

# NON-LINEAR EFFECTS AT IONIZATION OF HYDROGEN ATOMS IN THE STRONG PULSED LIGHT FIELD

*S.S. Starodub\* and S.P. Roshchupkin*

*Institute of Applied Physics, National Academy of Sciences of Ukraine, 40030, Sumy, Ukraine*

(Received October 28, 2011)

It is a model theoretical work of the applied character in which: outside the framework of the dipole approximation (with an accuracy of about  $v/c$ ) the effective interaction force between two atoms of hydrogen at their ionization in a pulsed field of two opposite laser wave is theoretically studied. It is shown that the effective interaction force between hydrogen ions (atoms after ionization), can become an attractive force on certain time intervals in the presence of the pulsed laser field. As a result the pulsed laser field can slow down backward motion of ions in 6 times.

PACS: 41.20.Jb, 41.75.Fr

## 1. INTRODUCTION

There are many works devoted to study of the interaction of similarly charged particles in the presence of electromagnetic field (see for example [1-9]). The classical electron interaction in plane electromagnetic wave was studied in [2, 3]. The possibility of the electron attraction in the presence of plane electromagnetic wave was firstly showed by Oleinik [1]. However, the theoretical proof of mentioned process was given by Kazantsev and Sokolov during investigation of classical relativistic electron interaction in the field of plane wave [2]. We also note paper [3]. It is very important to point out, that the classical nonrelativistic electron attraction in the field of plane monochromatic electromagnetic wave is impossible. The possibility of the nonrelativistic electrons (ions) attraction in the pulsed laser field was found out by authors in [4-7]. Moreover, the electrons (ions) interaction in the field of one pulsed laser wave was studied in [4, 5]. The nonrelativistic electrons interaction in the field of two pulsed laser waves propagating in opposite directions and normally to the initial direction of electrons motion was considered in [6]. And the attraction possibility of hydrogen ions moving as paraxial beam in a pulsed field of two opposite laser waves in parallel direction to beam was studied in [7]. Also the heavy nuclei interaction moving towards to each other in a pulsed field of two opposite laser waves which perpendicularly to the initial direction of nuclei motion was studied in [9]. In contrast to the papers, mentioned above, essential slow down backward motion of hydrogen ions (in initial they was atoms) moving towards to each other in a pulsed field of two opposite laser waves which perpendicularly to the initial direction of atoms motion is investigated in this article. The presence of the

second laser wave has allowed to considerably obtain the Coulomb repulsion compensation of particles.

## 2. THE EFFECTIVE INTERACTION FORCE

Let's investigate interaction of the two classical non-relativistic hydrogen atoms ( $q_1, q_2$ ) moving towards to each other along the axis  $x$  in the pulsed field of two opposite laser waves, extending along the axis  $z$ .

It is known [8], in the dipole approximation in the center-of-mass system the external field does not influence on the relative motion of particles, therefore we will study interaction of the hydrogen atoms out of the dipole approximation taking into account the relativistic correction  $v/c \ll 1$  ( $v$  is the relative transverse velocity of atoms,  $c$  is the velocity of light in free space). We assume that the electric and magnetic strengths of the pulsed field of two contradirectional laser waves can be written as:

$$\mathbf{E}_1(t, z) = E_{01} \cdot \exp(-t^2/t_1^2) \cos(\omega_1 \xi_-) \cdot \mathbf{e}_x, \quad (1)$$

$$\mathbf{E}_2(t, z) = E_{02} \cdot \exp(-t^2/t_2^2) \cos(\omega_2 \xi_+) \cdot \mathbf{e}_x, \quad (2)$$

$$\xi_{\pm} = t \pm \frac{z}{c}, \quad (3)$$

$$\mathbf{H}_1(t) = H_{01} \cdot \exp(-t^2/t_1^2) \cos(\omega_1 t) \cdot \mathbf{e}_y, \quad (4)$$

$$\mathbf{H}_2(t) = -H_{02} \cdot \exp(-t^2/t_2^2) \cos(\omega_2 t) \cdot \mathbf{e}_y. \quad (5)$$

