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The Management of the Weather Risk on the Electric Energy Market

The aim of this study is to present a new approach for analyzing the effect of temperatures, wind speed and heating degree days index on the variability of the daily demand for electric energy in Silesian region. Specific nature of the time series used in this paper concerning the electric energy consumption and the weather variables, such as temperature or heating degree day index, requires application of a special class of models —ARFIMAX—GARCH (Generalized Autoregressive Conditional Heteroscedastic model).

Рассмотрен новый подход к анализу влияния температуры, скорости ветра и индекса степени дневного нагрева на изменчивость суточного потребления электроэнергии в районе Силезии. Особенности используемых временных рядов связаны с потреблением электроэнергии и погодными переменными, такими как температура или индекс степени дневного нагрева, обусловливают применение специального класса моделей — ARFIMAX—GARCH (обобщенная авторегрессионная условная гетероскедастическая модель).

K e y w o r d s: energy market, weather risk management, time series models, ARFIMAX—GARCH.

The weather has a significant impact on the activity of numerous economic entities. Estimates of the International Weather Risk Management Association [1] indicate that approximately 60% of all companies are directly or indirectly dependent on weather conditions. Among the sectors that are exposed to the weather risk to the greatest extent are: power engineering, agriculture, construction industry, transport, retail trade, entertainment and tourism industry, and municipal security personnel. From among the eight sectors listed above, only in case of construction industry and agriculture a physical reduction of the weather risk is possible to a certain degree.

Weather risk analysis and management, seen as a possibility to achieve better or worse financial results due to weather variability, allows one to make the financial results of companies independent of the weather conditions in question. It also enables better financial planning, primarily thanks to the possibility of improving the sales volume forecast.

It should be emphasized that since it is impossible to store electric energy, companies dealing with its sales must precisely determine the energy purchase volume. Thus all kinds of methods of the analysis and modeling of the electric energy demand, taking into account the impact of weather conditions, have become so important. Therefore, the research objective of the authors is to verify the usability of the ARFIMAX—GARCH models for the description of development of electric energy consumption in a selected South Poland region with reference to weather variables.

Weather risk management. Every enterprise bears the risk characteristic of the economic activity it pursues. Companies selling gas, fuel oil or electric energy are exposed to weather deviations from conditions considered standard. These companies, therefore, require all-year-long protection against such cases. In winter, protection against temperatures higher than the standard ones is needed, as higher temperatures cause reduction in the sales of energy-related products. On the contrary, in summer the same companies need a protection against temperatures lower than the standard ones, since in this case it also results in the reduction of the sales volume. Companies providing electric energy are increasingly seeking for tools of protection against extremely sweltering heats when the demand for electric energy (used for air conditioning) may exceed the planned level and force the suppliers to purchase energy on a short-term market in the period of the highest prices [2].

The following basic types of risk may be distinguished in the energy sales [3]:

risk related to the price of electric energy is considerably dependent on the energy demand, it is also connected with the fuel price (in Poland mostly coal) and generally with the costs of production. In those cases, commodity exchange offers futures and options, allowing one to protect the production and sales;

general systems and political risk related to the economic situation of the country, social policy (e.g. mining industry) etc.;

transaction and credit risk; each transaction carried out through the commodity exchange, significantly reduces this risk due to the use of guaranteed deposit mechanisms;

natural environment-related risk, which in the face of the evident impact of the power station, but also that of numerous energy consumers on the environment is currently a subject of public pressure and rigorous legal regulations (green and red certificates);

risk (and opportunity) related to various price areas which may result in arbitration transactions of the use of price differences between the bidders in various areas;

volume-related risk, i.e. risk related to the lack of production and consumption balance, which is considerably dependent on weather conditions, commodity exchange offers options, whose purchasers may exercise the right to purchase or resign from the right (contracts allowing the salespersons of electric energy to protect themselves on the balancing market);

refinancing-related risk; options offered by commodity exchange allow one to control the cash flow and to plan revenue and expenditure more securely;

currency risk, covered with currency derivatives;

interest rate risk, similarly to currency risk, covered with interest rate derivatives;

other types of risk in various operational activities, in particular, those related to managing the risk of changes in the supplier's prices, which can be controlled in a manner similar to one's sales control.

