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Collective Computational Processes of Portfolio Management Using Cellular Automata

The paper presents simulation of selection elements of portfolio applying the classical Markowitz's portfolio analysis theory using parallel collective computational environment — cellular automata. The basics of classical portfolio analysis and theory of cellular automata and their collective work are presented. Structure of drawn up simulation and its results are presented. Simulations was performed application in Borland C++ Builder.

Представлено моделирование отдельных элементов портфеля инвестиций с применением классической теории анализа портфеля инвестиций Марковица с использованием параллельной коллективной вычислительной среды клеточных автоматов. Представлены основы классического анализа портфеля ценных бумаг и теории клеточных автоматов и их совместного использования, а также структура процедуры моделирования и его результаты. Моделирование выполнено в системе Борлэнд С++ Билдер.

Key words: portfolio analysis, portfolio management, cellular automata.

Portfolio analysis. In last few years, Polish stock market has strongly developed. About 500 stock companies are quoted on Warsaw Stock Exchange every working day. Stock market in Poland in spite of today's global capital markets crisis is a good object for researches on behavior of stock prices. Apart from stock, investors on Polish capital market can invest money in bonds, futures contracts and other financial instruments. The main aim for investors is to estimate correctly future value of securities and then choose the biggest profit ones. Problem is in high level of risk which those securities are characterized by very often. Investors can reduce that risk by investing in more than one security. Portfolio analysis is one of the basic techniques of capital market analysis.

According to the theory of portfolio analysis we can predict future values of stock basing on historical quotation. From this data the rate of return is counted interpreted as an expected profit and standard deviation which is a measure of dispersion and is interpreted as a risk connected with this expected profit. Investments are usually interested in shares with the large profit and low risk level. Portfolio analysis studies how these values will change if we will invest in more than one share. This theory also shows how to choose assets during constructing the portfolio to diversify a risk which means that the risk of portfolio would be lower than the risk of shares that are this portfolio's components.

H. Markowitz published in 1952 his first paper about the portfolio selection [1]. It has started a real revolution for capital markets and created completely new technique of making investment decisions today, called portfolio analysis. Although this theory showed how to choose the best stocks to get the highest income with the lowest level of risk, it was very hard to apply it in practice because of computational complexity. After few years another scientist W. Sharp has simplified this model and made the portfolio analysis able to be applied in practice [2]. He has added to this theory two coefficients giving investors a clear hint which stocks are giving better results than a general tendency on the considered market. He has suggested stock exchange index as a factor reflecting the market tendency. On stock market in Poland investors most commonly use Warsaw Stock Exchange Index for this purpose.

Rate of return and risk. In classical Markowitz's model of portfolio selection the most important profiles of assets are rate of return and standard deviation [3]. These two values have to be calculated for all shares taken by investor into consideration. Positive value of rate of return is interpreted as an expected profit and negative value — as an expected loss. This quantity is counted in a period of time t with following formula:

$$R_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}},$$

where R_t — the rate of return in period t; P_t — the price of securities in period t; P_{t-1} — the price of securities in period $t-1$; D_t — paid-out dividend in period t.

Rate of return is appointed for every period of time t, there for it becomes a function of time. The real value of income depends on many factors and investor cannot be sure that he will get calculated profit. This is the reason that we use expression: expected rate of return. It is counted with the following formula:

$$R_t = \frac{\sum_{t=1}^{N} R_t}{N},$$

where R — expected rate of return from securities; R_t — the rate of return in period t; N — the number of all analyzed rates of return.

