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HEMISPHERICAL AND ASPHERIC WGM DIELECTRIC RESONATORS WITH CONDUCTING ENDPLATES: RADIATION AND CONDUCTIVITY LOSSES DEPENDING ON SHAPE OF THE RESONATORS SURFACE

The measurement technique based on whispering gallery mode (WGM) dielectric resonators for surface microwave impedance characterization of high-T_c superconducting films is very accurate. However its sensitivity is limited by the radiation loss of these open electrodynamic systems and depends on effectiveness of interaction of microwave fields and the film (coefficient A_s). The aim of the work is to study the radiation and conductivity losses in WGM resonators of different shape of the body of rotation. In this work, electromagnetic properties of a number of WGM resonators with conducting endplates are studied. Losses are studied in Ka-band for two kinds of dielectric material, namely, isotropic (Teflon) and anisotropic (single-crystal sapphire). For the Teflon hemispherical resonator, the analytical approach is used and its results are compared with the results of experimental measurements. The resonators of other shapes are studied experimentally. Experiments show that aspheric sapphire resonator has maximum coefficient A_s. however the radiation Q-factor is larger in the hybrid sapphire resonator (aspheric+cylindrical disk). Thus, the hybrid resonator with A_s 25 % greater than A_s of the hemispherical resonator with approximately the same radiation loss is a better option for the (super)conductor measurement purpose in microwave range. Fig. 8. Tab. 2. Ref.: 10 titles.

Key words: dielectric resonators, whispering gallery modes, radiation Q-factor, coefficient A_s.

For quite a long time, approaches for measuring the surface impedance $Z_S = R_S + iX_S$ (where R_S and X_S are the surface resistance and reactance, respectively) high-temperature superconducting (HTS) films were developed based on whispering gallery mode (WGM) dielectric resonators in a form of cylindrical disk with conducting endplates (CEP) [1, 2]. Such resonators are in fact quasi-optical dielectric ones because several wavelengths are confined along at least one of the coordinate axes. Whispering gallery mode resonator is body of rotation. WGMs propagate along a curved surface. Thus, these waves have a character of surface waves pressed to this surface. As a rule the resonators are made of single crystal sapphire as dielectric material of very small loss tangent. Both factors, namely, WGM excited in the resonator and sapphire material allow achieving high Q-factor in millimeter wavelength range, which provides high accuracy and sensitivity of measuring Z_S.

Recently WGM dielectric resonators in the forms of truncated cone [3] and hemisphere [4] have been proposed. It should be noted, that the electromagnetic properties of isotropic hemispherical resonator were considered analytically earlier, however only for the case of perfect conductor plate [5]. A comparison of three types of the resonators, cylindrical disk, truncated cone and hemisphere, was performed and niche was defined for each of them as the sensors of surface impedance for HTS films [6]. From the viewpoint of maximum sensitivity the hemispherical resonator is the most appropriate because of the lowest radiation loss. Now this kind of resonator is used for physical and applied studies of cuprate HTS films [7, 8]. However, the interaction of WGM microwave fields with CEP, i. e. with HTS film, is weaker compared with the case of the truncated cone. Thus the question arises concerning strengthening interaction while maintaining the same radiation loss using transformation of the hemispherical surface in the surface of another shape. The answer to this question will allow in principle the possibility of further increasing the sensitivity of impedance measurements.

With this purpose in the present work a number of WGM resonators made of Teflon and single crystal sapphire with the transformation of the hemispherical surface as well as cylindrical surface in aspheric surface are considered. As electromagnetic problems concerning the proposed resonators have not been solved, the authors propose the following. At first, the analytical solution [4] of electromagnetic problem concerning an isotropic Teflon hemisphere on the conducting plane is used. In this case the characteristic equation allows finding the frequency spectrum and Q-factor, including the radiation Q-factor. Then the radiation Q-factor values obtained experimentally as a function of frequency (or azimuthal index) are compared with the calculated ones. A number of sapphire resonators with evolution of the resonator lateral surface are studied by means of simulation and experimental measurements. Analysis of radiation losses in the studied resonators depending on the surface evolution is performed.

