rho^0$, ω, ϕ и $\mathcal{P} \equiv \pi^0$, η. Подход основан на эффективной киральной теории поля с резонансами. Нормированный формфактор для перехода $\omega \to \pi\gamma^*$ подогнан к данным эксперимента NA 60 при помощи варьирования единственного свободного параметра σ_V. Результаты подхода сравниваются с имеющимися данными и предсказаниями других моделей.

РАДІАЦІЙНІ ПЕРЕХОДИ ВЕКТОРНОГО МЕЗОНА В ПСЕВДОСКАЛЯРНИЙ МЕЗОН В КІРАЛЬНІЙ ТЕОРІЇ З РЕЗОНАНСАМИ

С.А. Івашин

Формфактори для конверсійного переходу векторного мезона V в псевдоскалярний мезон $\mathcal{P}$ та лептонну пару $l^+ l^-$ представлено для випадків $V \equiv \rho^0$, ω, ϕ і $\mathcal{P} \equiv \pi^0$, η. Підхід засновано на ефективній кіральній теорії поля з резонансами. Нормований формфактор для переходу $\omega \to \pi\gamma^*$ підігнано до даних експерименту NA 60 за допомогою варіації єдиного вільного параметру σ_V. Результати підходу порівняно з наявними даними та передбаченнями інших моделей.