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# Mathematical Modelling as a First Stage for Logistics Systems Simulation

Mathematical modelling is presented as a principle for computer support usage in logistic processe modelling and simulation.

Математические модели рассмотрены как основа для использования компьютерных средств при моделировании логистических процессов.

*K e y   w o r d s: mathematical modeling, simulation, logistics system.*

Simulation is a technique for the all systems' functioning imitation or for particular situation (economical, military or mechanical) imitation. The technique use suitable models or tools in purpose of information achievement or didactic goals obtaining. Simulation is defined as an art and science as well. The main aim of simulation is whole system or particular process model creation, application of changes and in effect researched system functioning estimation in various circumstances. Simulation is a perfect analytical technique that can really facilitate problem-solving process (Fig. 1). Not every time simulation gives the best of possible solutions but its effects are precious because they are results of many repeated research cycles. Achieved results become more real because of statistic methods usage and computer graphics implication. It also helps non-professionals to understand the mechanisms of researched system.

Simulation is a technique that help determining which of possible solutions is optimal in context of particular requirements. Financial aspect of researches should not be forgotten, using simulation tools. It means that simulation usage lets for optimal solution choice almost with no investments — very often there is no necessity to build a physical system, previously. It is very important in case of design and optimizing extraordinary expensive Flexible Manufacturing Systems. Now a day simulation usage as a tool for problem solving is one of the most often used technique and all scientific spheres are practically the spheres of its application. One can make analysis using simulation methods in almost all spheres beginning from production systems, chemical and physical processes,

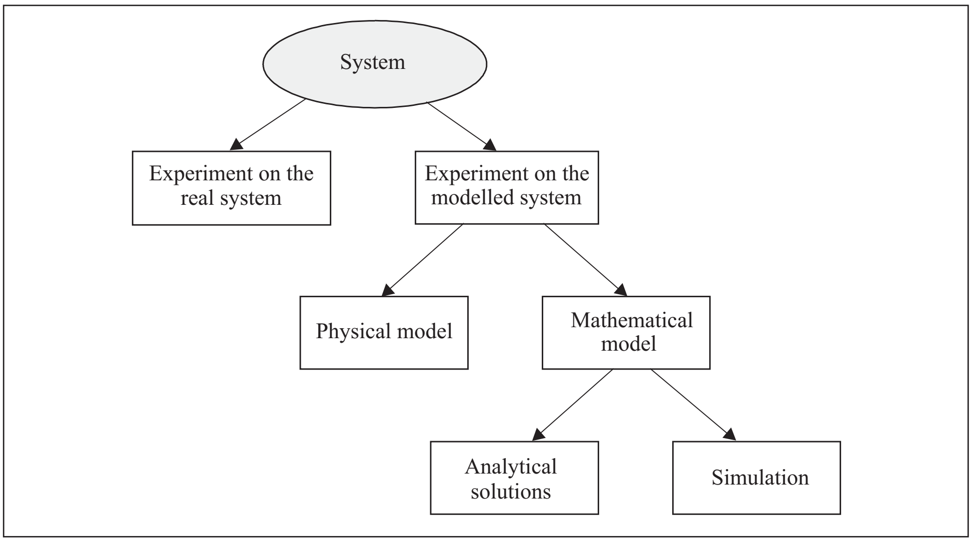


*Fig. 1.* System analysis techniques

distribution and transportation systems, computer nets, on military projects finishing.

The user imagination can be the only one barrier of simulation new application. However it should be remembered that various scientific domains require different research methods.

**Mathematical modelling for simulation experiment elaboration.** Distinguishing different ways in which a system and its' processes might be studied, it is rarely feasible to experiment with the actual system, because such an experiment would often be too costly or too disruptive to the system, or because the required system might not even exist. For these reasons, it is usually necessary to build a model as a representation of the real system and to study it as a surrogate for the real system. M. Pidd [1] defines a model as follows.

A model is an external and explicit representation of part of reality as seen by the people who wish to use that model to understand, to change, to manage, and to control that part of reality in some way or another.