1. Theoretical description of isotropic hemispherical WGM resonator with conducting endplate and simulation of the anisotropic ones. Analytical solution of the isotropic dielectric hemisphere placed on the conducting plate obtained in [4] allows determining eigen frequencies and radiation Q-factor, Q_{rad}. For the purpose of describing the microwave fields and calculating the frequency spectrum and quality factor of the resonator, the special program product was developed. The advantage of the product is the sufficiently increased performance.

The results obtained by using this program product are presented in section 4 together with the experimental data.

Electromagnetic problem of hemispherical anisotropic dielectric WGM resonator has not been analytically solved yet. Therefore we are forced to use Microwave Studio CST 2008 product for simulation of these and the other resonators studied in this work. The results of simulation and experimental measurements of anisotropic resonators are given in section 4.

Only modes with both zero radial components of electric field, i. e. HE (in cylindrical disk) and H (in hemisphere) modes, and one radial variation of field are considered.

2. Experimental details. A number of WGM resonators were fabricated of Teflon because this material is easy to process mechanically. Their shapes changed from the hemispherical of a radius R to the aspheric of a spherical part radius r and an eccentricity $2a$ (Fig. 1) and from cylindrical disk of a radius R and a height h (Fig. 2) to the same aspheric resonator.

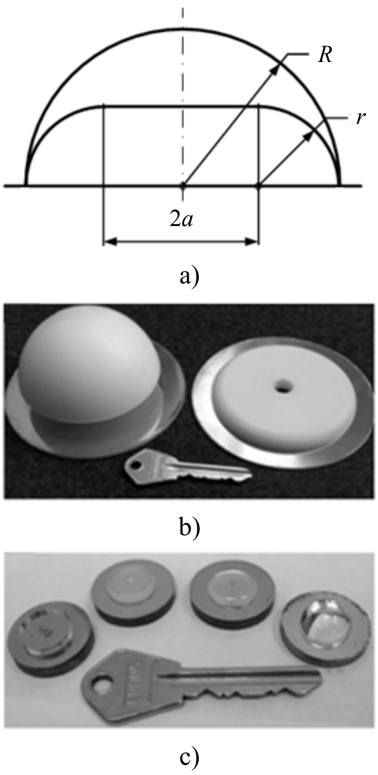

Fig. 1. Hemispherical and aspheric resonators with CEP: schematic drawing (a) and photo of Teflon (b) and sapphire (c) WGM resonators

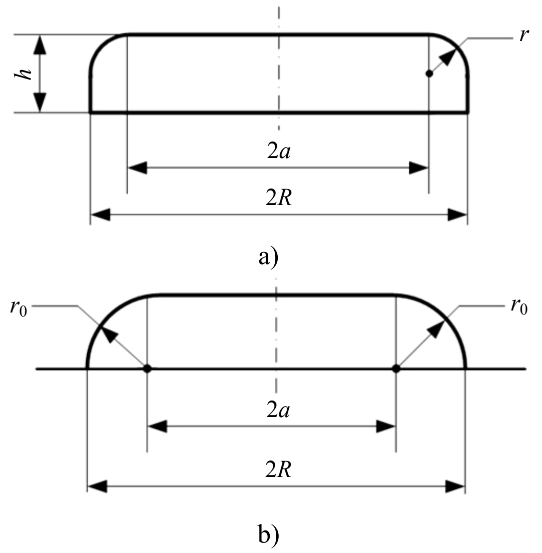

Fig. 2. Aspheric+cylindrical disk (a) and aspheric resonator (b) with CEP

In this case the resonator forms are varied in such a way that the same resonator of a radius $R = 78$ mm and a height $h = 7$ mm was obtained as a result of both evolutions. It is clear that $r_0 = h$ in the resonator. Here r_0 is a maximum meaning of radius at evolution from cylindrical disk and minimum meaning at evolution from hemisphere. A list of studied Teflon resonators is presented in Tab. 1.