Here  $E_{0j}$  and  $H_{0j}$  ( $j = 1, 2$ ) are the electric and magnetic field strengths at the laser pulse peak, respectively;  $\mathbf{e}_x$  and  $\mathbf{e}_y$  are the unit vectors directed along the  $x$  and  $y$  axes, respectively;  $t_{1,2}$  and  $\omega_{1,2}$  are the laser pulse durations and the frequencies of the first and second waves, respectively. Note, that the magnetic field was chosen in dipole approximation, as the correction  $v/c \ll 1$  is in the expression of Lorenz force (see, (6), (7)).

\*Corresponding author E-mail address: starodubss@mail.ru

Newton equations for hydrogen atoms motion in the presence of pulsed laser field of two contradirectional waves (1)–(5) are the following:

$$m\ddot{\mathbf{r}}_1 = q \left[ \mathbf{E}(t, z_1) + \frac{1}{c} \dot{\mathbf{r}}_1 \times \mathbf{H}(t) \right] - \frac{q^2}{|\mathbf{r}_2 - \mathbf{r}_1|^3} (\mathbf{r}_2 - \mathbf{r}_1), \quad (6)$$

$$m\ddot{\mathbf{r}}_2 = q \left[ \mathbf{E}(t, z_2) + \frac{1}{c} \dot{\mathbf{r}}_2 \times \mathbf{H}(t) \right] + \frac{q^2}{|\mathbf{r}_2 - \mathbf{r}_1|^3} (\mathbf{r}_2 - \mathbf{r}_1). \quad (7)$$

Here

$$\begin{aligned} \mathbf{E}(t, z) &= \mathbf{E}_1(t, z_j) + \mathbf{E}_2(t, z_j), \\ \mathbf{H}(t) &= \mathbf{H}_1(t) + \mathbf{H}_2(t). \end{aligned} \quad (8)$$

In (6), (7)  $q = q_j$ ,  $m = m_j$  ( $j = 1, 2$ ) are the charge and the mass of hydrogen atoms (the charge of atoms  $q = 0$ , but after ionization the charge of ions  $q = +e$ );  $\mathbf{r}_1$  and  $\mathbf{r}_2$  are the radius-vectors of the first and the second atoms, respectively. Let us turn to the center-of-mass system:

$$\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1, \quad \mathbf{R} = \frac{1}{2} (\mathbf{r}_2 + \mathbf{r}_1). \quad (9)$$

In this case, the equations for relative motion of particles do not depend on a motion of center-of-mass and are given by following expressions:

$$\frac{d^2 \xi_x}{d\tau^2} = \eta_1 f_1 (C_1 - \dot{\xi}_z a_1) - \eta_2 f_2 (C_2 + \dot{\xi}_z a_2) + \beta \frac{\xi_x}{\xi^3}, \quad (10)$$

$$\frac{d^2 \xi_y}{d\tau^2} = \beta \frac{\xi_y}{\xi^3}, \quad (11)$$

$$\frac{d^2 \xi_z}{d\tau^2} = \dot{\xi}_x (\eta_1 f_1 a_1 - \eta_2 f_2 a_2) + \beta \frac{\xi_z}{\xi^3}. \quad (12)$$

Here  $\boldsymbol{\xi} = (\xi_x, \xi_y, \xi_z) = \mathbf{r}/\sqrt{\lambda_1 \lambda_2}$  ( $\lambda_j = c/\omega_j$ ,  $j = 1, 2$ ) is the radius-vector of the relative distance between atoms in units of wave-length;  $\tau = \sqrt{\omega_1 \omega_2} t$ ; parameter  $\beta$ , the pulse envelopes  $f_j$  and the velocities of oscillation of hydrogen atoms  $\eta_j$  (in units of velocity of light) can be written as:

$$\beta = \frac{q^2}{\mu c^2 \sqrt{\lambda_1 \lambda_2}} \ll 1, \quad (13)$$

$$f_1 = \exp\left(-\frac{\tau^2}{\tau_1^2}\right), \quad f_2 = \exp\left(-\frac{\tau^2}{\tau_2^2}\right), \quad (14)$$

$$\eta_j = \frac{v_j}{c} = \frac{q E_{0j} \sqrt{\lambda_1 \lambda_2}}{\mu c^2} \ll 1, \quad \tau_j = \sqrt{\omega_1 \omega_2} t_j, \quad (15)$$

$$\beta \ll \eta_j, \quad j = 1, 2. \quad (16)$$

Here  $\mu = m/2$  is the reduced mass of atoms.

Coefficients  $C_j$ ,  $a_j$  are given by following expressions:

$$\begin{cases} C_1 = \sin\left(\frac{\xi_z}{2\sqrt{\phi}}\right) \sin\left(\frac{\tau}{\sqrt{\phi}}\right), \\ C_2 = \sin\left(\frac{\xi_z \sqrt{\phi}}{2}\right) \sin\left(\frac{\tau}{\sqrt{\phi}}\right); \end{cases} \quad (17)$$

$$\begin{cases} a_1 = \cos(\tau/\sqrt{\phi}), \\ a_2 = \cos(\tau\sqrt{\phi}); \end{cases} \quad (18)$$

$$\phi = \omega_2/\omega_1. \quad (19)$$

Note, that in the dipole approximation we must assume  $\xi_z = \dot{\xi}_z = \dot{\xi}_x = 0$  then the influence of an external field on the relative motion of hydrogen atoms, as one would expect, vanishes.

The effective force of atoms interaction is determined by the right parts of (10)–(12), and its projection to a direction of the relative particles motion is:

$$F_\xi = \mathbf{F} \cdot \mathbf{e}_\xi = \frac{1}{\xi} (\eta_1 f_1 a_- - \eta_2 f_2 a_+) + \beta \frac{1}{\xi^2}, \quad (20)$$

where

$$a_- = \xi_x (C_1 - \dot{\xi}_z a_1) + \xi_z a_1 \dot{\xi}_x, \quad (21)$$

$$a_+ = \xi_x (C_2 + \dot{\xi}_z a_2) + \xi_z a_2 \dot{\xi}_x. \quad (22)$$

We assume that the characteristic oscillation time ( $\sim \omega_j^{-1}$ ) is significantly less than the laser pulse duration, so that the following condition is satisfied:

$$\tau_j \gg 1, \quad j = 1, 2. \quad (23)$$

Thereby, we would be averaged the projection of the effective force (20) and the relative distance between atoms with respect to the period of fast oscillations:

$$\bar{F}_\xi = \frac{1}{2\pi} \int_0^{2\pi} F_\xi \cdot d\tau, \quad \bar{\xi} = \frac{1}{2\pi} \int_0^{2\pi} \xi \cdot d\tau. \quad (24)$$

Let's name the quantity  $\bar{F}_\xi$  as the average effective force of the hydrogen atoms (ions) interaction. One can see that condition  $\bar{F}_\xi > 0$  determines the particles repulsion and  $\bar{F}_\xi < 0$  attraction of the particles. We will set the initial relative coordinates and velocities of atoms as:

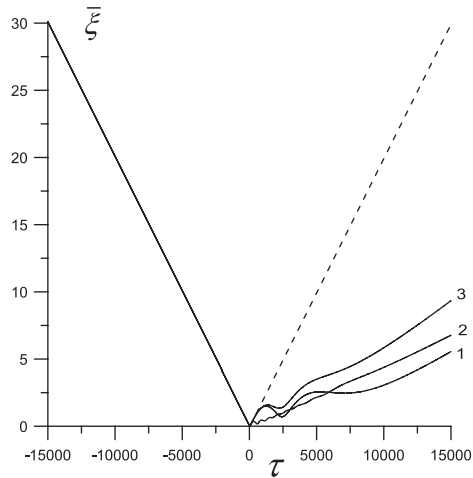
$$\begin{aligned} \xi_{x0} &= 200, & \xi_{y0} &= 0, & \xi_{z0} &= 0; \\ \dot{\xi}_{x0} &= -10^{-3}, & \dot{\xi}_{y0} &= 0, & \dot{\xi}_{z0} &= 0. \end{aligned} \quad (25)$$