For this reason, one of the most essential issues in the risk management policy is to determine the requirements relating to the expected security of the functioning of the power engineering companies and to search for methods and instruments allowing one to move the weather risk out of the company.

The purpose of risk management is not to generate profits, but to reduce losses. No risk management system ensures a complete elimination of the losses. However every effort should be made to minimize the risk of their occurrence. Risk management process should be designed in a manner which excludes the possibility of bearing losses which may threaten the company existence. Recognition of the risks and implementation of effective tools of their reduction allows one to take efficient preventive actions in the face of emergency [4].

Weather risk management is not a novelty, as it has been known for many years, primarily in the context of protection against the after-effects of hurricanes, floods or droughts (catastrophic risks). All companies from the energy sector are characterized by a regularity of the amount of energy consumption in relation to the air temperature observed outside. The research indicates that, in case of electric energy, such weather parameters as wind speed, change of cloudiness or precipitation have also a considerable impact on the volume [5].

Due to demonopolization in power engineering, the long-term solutions offered by insurance companies have ceased to meet the needs of the energy sector. The companies of this branch are more exposed to short-term temperature variations. Every deviation from the mean in the period of summer peak or winter peak may have a direct impact on the reduction in profits. A search for financial solutions and abandonment of the traditional insurance of many years have become the basis for the development of a new market of weather risk management [6].

Weather instruments belong to a dynamically developing part of the capital market. It appears that financial derivative instruments, i.e. contracts, according to which the profits (or losses) depend on the value of basic securities, are to a large extent suitable for the protection against inappropriate weather. Basic securities of weather contracts are generally indices. Basic types of weather indices have been presented in Table 1.

Two indices are most frequently used with reference to power engineering: heating degree days (HDD) and cooling degree days (CDD). These indices count up a sum of deviations (negative and positive, respectively) of the average daily temperature T (calculated as e.g. a mean of the minimum and maximum daily temperature of the ith day) from a reference temperature, assumed as a parameter of the contract, in a fixed number of days n.

In the United States, the commonly accepted reference temperature is 65°F, in Europe it is respectively 18°C — this is the temperature recognized as a conventional border between the period in which air conditioners are used and the heating period:

$$\mathrm{HDD} = \sum_{i=1}^{n} \max(0{,}18°C - T_i), \ \ \mathrm{CDD} = \sum_{i=1}^{n} \max(0, T_i - 18°C). \qquad (1)$$

Weather derivative instruments are offered, amongst others, by the Chicago Mercantile Exchange. However, these are futures contracts and options for fu-

Table 1. **Basic types of weather indices and their application** [7]

Weather index	Weather parameter constituting the basis of the weather index	Application of weather index in a specific economic activity
HDD	Air temperature	Wide application in various economic sectors, particularly in power engineering, agriculture, construction industry, recreational activity, hotel industry, transport
CDD	" "	
Energy degree day (EDD)	" "	Power engineering
Cumulative average temperature (CAT)	" "	Wide application in various economic sectors, particularly in power engineering, agriculture, construction industry, recreational activity, hotel industry, transport, etc
Average temperature (AT)	" "	
Growing degree day (GDD)	" "	Agriculture
Chilling degree gay (CDH)	" "	
Frost day (FD)	" "	
Wind power index (WPI)	Wind speed	Wind power stations
Critical temperature day (CTD)	Air temperature	Wide application in various economic sectors, particularly in power engineering, agriculture, construction industry, recreational activity, hotel industry, transport; they are also used by e.g. local governments
Critical precipitation day (CPD)	Precipitation level (snow, rain)	

ture contracts for almost 30 various locations worldwide, including selected regions in Europe. Derivatives issued for weather indices are also placed in the over-the-counter sales which is characterised by a greater variety of concluded transactions due to the underlying instrument and a structure of the derivative instrument.

Majority of forward transactions concluded on the over the counter (OTC) market show a great innovativeness, as they are tailored to the individual needs of the customers who make use of them in the process of weather risk management.