Table 1

Dimensions and forms of the studied teflon resonators

N	r, mm	$2a$, mm	h, mm	Shape
1	0	78	7	cylindrical disk
2	3.5	71	7	aspheric+ +cylindrical disk
3	7	64	7	aspheric disk $r = r_0 = h$
4	39	0	39	hemisphere
5	20	38	20	aspheric disk
6	30	18	30	aspheric disk

More practical importance belongs to WGM resonators made of single crystal sapphire. Therefore, the sapphire resonators were numerically simulated using the Microwave Studio CST 2008 package (transient and eigen solvers). The resonance frequencies and field distribution of the excited modes were calculated. An optical c-axis was directed along a geometric axis of rotation. Additionally, manufacturing high-Q sapphire resonators is a rather complicated and labor-consuming technique. Therefore, number of sapphire resonators with the surface evolution is some less in comparison with Teflon resonator case (see Tab. 2).

Table 2

Dimensions and forms of the studied sapphire resonators

N	r, mm	$2a$, mm	h, mm	Shape
1	0	14.5	2.5	cylindrical disk
2	1.25	12	2.5	aspheric+ +cylindrical disk
3	2.5	9.5	2.5	aspheric disk $r = r_0 = h$
4	7.25	0	7.25	hemisphere

Experimental study of WGM resonators made of both dielectrics was performed in Ka-band using network analyzer PNA-L N5230A.

3. Results and considerations. Experimental frequency spectra of Teflon hemispherical (a) and aspheric (b) WGM resonators are displayed in Fig. 3. The spectra of sapphire hemispherical (a), aspheric (b), aspheric+cylindrical disk (c) and cylindrical disk WGM are shown in Fig. 4.

One can see complication of the spectrum in the sapphire resonators in comparison with the Teflon resonators, which is obviously due to the presence of anisotropy in the former.

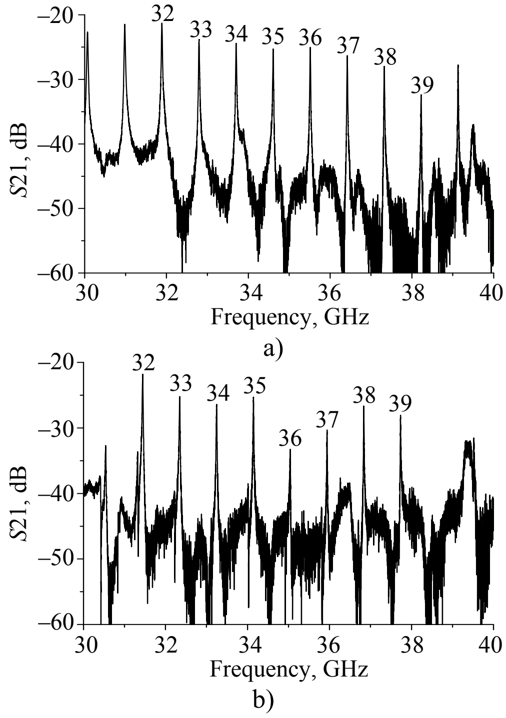

Fig. 3. Experimental spectra of Teflon hemispherical (a) and aspheric (b) WGM resonators; numbers at resonances are azimuthal indexes at one variation along radius of resonators.