The system of (10)–(12), (24) with initial conditions (25) was solved numerically. Thus, frequencies of waves were set by the equal:  $\omega_1 = \omega_2 = \omega = 3 \cdot 10^{19} \text{ s}^{-1}$  ( $\hbar\omega = 19.7 \text{ keV}$ ), the laser pulse durations correspond to femtoseconds lasers:  $t_1 = t_2 = 1.5 \cdot 10^{-16} \text{ s}$  ( $\tau_1 = \tau_2 = 5000$ ).

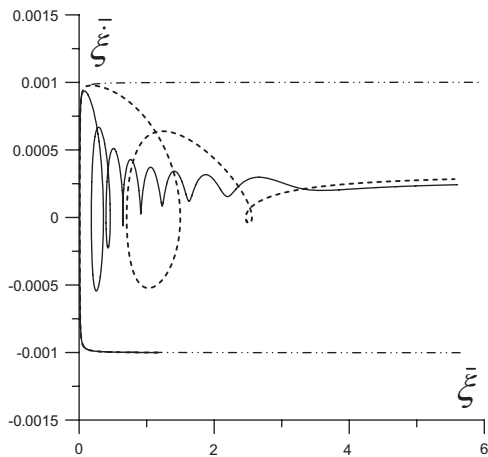
### 3. THE EFFECTS AT IONIZATION OF HYDROGEN ATOMS

The oscillation velocity of particles in the first or second laser wave  $\eta_j$ ,  $j = 1, 2$  (15) is the main parameter. Strength of an external field and possible character of particles interaction depends on its quantity. The oscillation velocity varied in the range from  $10^{-3}$  to  $10^{-2}$  magnitudes of velocity of light in free space. It corresponds to fields strength from  $10^{11}$  to  $10^{12} \text{ V/cm}$ . Calculations have shown that in all range

of the oscillation velocities we can observe the effect of an attraction of atoms after they ionization. However, it is valid provided that quantities  $\eta_1$  and  $\eta_2$  will differ from each other on quantity of the initial relative atoms velocity. We can see that at increase one of them the effect of an attraction weakens (Fig. 1).



**Fig. 1.** The average relative distance  $\bar{\xi}$  in units  $\lambda$  vs. time  $\tau$ . The dashed line - 0 in figure corresponds to interaction without external field influence ( $\eta_1 = \eta_2 = 0$ ). Full lines 1, 2, 3 correspond to oscillation velocities, respectively  $\eta_1 = 10^{-3}$ ,  $\eta_2 = 2 \cdot 10^{-3}$ ;  $\eta_1 = 3 \cdot 10^{-3}$ ,  $\eta_2 = 7 \cdot 10^{-3}$ ;  $\eta_1 = 8 \cdot 10^{-3}$ ,  $\eta_2 = 9 \cdot 10^{-3}$