The level of profit or loss defined in this way is always accompanied by the investment risk. The risk in portfolio analysis is calculated using statistics. Stan-

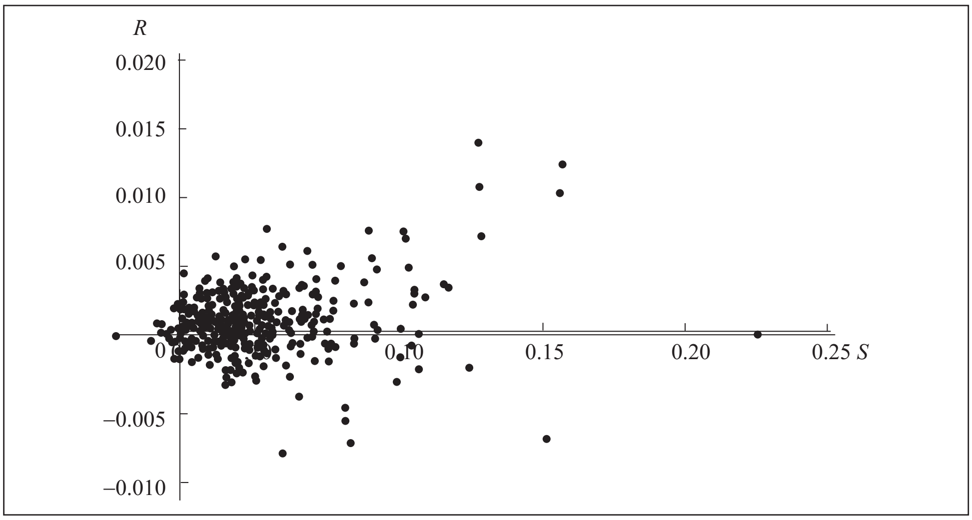

Fig. 1. Map of risk and income for companies on Warsaw Stock Exchange from 01.10.2008 to 01.10.2009

dard deviation is interpreted as the amount of risk. The formula to calculate variation of stock's rate of return is shown below:

$$S^2 = \frac{1}{n-1}\sum_{t=1}^{n}(R_t - R)^2,$$

where S^2 — the variation of the rate of return; R, R_t — as above; n — the number of all analyzed rates of return; S — standard deviation of the rate of return.

Map of risk and income. Both values: expected income and risk can be shown on a graph. It is called a map of risk and income. This graph makes possible retrieval companies with possibly the lowest risk and largest income. The following figure represents a map of risk and income for counted all companies' quotation from the Polish stock market in the period from October 1, 2008 to October 1, 2009.

As we can see on the map of risk and income, most of stock companies have achieved standard deviation much higher than a standard deviation of Warszawski Index Gieldowy (WIG) which is in the center of Fig. 1. There has been only 2.5 % of stock companies with dispersion of its quotation lower than WIG, and 97.5 % of stock companies with dispersion of its quotation higher than WIG. That means that values of quotations of stock companies have been changing more often than values of index WIG, so they have been more risky for investors. This situation happens very often on the Polish stock market.

At the same time 45.6 % of stock companies expected the income higher than index WIG, and 54.4 % of stock companies expected the income lower than index WIG. This situation does not occur too often on the Polish stock market. Usually situation is much worse for potential investors.

Two-element portfolios. The expected rate of return and standard deviation are two basic characteristics of individual securities. When we consider more than one security there is one more important value. It is the coefficient of correlation between two securities. It defines connection of rates of return of two stocks. The coefficient of correlation can be calculated by the following formula:

$$\rho_{12} = \frac{\sum_{i=1}^{n}(R_{1i} - R_1)(R_{2i} - R_2)}{S_1 S_2},$$

where ρ_{12} — coefficient of correlation of rates of return of stock 1 and 2; R_{1i} — rate of return in period i; R_1 — expected rate of return from the first stock; S_1 — standard deviation of rate of return of the first stock; R_{2i}, R_2, S_2 — the same for the second stock; n — the number of all analyzed rates of return.

Investors are seeking for the possibility of investing capital in stocks with high rate of return. However these securities are very often characterized by high level of risk. The investor is interested in such investments, in which with growth of rate of return, the risk will go down. The portfolio analysis gives us such a possibility [4—6].