Review of research in the scope of the impact of the climatic factors on the electric energy consumption. Identification and measurement of the weather risk is connected with the necessity to isolate from the observable electric energy consumption a part which is sensitive to the effects of climatic factors. While analysing historical time series relating to the electric energy demand, containing daily, weekly or monthly data from a dozen years or so, one may notice a strong long-term tendency, whose occurrence has been affected by social, demographic and economic factors. The factors might be as follows: population growth in a given region, technological progress, growth of industrialization, changes in the market share (measured by the number of customers) of the companies dealing with energy production and sales, electric energy market price and prices of alternative energy sources, monthly seasonality connected with a lower demand of the industry sector for electric energy in the summer holiday season.

The influence of the demographic factors on the energy demand may be eliminated by dividing the total energy consumption in a given region by the population, i.e. by considering the average energy consumption per capita [8, 9]. In order to isolate the electric energy demand which is sensitive to weather factors, various ways of data filtration may be used. In empirical research on modeling the above relation the following methods are used.

I. *Method of analytical trend function adjusted data* [10, 11]. Robinson suggested calculating the demand sensitive to the effects of weather factors as a fraction of a demand determined by economic and demographic factors, with specified weather conditions:

$$E_t = NE_t + NE_t w_p = NE_t(1+w_p) = NE_t W_p,$$

where E_t — total electricity load; w_p — weather factors (i.e. air temperature, wind speed); W_p — normalized weather sensitive load; NE_t — nonweather sensitive load, computed from analytic tendency function,

$$NE_t = \alpha_0 + \sum_{j=1}^{m} \alpha_j t^j + \xi_t.$$

On the other hand, Moral-Carcedo and Vicens-Otero modified Robinson's method by taking into account the seasonality occurring in the energy demand in the business sector, which is related to the reduction in or cessation of the production over the weekends and holiday seasons[1]:

$$E_t = \alpha_0 + \sum_{j=1}^{m} \alpha_j t^j + \delta I_{\text{Aug.},t} + \kappa WD_t + FE_t,$$

where $I_{\text{Aug.},t}$ is a dummy variable taking the value 1 for observations made in August, and the value 0 for others; introduced for the purpose of eliminating the effect of the reduced demand for energy in the business sector during the holiday season; WD_t — a variable that takes the value 1 for Wednesday, whereas for all other days of the week it equals a ratio of electric energy consumption on a given day in relation to the energy consumption on Wednesday (in a week in which the given observation was made); a variable describing the calendar effect of the «working day» on the energy demand; FE_t — electricity load, which is free of the influence of demographic and economic factors.

II. *Index-related equalization of the long-term tendencies which do not result from weather conditions in terms of the electric energy demand* [8, 9, 12]:

$$FE_t = \frac{E_{r,t}}{E_{\bar{r}} / \overline{E}},$$

where $E_{r,t}$ — monthly electricity consumption for month t in year r; $E_{\bar{r}}$ — monthly average electricity consumption in year r; $\overline{E}$ — monthly average electricity consumption in all analyzed period.

The suggested method of data filtration allows one to determine a monthly seasonal structure in the formation of the demand for the electric energy. It does not however eliminate all seasonal effects that do not result from climatic factors. Indices for daily data in relation to a weekly seasonal cycle may be constructed in a similar manner [12].

After the estimation of the electric energy demand which is sensitive to climatic factors, the strength and nature of the causa link between the weather variables and the electric energy consumption should be assessed. Previous research [13] has proved that among various weather variables, the air temperature has the most significant impact on the electric energy demand. Linear regression method may be used for the measurement of the weather risk:

$$FE_t = \phi_0 + \sum_{l=1}^{r} \phi_l x_{l_p t} + \varepsilon_t, \tag{2}$$

[1] Sailor D. J. and Munoz J. R. [8] estimated linear regression models for the whole period, as well as for the winter and summer period separately, due to the seasonality in the mean, typical of the climatology-related processes.

where $x_{l_p t}$ — the value of l's-weather index in the t period; ϕ_l — model parameter informing about the sensitivity of the electric energy demand to the effect of l's weather factor.

