ON 2D MASSLESS FIELD WAVE EQUATIONS

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The 2D wave equations of massless fields with arbitrary spin are obtained by application of the Extended Little Lorentz Group. The possibility of the existence of an interaction of charged 2D massless fields of spin one or higher with the external electromagnetic field is investigated.

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1. INTRODUCTION

The great success in the study of fundamental electronic properties of two-dimensional graphene [1] recently attracted attention to relativistic 2D massless field wave equations. In application to the graphene, usually only 2D Dirac-Weyl wave equations are considered but this restriction is not well founded and we want to clarify the question whether or not this restriction can be removed if possible. So our subject now is the investigation of the relativistic wave equations of 2D massless fields with arbitrary spin.

2. LITTLE LORENTZ GROUP, EXTENDED LITTLE LORENTZ GROUP AND CORRESPONDING WAVE EQUATIONS

In [2] we considered some properties of 2D Dirac-Weyl equations for charged massless spin 1/2 particles. Wave equations in 3D case for massless arbitrary spin particles can be obtained by application of the Little Lorentz Group [3, 4] or by application of the Extended Little Lorentz Group [5, 6]. The Little Lorentz Group [3] consists of the Lorentz transformations leaving invariant particle momentum ($i = 1, 2, 3$). The Extended Little Lorentz Group [5,6] is the Little Lorentz Group enlarged by dilatations. The Extended Little Lorentz Group includes Lorentz boosts of the plane-wave solutions of massless arbitrary spin equations. The wave equations of massless fields with arbitrary spin result from the very simple assumption: *the state of an elementary particle may be completely described by its momentum* [4]. In 3D case (4-dimensional space-time) in both cases the obtained wave equations are identical [5]. But this is not the case for relativistic 2D wave equations, when the Little Lorentz Group leads to the equations:

$$S_i \frac{\partial}{\partial x_i}\psi = 0 \qquad (1)$$

and the Extended Little Lorentz Group leads to the equations:

$$\left(S\frac{\partial}{\partial x_i} + i\varepsilon_{ikl}S_l\frac{\partial}{\partial x_k}\right)\psi = 0 \qquad (2)$$

(S_i are relativistic 2D spin operators, $S_iS_i = S(S{+}1)$, S is the value of spin, $S_iS_k - S_kS_i = i\varepsilon_{ikl}S_l$). These equations are equivalent only in the Dirac-Weyl case of $S = 1/2$. In this case the multiplication of (2) by S_i at once gives (1). The action on (2) by $\partial/\partial x_i$ immediately gives the Klein-Gordon equation

$$\frac{\partial^2}{\partial x_i^2}\psi = 0,$$

i.e. equation (2) describes the massless particle whereas equation (1) has only not massless solutions.

3. CAN 2D MASSLESS PARTICLES BE CHARGED?

It is well known that 3D massless particles of spin one or higher cannot be charged owing to inconsistency of corresponding wave equations [7]. It is of interest to consider this problem of equations inconsistency in 2D case. Let us consider charged massless particle which is described by the equation:

$$(SD_i + i\varepsilon_{ikl}S_lD_k)\psi = 0, \qquad (3)$$

where

$$D_i = \frac{\partial}{\partial x_i} - ieA_i,$$

and A_i is the relativistic 3-vector potential of the electromagnetic field. Note that

$$D_iD_k - D_kD_i = ie\varepsilon_{ikl}F_l, \qquad (4)$$

where relativistic 3-vector of electromagnetic field is

$$F_i = \frac{1}{2}\varepsilon_{imn}\left(\frac{\partial A_m}{\partial x_n} - \frac{\partial A_n}{\partial x_m}\right). \qquad (5)$$

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Instead of Klein-Gordon equation we have now the following equation:

$$(SD_i^2 - eS_iF_i)\psi = 0. \quad (6)$$

The multiplication of (3) by S_i gives

$$S_iD_i\psi = 0. \quad (7)$$

To verify the compatibility of the equations (7) and (3) we have to verify that

$$[S_jD_j, (SD_i + i\varepsilon_{ikl}S_lD_k)]\psi = 0. \quad (8)$$

After calculation we obtain the relation:

$$[S_iD_j^2 - SS_ie(S_jF_j) + e(S_jF_j)S_i(1+S) - S(S+1)eF_i]\psi = 0. \quad (9)$$

Excluding D_j^2 from (9) with the help of (6) we obtain

$$[(\frac{1}{S} - S)S_i(S_jF_j)S_i(1+S) - S(S+1)F_i]\psi = 0. \quad (10)$$

The latter equation is physically meaningless because according to (10) the arbitrarily small electromagnetic field puts the perceptible influence on the particle under consideration. Only in the case of spin 1/2 the equation (10) turns to identity. So we can conclude that 2D massless particles with spin $S > 1/2$ cannot be charged.

4. CONCLUSIONS

We investigated the wave equations of 2D massless fields with arbitrary spin obtained by application of the Extended Little Lorentz Group. We showed that 2D massless particles of spin one or higher cannot be charged due to inconsistency of corresponding massless particles wave equations.

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О ВОЛНОВЫХ 2D-УРАВНЕНИЯХ БЕЗМАССОВЫХ ПОЛЕЙ

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С помощью расширенной малой группы Лоренца получены 2D волновые уравнения для безмассовых полей произвольного спина. Выяснено, возможно ли существование взаимодействия безмассовых частиц со спинами, равными или большими единицы, с электромагнитным полем.

ПРО ХВИЛЬОВІ 2D-РІВНЯННЯ БЕЗМАСОВИХ ПОЛІВ

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За допомогою розширеної малої групи Лоренця одержані 2D хвильові рівняння для безмасових полів довільного спіну. З'ясовано, чи можливе існування взаємодії безмасових частинок зі спінами, рівними або більшими одиниці, з електромагнітним полем.