ELECTRON-POSITRON PAIR PHOTO-PRODUCTION WITH RADIATION OF A PHOTON IN MAGNETIC FIELD AT NONRESONANT REGIME

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(Received November 1, 2011)

Theoretical investigations of quantum electrodynamic processes in strong magnetic field are carried out. Such the processes may occur between colliding heavy ions. Magnetic fields of the nuclei are added and electric fields of nuclei mutually compensate one another in that region. The electron-positron pair production by a photon in the case when one additional photon is emitted in external magnetic field under nonresonant condition has been investigated. Kinematics of the process and the resonance conditions in approximation of strong magnetic field and weakly excited electron (positron) states (ultra-quantum approximation) have been studied. The resonant conditions have the place, when the photon energy is close to the splitting between Landau levels. The differential probability of nonresonant process has been obtained. The probability of the process is three order of magnitude less the resonant case.

PACS: 12.20.-m, 13.88.+e

1. INTRODUCTION

FAIR (Facility for antiproton and ion research project) is one of the largest research project today. It will be erected at GSI (Darmstadt) in the next few years. Quantum-electrodynamic test in strong electromagnetic fields, for example under heavy ions collisions, is one of the important applied task of this project.

The quantum-electrodynamic (QED) processes in the presence of strong magnetic field close to the critical value of about 10^{13} G may accompany fast heavy nuclei collisions. The magnetic field is generated by two colliding nuclei, that play role of two current. The magnetic field produced by colliding nuclei in the region between them at the moment of the closest approach has order of magnitude about 10^{12} G in the case, when that region has the size of Compton wavelength of electron. The electric fields of nuclei mutually compensate one another in that region.

We consider, that the series of quasi-equidistant narrow peaks in the electron-positron distribution of total energy, observed more then ten years ago in heavy ions collision at GSI, Darmstadt [1,2], is a result of movement of an electron-positron pair in such magnetic field in that region. Narrow lines are the resonant pair production on the Landau levels [3].

The first theoretical works for study of the process of electron-positron pair photoproduction in magnetic field were performed in the middle of the last century yet [4]. There are monographs that are devoted to the first order QED processes in magnetic field [5,6]. Spin polarization effects are considered in [7]. However, it should be noted, that similar quantum-electrodynamic processes can be accompanied by emitting of additional photon. Such a process in resonant condition has been considered in previous work [8].

This work is devoted to the study of such the process at nonresonant regime. In this work we use the relativism system of units: $\hbar = 1$, $c = 1$.

2. PROBABILITY AMPLITUDE AND RESONANT CONDITION

Probability amplitude of the process is described by Feynman diagrams that are shown in Fig. 1. Wave lines in the diagrams correspond to wave functions of photons. External and internal solid lines are wave functions and Green's functions correspondingly of electrons (positrons) in a homogenies magnetic field.

So amplitude of probability is

$$S_{fi} = -\frac{ie^2(2\pi)^4}{4SV}\frac{\delta^3(k-k'-p^+-p^-)}{\sqrt{\omega\omega' m^+ m^- \varepsilon^+ \varepsilon^-}}\times$$
$$\times\left[\sum_{n_g=0}^{\infty}\frac{e^{i\Phi_g}\sum_1^{16}Bg_i}{g_0^2-\varepsilon_g^2}+\sum_{n_f=0}^{\infty}\frac{e^{i\Phi_f}\sum_1^{16}Bf_i}{f_0^2-\varepsilon_f^2}\right]. \qquad (1)$$

Here,

$$\varepsilon^\pm = \sqrt{(m^\pm)^2 + (p^\pm)2}, \qquad (2)$$

$$m^\pm = m\sqrt{1+2l^\pm h}, \qquad (3)$$

$$\varepsilon_{g,f} = \sqrt{m^2 + 2n_{g,f}hm^2 + p_{g,f}^2}, \qquad (4)$$

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PROBLEMS OF ATOMIC SCIENCE AND TECHNOLOGY, 2012, N 1.
Series: Nuclear Physics Investigations (57), p. 111-114.

$$\begin{aligned} g_0^2 &= (\omega - \varepsilon^+)^2, \\ f_0^2 &= (\omega - \varepsilon^-)^2, \end{aligned} \tag{5}$$

$$h = H/H_c, \tag{6}$$

where $H_c = 4.41 \cdot 10^{13}$ G is the critical magnetic field strength, l^- and l^+ are Landau levels of electron (positron), Φ_g and Φ_f are phases of direct and exchange processes, Bg_i and Bf_i are the factors, that take into account spin-polarization properties of particles.