Those types of models were used in the research by [8] who made the electric energy consumption dependent, in a linear manner, on air temperature, wind speed and a relative air humidity as well as HDD and CDD indices[1]. Robinson, on the other hand, in addition to considering the linear regression function of the electric energy demand in relation to the air temperature, also considered the dependence of a non-linear nature of the form:

$$FE_t = \alpha_0 + \alpha_1 T + \alpha_2 \mathrm{T}^2 + \alpha_3 \mathrm{T}^3 + \zeta_t .$$

A summary of the research conducted by the above-mentioned authors is an index of elasticity of the electric energy demand in relation to a specified weather factor, suggested by [12]:

$$\eta_{\mathrm{HDD}} = \frac{\mathrm{HDD}}{E_{\mathrm{HDD}}} f'(\mathrm{HDD}),$$

where η_{HDD} is elasticity of energy consumption in relation to the HDD [2] index; $E_{\mathrm{HDD}} = f(\mathrm{HDD})$ — theoretical value of the function of the energy demand for a determined value of the HDD index.

A description of the seasonal structure (for seasonal cycles of various length) occurring in the mean of the climatic processes may be obtained by introducing (2) harmonic elements or dummy variables to the equation [3]:

$$FE_t = \phi_0 + \sum_{l=1}^{r} \phi_l x_{l_p t} + \sum_{i=1}^{6} \delta_i D_{it} + \sum_{k=1}^{11} \varphi_k M_{kt} + \varpi H_t + \varepsilon_t , \qquad (3)$$

where D_{it} — dummy variable for daily data ($D_{1t} = 1$ for Monday, $D_{1t} = 0$ for other days of the week); M_{it} — dummy variable for monthly data ($M_{1t} = 1$ for January, $M_{1t} = 0$ for other months of the year); H_t — dummy variable for holidays ($H_t = 1$ for holidays, $H_t = 0$ for other days of the year).

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[1] Sailor D. J. and Munoz J. R. [8] estimated linear regression models for the whole period, as well as for the winter and summer period separately, due to the seasonality in the mean, typical of the climatology-related processes.

[2] Indicator of the flexibility of the electric energy demand can be estimated for any weather factor which has a significant impact on the analysed phenomenon.

[3] Due to the occurrence of collinearity between the dummy variables and the constant term, the variables for Sunday and December were omitted in the equation (3). Lacking estimations of the parameters are calculated from applicable identities (compare to [14]).

Due to specific properties of meteorological time series, such as [1] trend and seasonality in the mean, long process memory, seasonality in variation, as well as the autoregressive conditional heteroscedastic (ARCH) effect, models of the ARFIMAX(P, D, Q, r) —FIGARCH (p, d, q) class dependencies between the weather variables and the energy demand [2]:

$$\phi(B)\,\Delta^D(E_t-\mu_t)=\theta(B)\,\varepsilon_t, \tag{4}$$

$$\mu_t=\phi_0+\sum_{l=1}^{r}\phi_l x_{l_p t}+\sum_{i=1}^{6}\delta_i D_{it}+\sum_{k=1}^{11}\varphi_k M_{kt}+\varpi H_t, \tag{5}$$

$$\varepsilon_t=z_t\sqrt{h_t},\;\; z_t\sim IID\,(0{,}1)\,, \tag{6}$$

$$\varphi(B)\,\Delta^d\varepsilon_t^2=\omega+\sum_{i=1}^{6}\delta_i D_{it}+\sum_{k=1}^{11}\varphi_k M_{kt}+\varpi H_t+[1-\beta(B)](\varepsilon_t^2-h_t), \tag{7}$$

where Δ^D — difference filter of D order ($-1 < D < 0.5$),

$$\Delta^D=(1-B)^D=\sum_{j=0}^{\infty}\binom{D}{j}(-1)^j B^j;$$

Δ^d — difference filter of d order ($0 < d < 1$),

$$\Delta^d=(1-B)^d=\sum_{s=0}^{\infty}\binom{d}{s}(-1)^s B^s;$$

B — lag operator; $B^s y_t=y_{t-s}$; $\phi(B)=1-\phi_1 B-\ldots-\phi_P B^P$, $\theta(B)=1+\theta_1 B+\ldots$ $\ldots+\theta_Q B^Q$, $\varphi(B)=1-\varphi_1 B-\ldots-\varphi_q B^q$, $\beta(B)=\beta_1 B+\ldots+\beta_p B^p$.