A portfolio is a set of stock, which we have or we want to buy. The rate of return of the portfolio is the sum of rates of return individual values multiplied by their parts in investment:

$$R_p = x_A R_A + x_B R_B,$$

$$0 \le x_A \le 1, \quad 0 \le x_B \le 1, \quad x_A + x_B = 1,$$

where R_p — the rate of return from two component portfolio; x_A — the part of share A in portfolio; R_A — the rate of return of stock A; x_B, R_B — the same for stock B.

Calculations of risk for two-element investment portfolio are more complicated. Variation of two-component portfolio is defined as:

$$S_p^2 = x_A^2 S_A^2 + x_B^2 S_B^2 + 2 x_A x_B S_A S_B r_{AB},$$

where S_p^2 — variation of investment portfolio; x_A, x_B, R_A, R_B — as before; r_{AB} — the coefficient of correlation between the rate of return of stock A and B; S_p — standard deviation of two-component portfolio.

Many-element portfolio. For more than two-element portfolio, the formulas are as follows:

$$R_p = \sum_{i=1}^{N} x_i R_i,$$

$$S_p^2 = \sum_{i=1}^{N} x_i^2 S_i^2 + 2\sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j S_i S_j r_{ij}.$$

Formulas above show computational complexity of the problem of selection of effective portfolios. The portfolio is effective when the rate of return is higher than any other portfolio with the same risk, or the level of risk is the lowest of the portfolios with the same rate of return.

The Sharpe measure of portfolio effectiveness value is also called the Sharpe efficiency of the portfolio. It is quotient of risk premium achieved by the portfolio and risk of the total portfolio which has been taken. It must be calculated with the following formula:

$$SH_J = \frac{E(R_J) - R_F}{E(S_J)},$$

where SH_J — Sharpe measure of portfolio effectiveness value; $E(R_J)$ — the value of the expected rate of return on portfolio J; R_F — risk-free rate of return; S_J — standard deviation of portfolio returns J.

The higher is the value of this measure, the better is the quality of portfolio management. Portfolios lying on the line of the greatest slope in the total system risk are preferred as those of additional income.

Problem of selection of effective portfolio is not simple in spite of existence of theoretical solution. With choosing portfolio the investor has first to choose securities in which he wants to invest, and then establish how the invested capital will be divided among securities. Very often the best solution we got as a result of watching nature. The most popular usage of such processes are genetic algorithms and neural networks. Another one is a cellular automaton. Cellular automata were created in the fortieth of the last century to emulate processes occurring in nature. Soon after this invention, cellular automata proved to be very interesting and useful. There are still some researches done to find new application for cellular automata. One of them can be a problem of selection of portfolio in the stock market.

Cellular automata (CA) are discrete models used mainly in physics, mathematics and computability theory. They were created in the 40's of the last century by Stanisław Ulman and later developed by his colleague John von Neumann who was also working at Los Alamos National Laboratory. CA are structures of the same cells put into lattice. It is usually one, two-or three-dimensional and involves the great number of cells (theoretically this number is infinite but it is impossible to implement such a model). Every cell has a defined type and starting value. It also has its algorithm called a function of conversion. This function defines what will be future value depending on the values of neighboring cells in current time [7].

There are two types of neighborhoods in CA: von Neuman's and Moore's. Von Neuman's neighborhood is made up of four cells adjacent vertically and horizontally. Eight cells surrounding the given cell from right, left, top, bottom and on a slant make up Moore's neighborhood. In every iteration a current value of every cell is calculated on the basis of values of adjacent cells from previous iteration [8].

The best known example of CA is a game «Life» created by John Conway. In this game every cell receives starting state: it can be active or unactive. This game simulates real environment where animals can be born or day when they do not have enough food. In this game there are settled rules of behavior of every cell. If it is unactive and three other active cells surround it, it comes to life, and so becomes active. If a cell is active and in its surrounding has two or three other active cells, it stays active too. In every different case a cell is unactive — it stays unactive if such it was earlier or if earlier conditions are not fulfilled. So simple rules lead to astounding solutions. The final effect of CA can be systematized as following: CA achieves stable state, in which nothing changes it (all cells stay in determinate state); state of CA changes cyclically after some iterations; CA achieves a chaotic state in which it is hard to find any order; in CA we can find stable local configurations with long life.