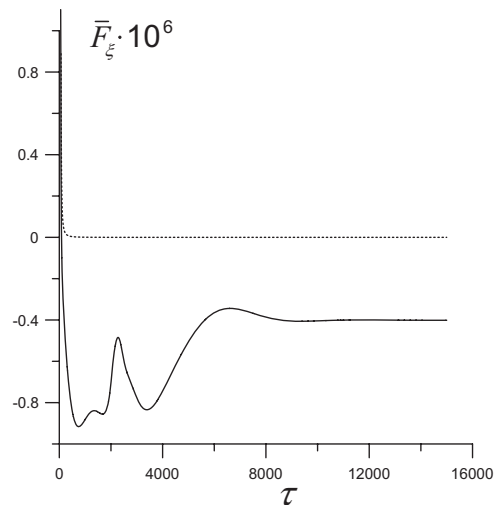


**Fig. 2.** The average relative velocity  $\bar{\xi}$  vs. average relative distance  $\bar{\xi}$ . The full line with dots on figure corresponds to interaction without external field influence ( $\eta_1 = \eta_2 = 0$ ). Full line correspond to oscillation velocities  $\eta_1 = 3 \cdot 10^{-3}$ ,  $\eta_2 = 7 \cdot 10^{-3}$ . The dashed line -  $\eta_1 = 10^{-3}$ ,  $\eta_2 = 2 \cdot 10^{-3}$

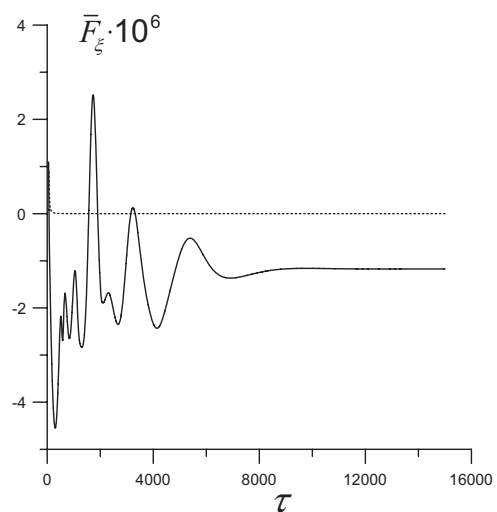
One can see that till the moment of the closest approach of particles, influence of an external field on a its motion very little (see the left part of Fig. 1). It is caused by a select of initial conditions (25) at which the external field does not resist to closest approach of atoms on distances, comparable with ‘‘Coulomb’’, i.e. such on which particles can approach without an external field. After approach to the several atoms distance the external field is on and we have ionization of atoms. After actually a stopping of ions, the

force of Coulomb repulsion decreases much rather, than the force caused by an external laser field. There is a magnetic current generated by strength of a magnetic field of the first and second wave. It compensate the Coulomb repulsion and does not allow hydrogen ions promptly to go away from each other (see the right part of Fig. 1). It is visible that in the presence of a pulsed field the ions leave from each other much more slowly. Deceleration can reach six times.

From the figure of average relative velocity (Fig. 2) it is visible that velocity changes a sign on far distances, i.e. just there where the magnetic current of particles is shown. It leads to an attraction (repulsion) of particles. The given effect confirms also a view of averaged effective force of hydrogen ions interaction (Figs. 3 and 4).



**Fig. 3.** The average effective force of the ions interaction  $\bar{F}_\xi$  in units  $\mu\omega$  vs. time  $\tau$ . The dashed line corresponds to interaction without external field influence. Full line correspond to oscillation velocities  $\eta_1 = 10^{-3}$ ,  $\eta_2 = 2 \cdot 10^{-3}$



**Fig. 4.** The average effective force of the ions interaction  $\bar{F}_\xi$  in units  $\mu\omega$  vs. time  $\tau$ . The dashed line corresponds to interaction without external field influence. Full line correspond to oscillation velocities  $\eta_1 = 3 \cdot 10^{-3}$ ,  $\eta_2 = 7 \cdot 10^{-3}$

From Figures 3 and 4 follows that averaged effective force of ions interaction on certain intervals of time has the negative value, i.e. ions start to effectively attract to each other. Let's underline that the greatest effect of ions confinement is observed when initial relative velocity and oscillation velocities have one order of magnitude. At a deviation from this requirement the confinement effect weakens (see lines 1-3 in Fig. 1).

The average effective force of particles interaction is more often has reverse sign at essential difference of the oscillation velocities  $\eta_1$  and  $\eta_2$  (see Fig. 4). It result to decrease of a magnetic current on ions and the particles go away from each other more promptly.