The assumed specification of equations (4), (5) allows one to describe the effect of the long and short memory in conditional mean process, seasonality of various length of the cycle in the time series of electric energy consumption which result from, among other things, changeability of weather conditions or different structure of energy consumers on working days and holidays. Moreover, relations (6), (7) allow one to consider in the process of modelling the electric energy demand the heteroscedasticity effect as well as stochastic seasonal-

[1] More information about modeling the meteorological data can be found in the paper of [15].

[2] In order to ensure the stationarity of the analysed time series models one assumes that the polynomial roots $\phi(B)=0$, $\varphi(B)=0$ are beyond the unit circle (compare to [5, 16]).

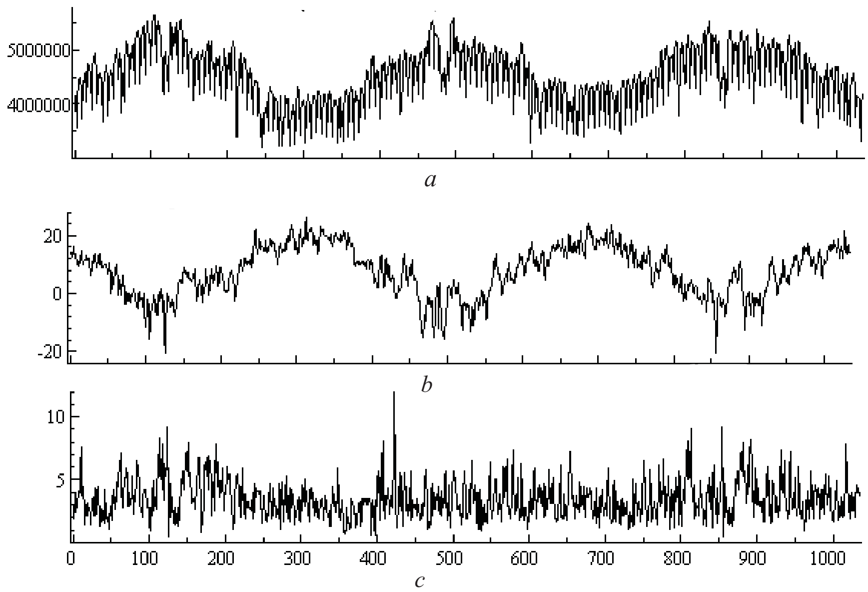

Fig. 1. Daily consumption of electric energy, kWh, (*a*), air temperature, °C, (*b*) and wind speed, m/s, (*c*) in the South Poland region in the period of 1000 days (time from September 1 2005 to June 30 2008)

ity, long memory of the conditional variance process and the «thick tails» effect of the distribution of random component [1]. In particular, when $D = 0$ and $d = 0$, the process of the electric energy consumption is described by the ARMAX(P, Q, r) — GARCH (p, q) models [17].

Statistical analysis of properties of the analysed time series. The following information was used in this study: information related to electric energy consumption (in kWh), air temperature (in °C) and wind speed (in m/s) in one of the South Poland regions. The initial time series connected with particular variables contained observations from a period of time from September 1, 2005 to June 30, 2008 with the reading frequency of 1 hour. Analyses and further calculations were based on the daily data (1034 observations) created from the initial database (the hour one) in the following way:

the energy consumption on each day constitutes a sum of 24 information from each hour of the day;

[1] While modeling high frequency data one assumes that scaled residuals $\{z_t\}$ are subject to «thick tail» distribution which includes generalized error distribution (GED), t-Student distribution and α-stable distribution.