Mathematical models are ones of the models used in logistics processes modelling and simulation. They are especially helpful because they make directly basis for usage of the computer tools in process of solution discovering. They have the advantage of providing the possibility to evaluate numerous logistics processes' alternatives. One of the main disadvantages of mathematical models is that they cannot precisely predict the feasibility of the modelled logistics processes in real-life (in addition to the fact that such a model is only reliable if the right assumptions underlying the model are made).

**Organization of modeling framework. 1.** ***Define a modelling framework.*** Starting with the real system, we first form a conceptual model of the system that contains the elements of the real system that we believe should be included in the model [2]. That is, one should identify all facilities, equipment, events, operating rules and descriptions of behaviour, state variables, decision variables, measures of performance, and so on, that will be part of the model. Other authors use other terms. Van Hee [3] refer to ontology, defined as a system of clearly defined concepts describing a certain knowledge domain. D.W.Wilson [4] refers to the world view or Weltanschauung, i.e. that view of the world which enables each observer to attribute meaning to what is observed.

Consecutively, we must identify the relationships between the elements identified. From the conceptualisation of the system a logical model (or flow chart model) is formed that contains the classification of and logical relationships among the elements of the system, as well as the exogenous variables that affect the system. A modelling framework is defined as a set of basic modelling constructs and their possible relationships required to model the behaviour of the SC completely.

**2.** ***Devise a set of symbolic objects.*** The next step is to devise a set of corresponding symbolic (i.e. formal) objects that can be used to represent the foregoing modelling constructs [5]. This includes identifying the integrity rules that go along with those formal objects (i.e. how to use the objects).

**3.** ***Building the simulation model.*** Using the modeling framework and the symbolic objects defined we develop a computer model, in a specified simulation language, which will execute the logic described. Which aspects of the SC are modelled depends on the demarcation of the SC. The decision on how much of the real system should be included in the conceptual model to bring about a valid representation of the real system must be jointly agreed upon by the simulation analyst and the decision-makers.

Developing a simulation model is an iterative process with successive refinements at each stage. The basis for iterating between the different models is the success or failure we have when verifying and validating each of the models.

**Mathematical model** [7]. Let us take a system of rolling mill into consideration. It is assumed it consists of $I$ assemblies and $J_i$ passes in the $i$-th assembly. The structure of the rolling mill can be presented in the matrix form: $E = \lfloor e_{i,j} \rfloor$, $i = 1, ..., I$, $j = 1, ..., J_i$, where $J = \max_{1 \le i \le I} J_i$. Matrix $E$ elements are defined in the following way:

$$e_{i,j} = \begin{cases} j \text{ for } 1 \le j \le J_i, \\ -j \text{ for } J_i \le j \le J. \end{cases}$$

Consequently, matrix $E$ elements are numbers of the passes. Non-existing passes are marked by negative numbers. A pass parameter is its life which equals the number of tons of material that can be passed through a new pass.

Let us assume the passes life matrix is given: $G = \lfloor g_{i,j} \rfloor$, $i = 1, ..., I$, $j = 1, ..., J_I$. Elements $g_{i,j}$ for $j \le J_i$ have practical meaning in the above shown matrix. Since for $e_{i,j} = -j$ there are non-existent adequate passes, it can be assumed that for such passes: $e_{i,j} = -j$ . Rolling process consists in passing material through the passes of the successive assemblies.

There are routes given for manufacturing certain products written in the matrix form: $D = \lfloor d_{i,n} \rfloor$, $n = 1, ..., N$, $i = 1, ..., I$, where $d_{i,n}$ — the $i$-th assembly pass number; $N$ — the number of products. A route can omit some assemblies in a general case. Should the $n$-th route not include an $i$-th assembly pass, then it is assumed that $d_{i,j} = 0$. A route consists of number of passes of the following assemblies from $i = 1$ to $i = I$. Some routes are shorter, which means they do not include passes from certain assemblies. The last pass is the decisive one for the product type. Therefore, the maximal number of products can be the sum of all the passes in all assemblies.