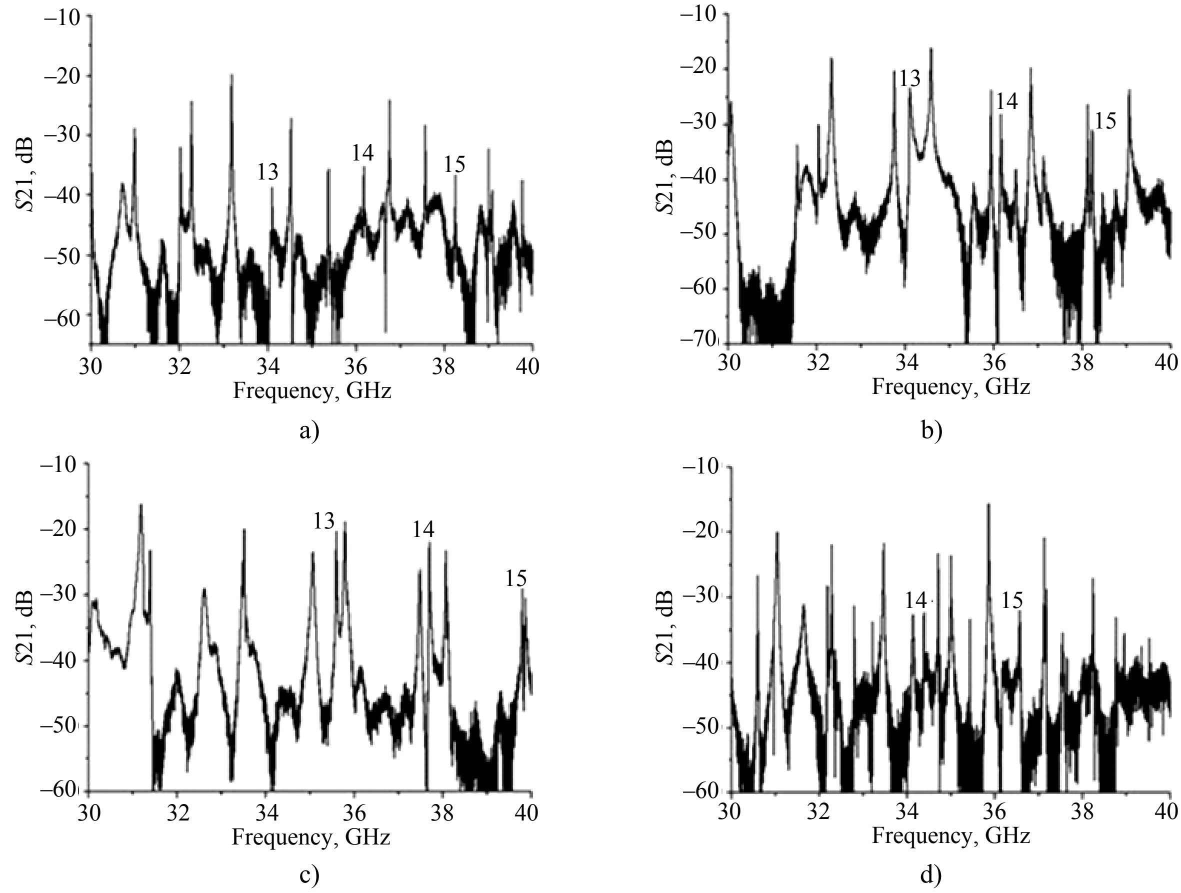

Fig. 4. Experimental spectra of sapphire cylindrical disk (a), aspheric+cylindrical disk (b), aspheric (c) and hemispherical (d) WGM resonators; numbers at resonances are azimuthal indexes at one field variation along radius

An important peculiarity is found here, namely a spectrum of aspheric+cylindrical disk resonator is the most rarefied, which is seen especially well in Fig. 4 for sapphire resonators. One can see change in the resonance frequency of the studied cavities in Fig. 5, physical nature of which is obvious.