#### 4. CONCLUSIONS

The research made in the present work allows to extract the following results:

- outside the framework of the dipole approximation (with an accuracy of about  $v/c$ ) the effective interaction force between hydrogen atoms (ions after ionization), moving towards to each other in a pulsed field of two opposite laser waves which perpendicularly to the initial direction of atoms motion was studied theoretically;
- possibility of the attraction force amplification was found out theoretically after atoms ionization for the oscillation velocities which differ from each other on quantity of the initial relative atoms velocity. The effect is observed at identical frequencies of waves and is stable to minor change of the laser parameters.

#### References

1. V.P. Oleinik. Resonant effects in the field of an intensive laser beam // *Zh. Eksp. Teor. Fiz.* 1967, v. 52, p. 1049-1067 (in Russian).
2. A.P. Kazantsev, V.P. Sokolov. Interaction of electrons in a light field // *Zh. Eksp. Teor. Fiz.* 1984, v. 86, p. 896-905 (in Russian).
3. S.T. Zavtrak. Radiative interaction of charges // *Letter in Zh. Eks. Teor. Fiz.* 1989, v. 15, p. 14-16 (in Russian).
4. S.S. Starodub and S.P. Roshchupkin. Interaction of the nonrelativistic electrons in the presence of a strong pulsed laser field // *Laser Phys.* 2003, v. 13, p. 1422-1425.
5. S.S. Starodub and S.P. Roshchupkin. The coulomb repulsion compensation between the ions of the beam in the presence of a strong pulsed laser field // *Laser Phys. Lett.* 2005, v. 2, p. 407-411.
6. S.S. Starodub and S.P. Roshchupkin. Interaction of the nonrelativistic electrons in the pulsed field of two laser waves // *Eur. Phys. J. D.* 2007, v. 44, p. 401-405.
7. S.S. Starodub and S.P. Roshchupkin. The hydrogen ions attraction effect in the pulsed field of two laser waves propagating in the opposite directions // *Laser Phys. Lett.* 2008, v. 5, p. 691-695.
8. S.P. Roshchupkin and A.I. Voroshilo. *Resonant and Coherent Effects of Quantum Electrodynamics in the Light Field*, Kiev: "Naukova Dumka", 2008, 340 p. (in Russian).
9. S.S. Starodub and S.P. Roshchupkin. Heavy nuclei confinement effect in a pulsed light field // *Laser Phys.* 2011, v. 21, p. 769-773.

#### НЕЛИНЕЙНЫЕ ЭФФЕКТЫ ПРИ ИОНИЗАЦИИ АТОМОВ ВОДОРОДА В СИЛЬНОМ ИМПУЛЬСНОМ СВЕТОВОМ ПОЛЕ

*С.С. Стародуб, С.П. Рошчупкин*

Вне рамок дипольного приближения (с учетом поправок  $v/c$ ) теоретически изучена эффективная сила взаимодействия между двумя атомами водорода при их ионизации в присутствии внешнего импульсного поля двух лазерных волн, распространяющихся навстречу друг другу. Исследуется возможность максимального сближения атомов водорода и удержание атомов за счет импульсного внешнего поля.

#### НЕЛІНІЙНІ ЕФЕКТИ ПРИ ІОНІЗАЦІЇ АТОМІВ ВОДНЮ В СИЛЬНОМУ ІМПУЛЬСНОМУ СВІТЛОВОМУ ПОЛІ

*С.С. Стародуб, С.П. Рошчупкін*

Поза рамками дипольного наближення (з урахуванням поправок  $v/c$ ) теоретично вивчена ефективна сила взаємодії між двома атомами водню за їх іонізації в присутності зовнішнього імпульсного поля двох лазерних хвиль, що розповсюджуються назустріч одна одній. Досліджується можливість максимального зближення атомів водню і утримання їх за рахунок зовнішнього імпульсного поля.