Let us assume there are $M$ types of materials (charges) from which $N$ types of products can be manufactured. To allocate a charge to a product the allocation matrix is given: $A = \lfloor a_{m,n} \rfloor$, $m = 1, ..., M$, $n = 1, ..., N$, where

$$a_{m,n} = \begin{cases} 1 & \text{if the } n\text{-th product can be manufactured from the } m\text{-th } ch \arg e, \\ 0 & \text{otherwise.} \end{cases}$$

Let us also introduce the charge vector $W = [w_m]$, $m = 1, ..., M$, where $w_m$ — the number of tons of the $m$-th charge type.

Let us introduce the rolling rate vector $V = [v_n]$, $n = 1, ..., N$, where $v_n$ — the number of the $n$-th product tons manufactured in a time unit.

Let us introduce the order vector $Z = [z_n]$, $n = 1, ..., N$, where $z_n$ — the number of tons of the $n$-th type product in state $k-1$.

The order vector changes after each decision about production:

$$z_n^k = \begin{cases} z_n^{k-1} - x_n^k, \text{if } n = a, \\ z_n^{k-1}, \quad \text{if } n \ne a, \end{cases}$$

where $x_n^k$ — the number of tons of product $a$.

The billet mill state is defined as the matrix $S = \lfloor s_{i,j} \rfloor$, $i = 1, ..., I$, $j = 1, ..., J_i$, where $s_{i,j}$ — the number of tons of material passed through the $j$-th pass of the $i$-th assembly and for $J_i < j \le J$ we can write $s_{i,j} = -j$. State matrix elements must satisfy the condition: $0 \le s_{i,j} \le g_{i,j}$, $1 \le j \le J_i$. Initial state $S^0$ is given. The equation of state of the production line takes a general form: $S^k = f(S^{k-1}, x_n^k, b)$, where $b$ — the number of an assembly assigned to replacement.

The equation of state in case of production can be presented as follows:

$$s_{i,j}^{k} = \begin{cases} s_{i,j}^{k-1}, \text{if material is not passed through the } j\text{-th pass} \\ \qquad \text{of the } i\text{-th assembly of rolls,} \\ s_{i,j}^{k-1} + \min(p_a^{k-1}, z_n^{k-1}) \text{ otherwise.} \end{cases}$$

In case of the $b$-th assembly replacement the equation of state takes the form:

$$s_{i,j}^{k} = \begin{cases} s_{i,j}^{k-1} - x_n^k, \text{if } i \neq b, \\ 0, \quad \text{if } i = b. \end{cases}$$

Replacement brings about the opportunity for restarting production. To be able to carry out further calculations the pass matrix of the rolling mill is introduced: $P = [p_{i,j}]$, $i = 1, ..., I$, $j = 1, ..., J_i$, where $p_{i,j} = g_{i,j} - s_{i,j}$, $i = 1, ..., I$, $j = 1, ..., J_i$; $p_{i,j} = -j$, $j = J_i + 1, ..., J$.

Having finished the rolling stage the billet mill flow capacity is low. Most often, the route roll flow capacity of each product, where $n = 1, \ldots, N$, equals zero. To start the rolling stage the most worn out rolls are to be replaced by new ones. On the basis of state $S^{k-1}$ the roll pass matrix can be calculated: $P^{k-1} = [p_{i,j}^{k-1}]$. Residual pass of assemblies will be calculated as shown:

$$R_i = \sum_{j=1}^{J_i} p_{i,j}.$$

According to the heuristic algorithm the $l$-th assembly is to be replaced on condition that

$$\exists_j (p_{i,j}^{k-1} = 0) \wedge (R_l = \min R_i).$$

During a rolling process, rolling time is determined: $t_n = x_n / v_n$. Each assembly replacement time $c_r$ is given as an element of the vector $C$.