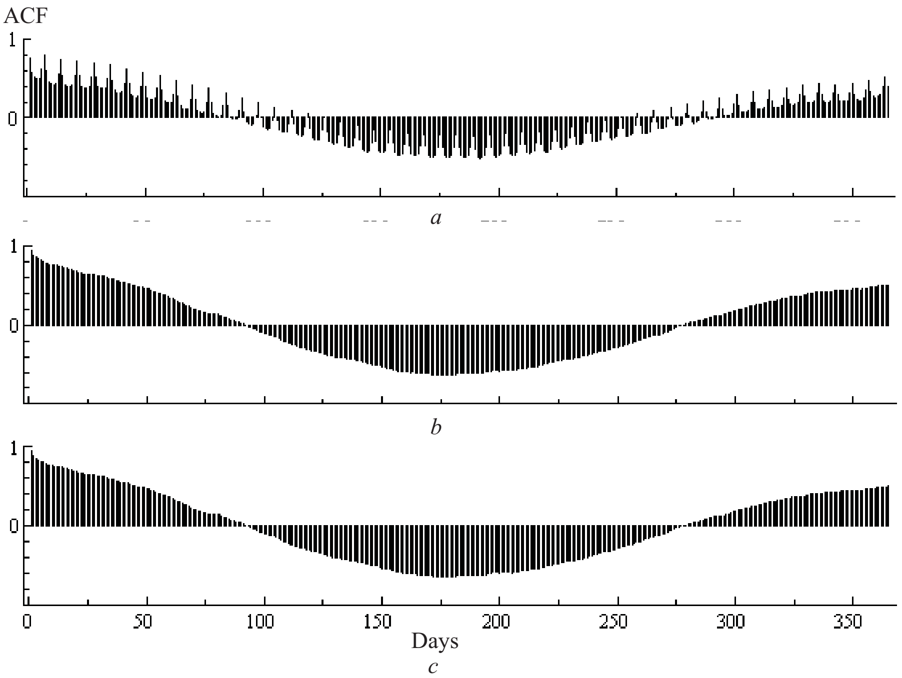

Fig. 2. ACF functional (delay k=365) for consumption of electric energy (*a*), ACF-HDD (*b*), air temperature (*c*)

air temperature is expressed as the average temperature calculated from 24 information from each hour of the day;

wind speed is expressed as the average speed calculated from 24 information from each hour of the day;

HDD (heating degree days) were calculated from the relation (1) where T_i is the average daily air temperature on the *i*th day;

CDD (cooling degrees days) were calculated from the relation (1) where T_i is the average daily air temperature on the *i*th day.

Fig. 1 proves the occurrence of similar periodic structure (for periodic cycles of various length) in the mean of analysed processes and the effect of volatility clustering. On figures 1—6 we have development of daily consumption of electric energy (in kWh) shown on the axes abscissas.

In order to analyse the properties of electric energy consumption distribution and particular weather variables distribution, the basic descriptive statistics were identified (Table 2).

While analysing the results presented in Table 1 one can notice that the relation between the standard deviation and the mean for each weather variable is very high, especially in case of air temperature. This proves that the daily volatitlity of climate factors and thus the weather risk to which a power company is exposed, is very big. Values of skewness and kurtosis as well as the conducted Jarque-Bera normality test indicate significant differences between empirical distribution of the analysed variables and the normal distribution[1]. Moreover, statistics in the ADF test examining the occurrence of unit root for each variable are significant at the significance level $\alpha = 0.01$, which allows one to reject the zero hypothesis in favour of the alternative hypothesis, so the analysed time series are stationary in variance.

A characteristic phenomenon in the electric energy market is the periodicity (at various cycle lengths: daily, weekly, yearly) in the development of the electric energy demand. Meteorological data have a similar property (compare to [15]). For the electric energy demand and the weather variables considered in this paper, the diagrams of ACF functions were prepared in order to identify significant autocorrelation dependencies and to assess the occurrence of the effect of the long process memory (compare to Fig. 2). The significance of autocor-

Table 2. **Descriptive statistics for the analysed variables**

Statistics	Load	Temperature	Wind	HDD	CDD
Average	4532400	7.5579	3.4495	10.670	0.22747
Standard deviation	504420	8.4172	1.5052	8.0799	0.84236
Skewness	–0.23758	–0.37406	0.90855	0.53073	4.5151
Kurtosis	–0.46885	–0.39923	1.3241	–0.39649	22.787
Minimum	3204400	–20.208	0.45833	0.00000	0.00000
Maximum	5657800	26.042	11.917	38.208	8.0417
L– B(36)	10188.3	19372.2	539.61	19103.6	3401.58
L– B^2(36)	10402.1	16759.9	498.929	12120.5	1334.8
J– B	19.198^{**}	30.980^{**}	217.79^{**}	55.316^{**}	25883^{**}
ADF	-12.09^{**}	-5.278^{**}	-18.19^{**}	-5.257^{**}	-13.22^{**}

Source: own calculations in PcGive package; symbol ** indicates the significance of the result at the 0.01 level.