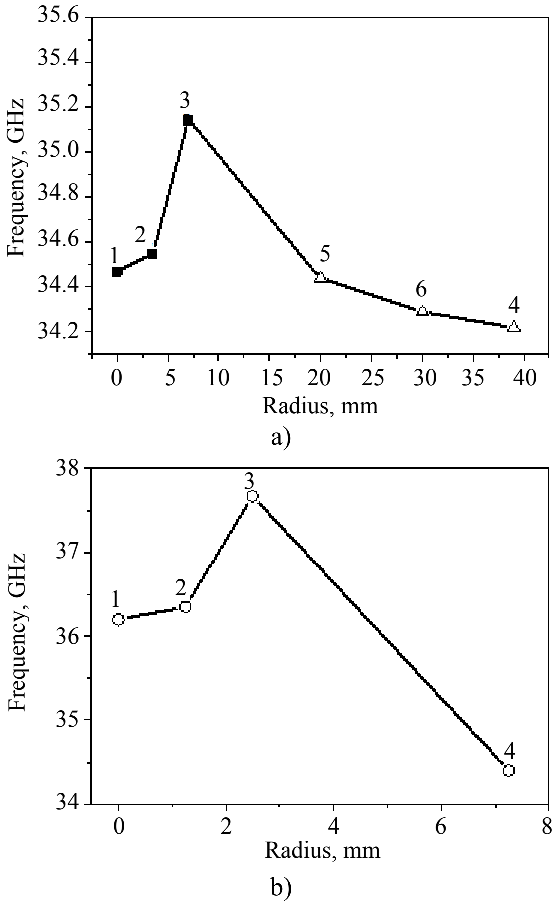

Fig. 5. Resonance frequency as function of radius r in Teflon WGM resonators (see Fig. 1 and Tab. 1) at azimuthal index $n = 35$ (a) and sapphire WGM resonators (see Fig. 1 and Tab. 2) at azimuthal index $n = 14$ (b)

The measurements results shown in Fig. 5 are connected by straight lines only to facilitate visual perception of the obtained dependencies. The radiation loss was found using the experimental data and the known relationship (see, e. g. [1]).

$$Q_0^{-1} = k \tan\delta + A_s^{\mathrm{CEP}} R_s^{\mathrm{CEP}} + Q_{rad}^{-1}, \qquad (1)$$

where k and A_s are coefficients describing interaction of fields with dielectric and conducting endplate, and $\tan\delta$ is dielectric loss tangent. Using the known k, A_s and the measured value of Q_0, the Q_{rad} can be found accordingly (1).

A coefficient k is very close to 1 and $k \tan\delta << A_s R_s$, therefore we can leave $k = 1$ for all resonators. However A_s is unknown for aspheric resonators. It can be found using two equations (1) and two measured Q_0 values for two conductors with the known surface resistance. We used copper with $R_s = 0.05$ Ohm and steel with $R_s = 0.12$ Ohm pre-measured by means of cylindrical disk WGM resonator [1]. Such an approach enables us to determine A_s and Q_{rad} for all resonators presented in Tab. 1 and 2. It is worth noting that coefficient A_s can be calculated analytically for cylindrical disk resonators (isotropic and anisotropic) with two conducting endplates and isotropic hemispheric resonator with CEP.

In Fig. 6 it is shown that Q_{rad} values in Teflon hemispherical resonator determined experimentally are lower than the ones calculated analytically. On the other hand, Fig. 6 displays experimental data concerning Q_{rad} in hemispherical, aspheric, aspheric+cylindrical and cylindrical disks. One can see that despite expectation a radiation loss is the largest for aspheric resonator. Of course this question is cleared easier for the resonators of higher Q-factor. In addition, the losses in Teflon cylindrical disk and aspheric+cylindrical disks are almost the same within the measurement error.

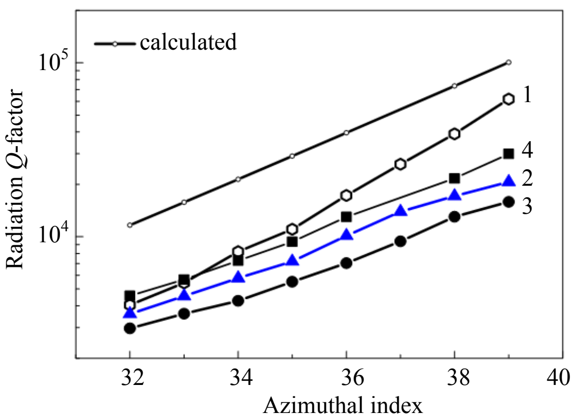

Fig. 6. Experimental and calculated data on radiation Q-factor for a number of Teflon WGM resonators. 1–4 are numbers in Tab. 1

Fig. 7 presents a coefficient A_s as a function of azimuthal number n for hemispherical, aspheric, aspheric+cylindrical and cylindrical disks. The data demonstrate that the interaction of microwave fields with conductor endplate is the strongest in the aspheric+cylindrical disk. This result was rather unexpected and demanded a further studying using a number of sapphire resonators. The authors carried out a thorough study of Q-factor and coefficient A_s for the mentioned number of sapphire resonators (Tab. 2) and compared them with that of Teflon resonators (Tab. 1) in Fig. 8.