Let us introduce the tolerance matrix $H = \lfloor h_{i,j} \rfloor$, $i = 1, ..., I$, $j = 1, ..., J$, where $h_{i,j}$ — the tolerance of the $j$-th pass of the $i$-th assembly of rolls. In case of allowing for the tolerance matrix, the equation of state takes the form:

$$s_{i,j}^{k} = \begin{cases} s_{i,j}^{k-1}, \text{ if } (i \neq b) \wedge (p_{i,j}^{k-1} \geq h_{i,j}), \\ 0, \quad \text{if } (i = b) \wedge (p_{i,j}^{k-1} < h_{i,j}). \end{cases}$$

The assembly regeneration time coefficient $\psi$ is introduced. It means how many units of the assembly flow capacity can be regenerated within the time

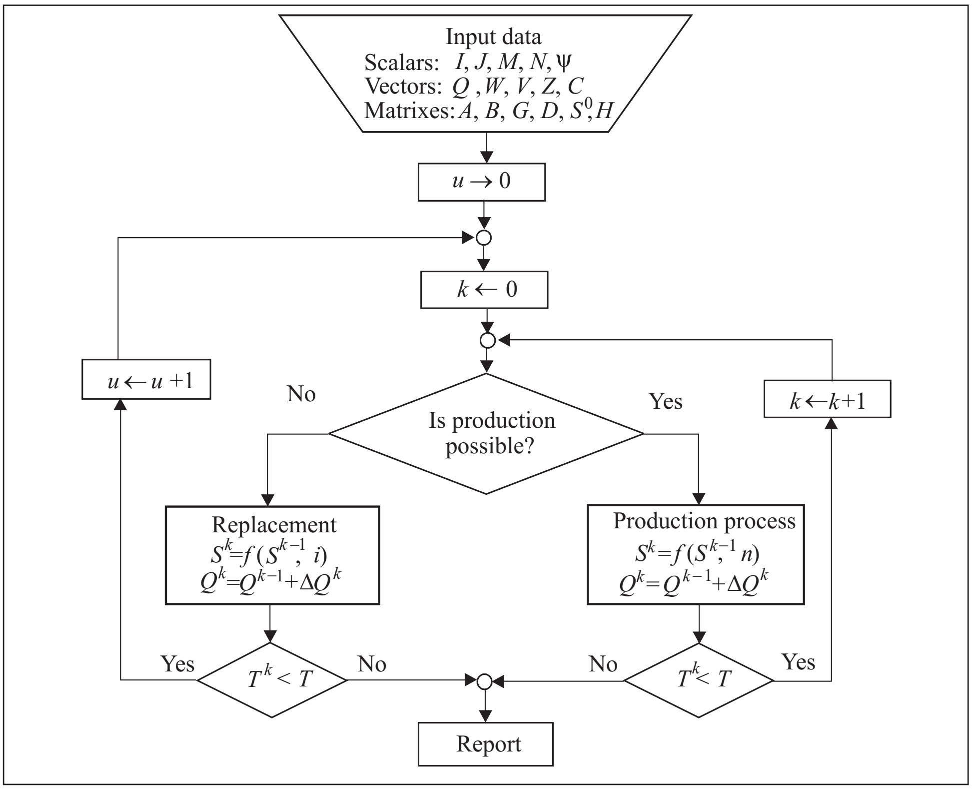


*Fig. 2.* The block diagram of production and replacement

unit. This can be written $\psi = \dfrac{\max P_i}{\tau_i}$, $i = 1, ..., I,$ where $\tau_i$ — the *i*-th assembly regeneration time.

**Simulators.** The above assumptions are taken into account. They are the basis for creating simulators. Fig. 2 presents a general simulator of production and replacement of rolls. The production process may be simulated in detail by means of a production simulator shown in Fig. 3. The replacement process itself will be checked by a simulation device.

To built simulators the following data must be input: $I, J, M, N, \psi, A, E, H, G, D, S^0, W, Z, V, C$ as well as the vector of optimization indexes $Q$ and the allowable time of the order realization $T$.