The cellular automata are already applied for a long time in many fields of science. In every issue it is necessary to establish three basic parameters of CA as: type of cells creating CA, which means to establish what kind of information they have to contain; starting value of every cell; function of passage which is an algorithm deciding what will be the state of cells in a current iteration on the base of values of neighboring cells in the previous iteration.

It has been proved that CA can simulate mechanisms occurring at the stock market and show how this simulation can be useful for potential capital market investors using its collective work to choose effective portfolios [3, 5], although further researches are needed, especially in time of capital markets crisis. Structure of CA simulating stock market using collective work of all cells can be organized in many ways, as it was shown before [9].

Simulation results. To find an effective portfolio we have to establish work of CA and their three basic parameters: type of cells, starting value and function

Multielement portfolios' characteristics and WIG characteristics

Number	Two-element			Three-element			Five-element			Ten-element		
	R	S	SH_J	R	S	SH_J	R	S	SH_J	R	S	SH_J
1	0.007536	0.057708	0.061271	0.006387	0.047457	0.050290	0.005728	0.038461	0.044931	0.001480	0.020040	–0.125770
2	0.003120	0.030226	–0.029113	0.005298	0.030660	0.042346	0.003270	0.003270	–0.021317	0.003299	0.020797	–0.033695
3	0.006291	0.078914	0.029034	0.004031	0.037111	0.000833	0.003415	0.031080	–0.018828	0.001390	0.010627	–0.245588
4	0.006223	0.050309	0.044177	0.004448	0.046314	0.009683	0.003420	0.025089	–0.023113	0.002145	0.014026	–0.132251
5	0.005748	0.044643	0.039162	0.004661	0.050315	0.013134	0.006629	0.040605	0.064752	0.001542	0.013875	–0.177122
6	0.006123	0.056502	0.037565	0.004151	0.021038	0.007201	0.005116	0.027682	0.040306	0.002297	0.020496	–0.083088
7	0.005911	0.062542	0.030551	0.004521	0.027484	0.018953	0.004063	0.021059	0.003008	0.001898	0.015732	–0.133604
8	0.010551	0.056892	0.115147	0.006127	0.040485	0.052529	0.003315	0.033345	–0.020529	0.002012	0.016467	–0.120735
9	0.005317	0.043450	0.030322	0.007111	0.036084	0.086211	0.003703	0.026313	–0.011295	0.001639	0.012481	–0.189164
10	0.004811	0.047673	0.017014	0.006032	0.034586	0.058757	0.004090	0.038234	0.002346	0.002336	0.018017	–0.092343
WIG	0.001188	0.013148	–0.213854									