The process is studied in the lowest Landau levels approximation (LLL or ultraquantum approximation) and the following conditions are true [9]:

$$\begin{aligned} h &\ll 1, \\ l &\sim 1. \end{aligned} \tag{7}$$

For a charged particle moving in a uniform magnetic field, the laws of conservation of energy ε and the longitudinal external field component of the momentum p (for definiteness, the magnetic field is directed along the z), so the following relations have place for this process:

$$\begin{aligned} \omega &= \omega' + \varepsilon^- + \varepsilon^+, \\ k_z &= k'_z + p^- + p^+, \end{aligned} \tag{8}$$

where ω and k_z are frequency and the longitudinal momentum of the initial photon, the primed variables correspond to the final photon, ε^- and p^- are energy and longitudinal momentum of the electron, for positron the same variables are ε^+, p^+. Analysis of these expression gives kinematics of the process.

In order to avoid resonance it is necessary to investigate resonant conditions. Such situation has the place at the poles of Green's function of electron, when intermediate state goes to mass shell. For electron in magnetic field these conditions have the form

$$\begin{aligned} \omega' &= mh(n_g - l^-), \\ \omega' &= mh(n_f - l^+). \end{aligned} \tag{9}$$

It means that the frequency of the final photon is equal to the distance between Landau levels. So nonresonant case is located between two neighboring resonances.

Evidently, the process has a threshold defined by the next expressions:

$$\begin{aligned} \omega &= 2m, \\ l^+ &= l^- = 0, \\ \mu^+ &= -\mu^- = 1, \end{aligned} \tag{10}$$

where μ^- (μ^+) is the sign of spin projection of electron (positron).

We consider the process near the threshold. Let the frequency of the initial photon be

$$\omega = 2m + amh^2, \tag{11}$$

where $a \sim 1$. In this case, the frequency of the final photon takes on the form

$$\begin{aligned} \omega' &= \kappa m h^2, \\ 0 &< \kappa < a. \end{aligned} \tag{12}$$

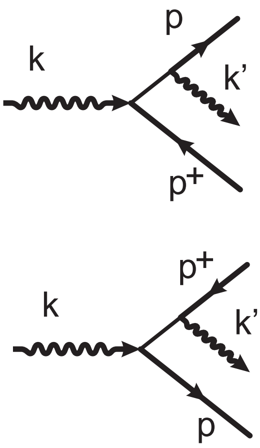

Fig. 1. *Feynman diagrams of the process of e^+e^--pair production by a photon with photon emission in a magnetic field. Solid lines represent solutions of Dirac equation for an electron in a magnetic field*

3. PROBABILITY PER UNIT OF TIME

Standard rules of quantum electrodynamics give expression for process probability in time unit in the form

$$dW = \frac{1}{T}|S_{fi}|^2 \frac{VS^2}{(2\pi)^7} d^3k d^2p d^2p^+, \tag{13}$$

where V, S, T are normalizing constants. Taking into account the condition (7) the process probability can be written as follows:

$$dW = \frac{\alpha^2 \pi h^2 e^{-2/h}}{\sqrt{a-\kappa}} \kappa |e_z|^2 |Y|^2 d\omega' d\Omega, \tag{14}$$

where

$$|Y|^2 = \frac{a-\kappa}{\kappa^2} K + \frac{\Delta\sqrt{a-\kappa}}{\kappa} L + M, \tag{15}$$

$$K = \frac{1}{2}(1 + \xi'_3)(1 - u^2), \tag{16}$$

$$\begin{aligned} L = \frac{1}{2}(1 + \xi'_3) \sin(2\theta) \left(\cos(\Delta\phi) - \cos(\Delta\phi - \Lambda)\right) + \\ + \xi'_2 \sin(\theta') \left(\cos(\Delta\phi) - \cos(\Delta\phi - \Lambda)\right) - \\ - \xi_1 \left(\sin(\Delta\phi) - \sin(\Delta\phi - \Lambda)\right), \end{aligned} \tag{17}$$

$$\begin{aligned} M = (1 + u^2) - \xi'_3(1 - u^2) + \\ + \left((1 - u^2) - \xi'_3(1 + u^2)\right) \cos(2\Delta\phi - \Lambda) + \\ + 2\xi'_2 u \sin(2\Delta\phi - \Lambda), \end{aligned} \tag{18}$$

$$\begin{aligned} u &= \cos(\theta'), \\ \Delta\phi &= \phi' - \phi, \\ \Lambda &= 2\kappa h \sin(\theta') sin(\Delta\phi). \end{aligned} \tag{19}$$

Here ξ'_1, ξ'_2, ξ'_3 are the Stokes parameters of the final photon, θ' and ϕ' are the final photon polar and azimuthal angles

The angular dependence of the quantity $|Y|^2$, that is the differential process probability in relative units is shown in Fig. 2 for linear polarized final photon when Stokes parameter $\xi'_3 = 1$.