In a Fig. 8 it is shown that the sapphire resonator with aspheric, not hybrid (i. e. aspheric+cylindrica disk), lateral surface has the maximum coefficient A_s. However the radiation quality, vice versa, is larger in the hybrid resonator. Thus, the hybrid, namely, aspheric+cylindrical disk sapphire resonator with A_s 25 % greater than A_s of the hemispherical resonator with approximately the same radiation loss is a better option for the microwave measurement purpose. It is useful for developing sensors of the surface impedance of superconductors or other types of conductors, such as graphene, where high sensitivity of contactless measurement of complex conductivity is especially important [10].

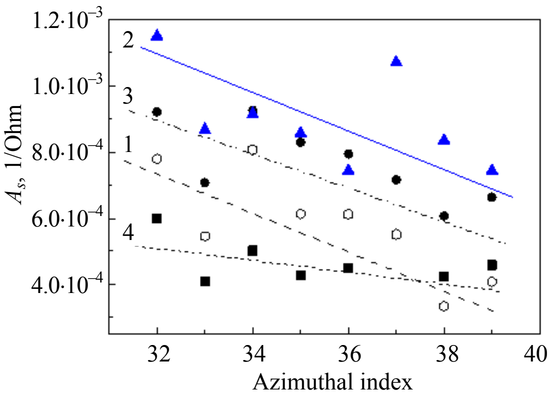

Fig. 7. Coefficient A_s for a number of resonators depending on azimuthal index. 1–4 are numbers in Tab. 1

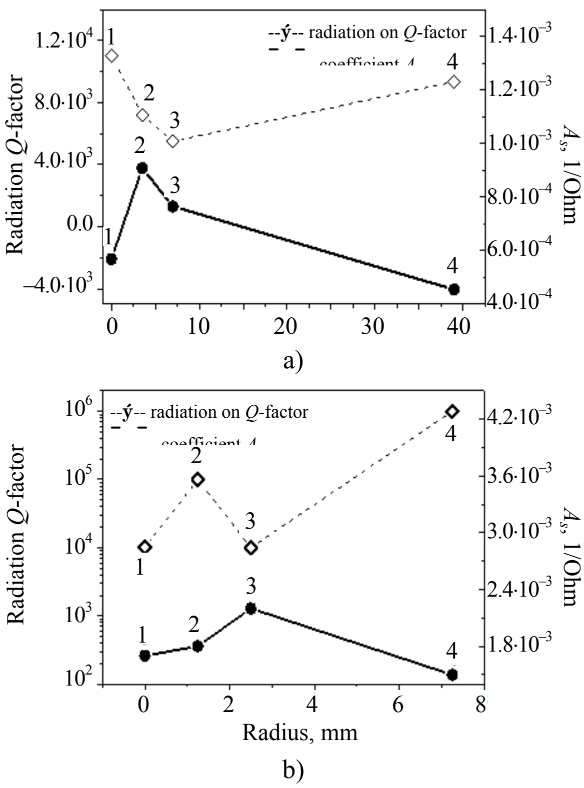

Fig. 8. Data on radiation Q-factor and coefficient A_s for Teflon WGM resonators 1–4 (Tab. 1) (a) and for sapphire resonators 1–4 (Tab. 2) (b)

Conclusion. In this work radiation and conductivity losses, which determined by coefficient A_s, in a number of WGM dielectric resonators of various shapes (of rotation body surface) made of isotropic (Teflon) and anisotropic (single crystal sapphire) dielectrics placed on conducting endplates are studied. For the hemispherical Teflon resonator the results of analytical calculation and experimental measurements of radiation losses are correlated, although the experimental losses are higher. Comparative analysis of sapphire WGM resonators shows that, on the one hand, resonator in shape of aspheric+cylindrical disk has higher radiation Q-factor and higher coefficient A_s compare to disk resonator. On the other hand, aspheric disk shows the highest coefficient A_s, but its radiation loses approximately equal to radiation loses in disk resonator. However, last ones much inferior in radiation Q-factor to the hemisphere. In this way, aspheric resonators are promising for surface impedance and complex conductivity characterization of superconductors and other types of conductors.