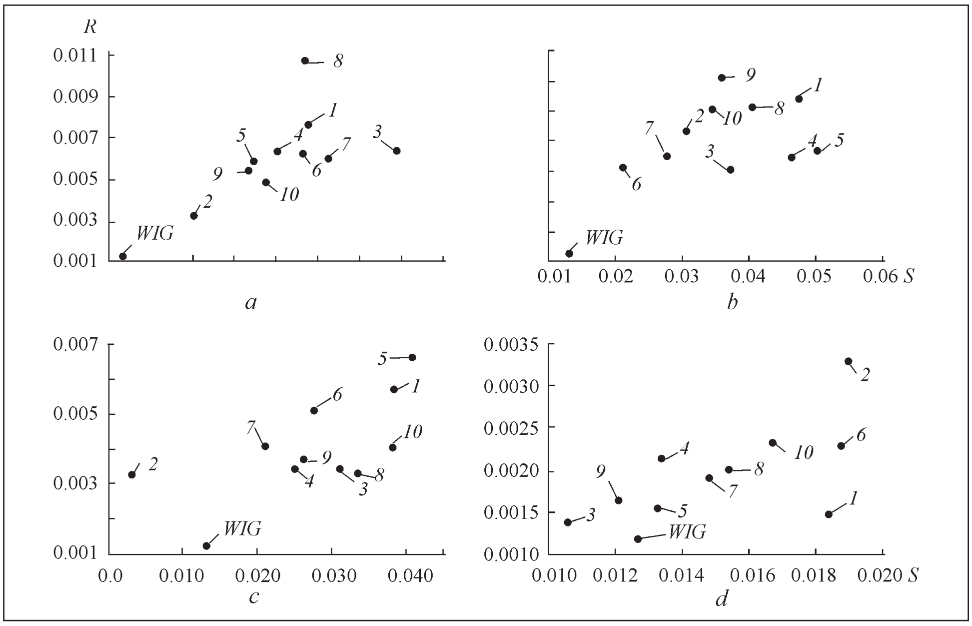

Fig. 2. Map of risk and income for two-element (*a*), three-element (*b*), five-element (*c*), ten-element (*d*) portfolios

of passage from one iteration to the other. In suggested solution all cells of cellular automata work collectively exchanging information between each other [3]. All cells of CA have one aim: to choose an effective portfolio, so they have to include information about composition of this portfolio and during iterations improve it. To be able to choose the portfolio, they should also have access to all input data. Every cell can be interpreted as an artificial investor choosing the most efficient portfolio. This artificial investor is working on his own choosing the best result but at the same time he exchanges all information with other artificial investors which are in CA arranged in grid [9]. At the beginning of simulation all cells of CA get randomly chosen portfolio. Then they calculate the basic characteristics of portfolio: rate of return and standard deviation. Then every cell communicates with its neighbors with Moore's or von Neumann's neighborhood (it is that moment when they work collectively). If in another cell there is a portfolio with better characteristics, then the cell gets this portfolio from its neighbor. If portfolios in adjoining cells have worse characteristics, then the cell stays with its original portfolio. At the next step all cells work individually, looking for a new, better portfolio. They get this portfolio as completely new, randomly chosen one. If the new portfolio is better, it stays as current one if not, then it is passed over. This is repeated until all cells in CA get the same effective portfolio.

Table includes characteristics of portfolios achieved in independent simulations. Table includes characteristics of two-, three-, five-, ten-element portfolios. Additionally characteristics of index WIG for two-element portfolios are shown in Table. These characteristics are rate of return R, S standard deviation and Sharpe measure of portfolio effectiveness SH_j.

All realizations of two-, three-, five- and ten-elements portfolios are shown in Fig. 2. Precise data for the maps of risk and income are in Table. On all maps of risk and income, there is also additionally index WIG as a mark of reference. Data of index WIG characteristics are in Table.

Conclusions. The conducted simulations confirmed that cellular automata can select portfolio on stock market, basing on classical Markowitz's model of portfolio analysis using collective work of cells. All portfolios obtained in simulation were different in every simulation. There were shown ten random portfolios gained in simulations for two-, three-, five- and ten-element portfolios.

On Fig. 2 we can see maps of risk and income for two-, three-, five- and ten-element portfolios. All of those portfolios have the rate of return higher than that calculated for index WIG and almost all of those portfolios have higher standard deviation, so they are more risky for investors. With this two characteristics it is hard to rate those portfolios that are results of collective work of cellular automata.

The third calculated characteristic of portfolios — which is SH_J — Sharpe measure of portfolio effectiveness — is helpful when we want to compare results of simulations. In all cases SH_J is higher than SH_J calculated for index WIG. It is a clear proof, that portfolios chosen by cellular automata are the effective ones.

Наведено моделювання окремих елементів портфеля інвестицій з застосуванням теорії аналізу інвестиційного портфеля Марковіца з використанням паралельного обчислювального середовища — кліткових автоматів. Наведено основи класичного аналізу портфеля цінних паперів і теорії кліткових автоматів та їхнього сумісного використання, а також структура процедури моделювання та його результати. Моделювання виконано у системі Борленд С++ Білдер.

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