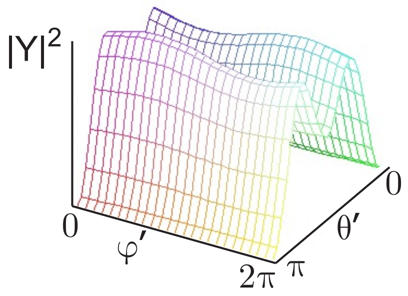

Fig. 2. *The angular dependence of the differential process probability in relative units*

After integrating over angles the differential probability has the form

$$\frac{dW}{d\omega'} = \alpha^2 \frac{2\pi^2}{3} h^2 e^{-2/h} (1+\xi_3) Z, \qquad (20)$$

where ξ_3 is the Stokes parameter of the initial photon and

$$Z = \frac{\sqrt{a-\kappa}}{\kappa}(1+\xi_3') + \frac{2\kappa}{\sqrt{a-\kappa}}(2-\xi_3'). \qquad (21)$$

Figure 3 shows the spectral distribution of the process probability.

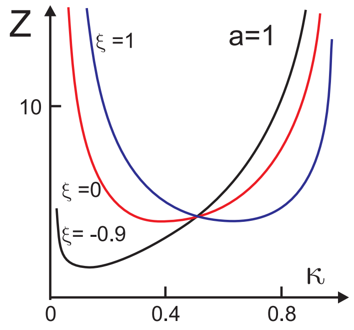

Fig. 3. *Spectral distribution of the probability for various values of Stokes parameter ξ_3'*

In order to estimate obtained probability it is necessary to consider unpolarized final photons.

$$\frac{dW}{d\kappa} = \alpha^2 \frac{4\pi^2}{3} m h^4 e^{-2/h} \left(\frac{\sqrt{a-\kappa}}{\kappa} - \frac{4\kappa}{\sqrt{a-\kappa}} \right). \qquad (22)$$

The total probability

$$W = \int_0^a \frac{dW}{d\kappa} d\kappa \qquad (23)$$

diverges logarithmically at the lower limit of the integral. The reason of the infrared divergence is radiation of ultrasoft photons [10]. After elimination of this divergence we have for the total probability of the process the next form

$$W = \alpha^2 \frac{4\pi^2}{3} m h^4 e^{-2/h} \sqrt{a} \left(\ln \frac{a}{\kappa_{min}} + \frac{16a}{3} \right). \qquad (24)$$

The numeric value of total probability in time unit is

$$W \approx 10^6 \text{ s}^{-1}, \qquad (25)$$

when $a = 1$ and $h = 0.1$, what means $H = 4.41 \cdot 10^{12}$ G and $\omega - 2mc^2 = 0.01\, mc^2$.

4. CONCLUSIONS

Nonresonant probability of photon emission in the process of photon pair production with photon emission strong depend on the polarization of the final photon and on its motion direction also; the total probability diverges logarithmically by reason of infrared divergence in the process of radiation of ultrasoft photons. Total probability for the nonresonant process of photon emission 10^6 s^{-1} is three order of magnitude less, than resonant one.

We thank V. Yu. Storizhko, S. P. Roshchupkin and O. P. Novak for useful discussions.

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РОЖДЕНИЕ ЭЛЕКТРОН-ПОЗИТРОННОЙ ПАРЫ ФОТОНОМ С ИЗЛУЧЕНИЕМ ФОТОНА В МАГНИТНОМ ПОЛЕ В НЕРЕЗОНАНСНОМ РЕЖИМЕ

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Проводятся теоретические исследования квантово-электродинамических процессов в сильном магнитном поле. Такие процессы могут происходить между сталкивающимися тяжелыми ионами. В этой области магнитные поля ядер складываются, а электрические поля взаимно компенсируются. Исследуется процесс рождения электрон-позитронной пары фотоном в случае, когда излучается один дополнительный фотон во внешнем магнитном поле при нерезонансных условиях. Изучаются кинематика процесса и условия резонанса в приближении сильного магнитного поля и слабо возбужденных состояний электронов (позитронов). Резонансные условия имеют место, когда энергия фотона близка к расстоянию между уровнями Ландау. Получена дифференциальная вероятность нерезонансного процесса в единицу времени. Вероятность такого процесса на три порядка меньше резонансного случая.

НАРОДЖЕННЯ ЕЛЕКТРОН-ПОЗИТРОННОЇ ПАРИ ФОТОНОМ З ВИПРОМІНЮВАННЯМ ФОТОНА В МАГНІТНОМУ ПОЛІ В НЕРЕЗОНАНСНОМУ РЕЖИМІ

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Проводяться теоретичні дослідження квантово-електродинамічних процесів в сильному магнітному полі. Такі процеси можуть відбуватися при зіткненні важких іонів. В області між ними магнітні поля ядер складаються, а електричні поля взаємно компенсуються. Досліджується процес народження електрон-позитронної пари фотоном у випадку, коли випромінюється один додатковий фотон в зовнішньому магнітному полі при нерезонансних умовах. Вивчаються кінематика процесу і умови резонансу в наближенні сильного магнітного поля і слабо збуджених станів електронів (позитронів). Резонансні умови мають місце, коли енергія фотона близька до відстані між рівнями Ландау. Знайдено диференційну ймовірність нерезонансного процесу в одиницю часу. Ймовірність такого процесу на три порядки менша за резонансний випадок.