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ПОЛУСФЕРИЧЕСКИЙ И АСФЕРИЧЕСКИЙ ДИЭЛЕКТРИЧЕСКИЕ РЕЗОНАТОРЫ С ВОЛНАМИ ШЕПЧУЩЕЙ ГАЛЕРЕИ С ПРОВОДЯЩИМИ ТОРЦЕВЫМИ СТЕНКАМИ: РАДИАЦИОННЫЕ ПОТЕРИ И ПОТЕРИ В ПРОВОДНИКЕ

Метод измерений, основанный на использовании диэлектрических резонаторов с волнами шепчущей галереи (ШГ), для нахождения поверхностного микроволнового импеданса пленок высокотемпературных сверхпроводников, является достаточно точным. Однако чувствительность его ограничивается радиационными потерями в этих открытых электродинамических системах и зависит от эффективности взаимодействия микроволновых полей и пленки (коэффициент A_s). Целью работы является исследование радиационных потерь и потерь, обусловленных проводимостью в резонаторах различной формы вращения с волнами ШГ. В работе изучаются электромагнитные свойства ряда резонаторов с волнами ШГ с торцевыми проводящими стенками. Потери изучаются в K_a-диапазоне для двух видов диэлектрического материала, а именно изотропного (тефлон) и анизотропного (монокристаллический сапфир). Для тефлонового полусферического резонатора используется аналитический подход, и его результаты сравниваются с результатами экспериментальных измерений. Результаты для других форм исследуются экспериментально. Эксперименты показывают, что асферический сапфировый резонатор имеет наибольший коэффициент A_s, однако радиационная добротность выше для гибридного сапфирового резонатора (асферический+цилиндрический диск). Таким образом, гибридный резонатор с A_s на 25 % более высоким, чем A_s полусферического резонатора с примерно теми же радиационными потерями, является лучшим выбором для измерений свойств (сверх) проводников в микроволновом диапазоне.

Ключевые слова: диэлектрические резонаторы, волны шепчущей галереи, радиационная добротность, коэффициент A_s.

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НАПІВСФЕРИЧНИЙ ТА АСФЕРИЧНИЙ ДІЕЛЕКТРИЧНІ РЕЗОНАТОРИ З ХВИЛЯМИ ШЕПОЧУЧОЇ ГАЛЕРЕЇ З ПРОВІДНИМИ ТОРЦЕВИМИ СТІНКАМИ: РАДІАЦІЙНІ ВТРАТИ ТА ВТРАТИ У ПРОВІДНИКУ

Метод вимірювань, що ґрунтується на використанні діелектричних резонаторів з хвилями шепочучої галереї (ШГ), для визначення поверхневого мікрохвильового імпедансу плівок високотемпературних надпровідників, є досить точним. Однак чутливість його обмежується радіаційними втратами в цих відкритих електродинамічних системах і залежить від ефективності взаємодії мікрохвильових полів і плівки (коефіцієнт A_s). Метою роботи є дослідження радіаційних втрат і втрат, зумовлених провідністю в резонаторах різної форми обертання з хвилями ШГ. У роботі досліджуються електромагнітні властивості ряду резонаторів з хвилями ШГ з торцевими провідними стінками. Втрати вивчаються в K_a-діапазоні для двох видів діелектричного матеріалу, а саме ізотропного (тефлон) і анізотропного (монокристалічний сапфір). Для тефлонового напівсферичного резонатора використовується аналітичний підхід, і його результати порівнюються з результатами експериментальних вимірювань. Результати для інших форм досліджуються експериментально. Експерименти показують, що асферичний сапфіровий резонатор має найбільший коефіцієнт A_s, проте радіаційна добротність вища для гібридного сапфірового резонатора (асферичний+циліндричний диск). Таким чином, гібридний резонатор з A_s на 25 % вищим, ніж A_s напівсферичного резонатора з приблизно тими ж радіаційними втратами, є кращим вибором для вимірювань властивостей (над) провідників в мікрохвильовому діапазоні.

Ключові слова: діелектричні резонатори, хвилі шепочучої галереї, радіаційна добротність, коефіцієнт A_s.