**Summary.** Modelling and simulation can make the research time and solution achieving shorter and more effective. Because of their possibilities and ability for verification of many alternative solutions for logistics processes design and managing it is possible to choose the best solution without building the

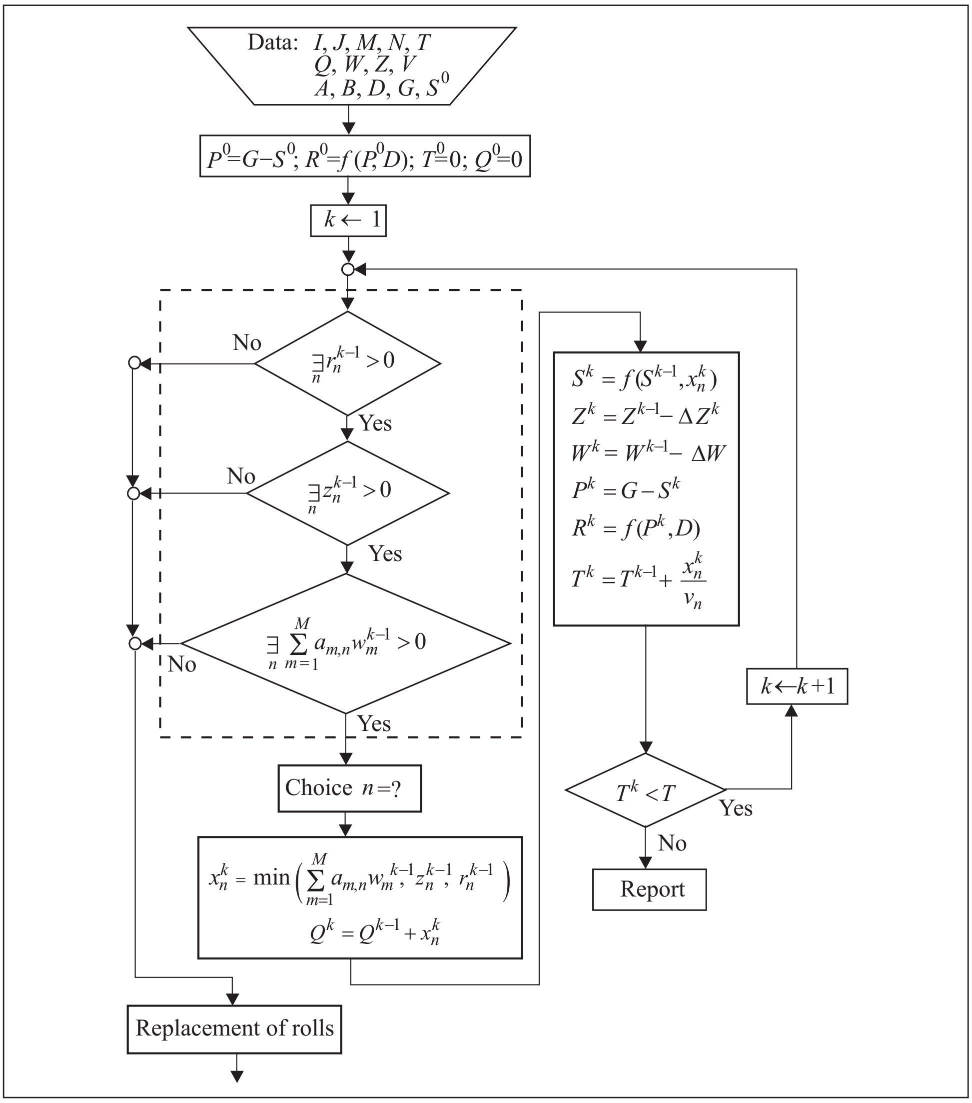


*Fig. 3.* The block diagram of the production process

structures that is very expensive and time consuming. The computer support enable verification of processes and plant structures in circumstances very similar to reality that allows for avoiding many expected and unexpected situations in future operations. It can be stated that modelling and simulation are useful and effective using computer support for processes improvement but It should be remembered that there is some disadvantages. They are as follows: user large experience, reality simplification or problems with results' credibility estimation.

Математичні моделі розглянуто як основу для використання комп'ютерних засобів при моделюванні логістичних процесів.

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