[1]Descriptive statistics for the CDD variable take atypical values due to relatively small number of non-zero values of this index. It is advisable to conduct subsequent research related to another way of calculating the HDD and CDD index for Poland — for temperatures lower than the assumed 18 °C threshold value of the air temperature within the definition of this index. Moreover, the research conducted by other authors [9] prove that in the climatic zone in which Poland is located, the heating season effect in the development of electric energy demand is more visible than the effect connected with the use of air-conditioning devices.

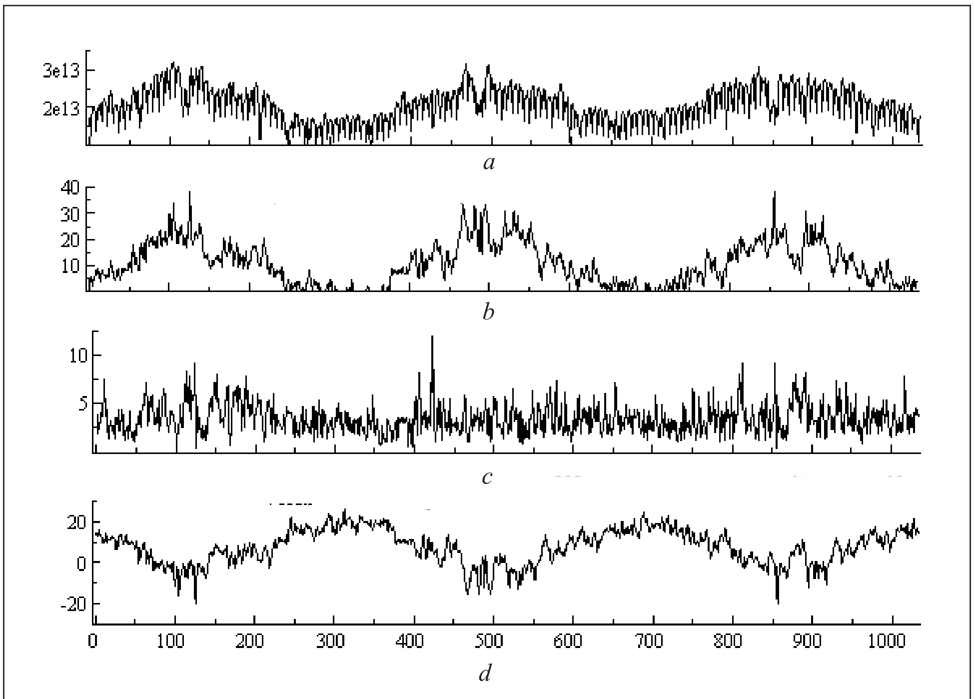

Fig. 3. Daily variations of electrical load, kWh, (*a*), HDD index, °C, (*b*), wind speed, m/s, (*c*) air temperature, °C, (*d*) in a South Poland region in the period of 1000 days time from September 1, 2005 to June 30, 2008

relation factors to the level 36 was examined with the use of Ljung-Box test (Table 2). All determined test statistics were significant at the significance level 0.001 [18].

Another property characteristic of the considered time series is the notion of volatility clustering, which is indicated by strong autocorrelation of squares of values of a given series (compare to Fig. 3) and the significance of the statistics in the McLeod and Li test (compare to the values $L\text{-}B^2(36)$ in Table 2). On the basis of plots presenting the daily volatility of the energy demand and the HDD index one can also formulate an assumption about the occurrence of a similar periodicity in the volatility of these time series.

In the next stage of the analysis, the Pearson correlation coefficients were assessed in order to point out which of the initially proposed weather variables has a significant impact on the development of the electric energy demand and what is the direction of such correlation (compare to Table 3).

The temperature and HDD have the biggest impact on the energy consumption, whereas the wind speed and CDD were meaningless. Thus only the temper-

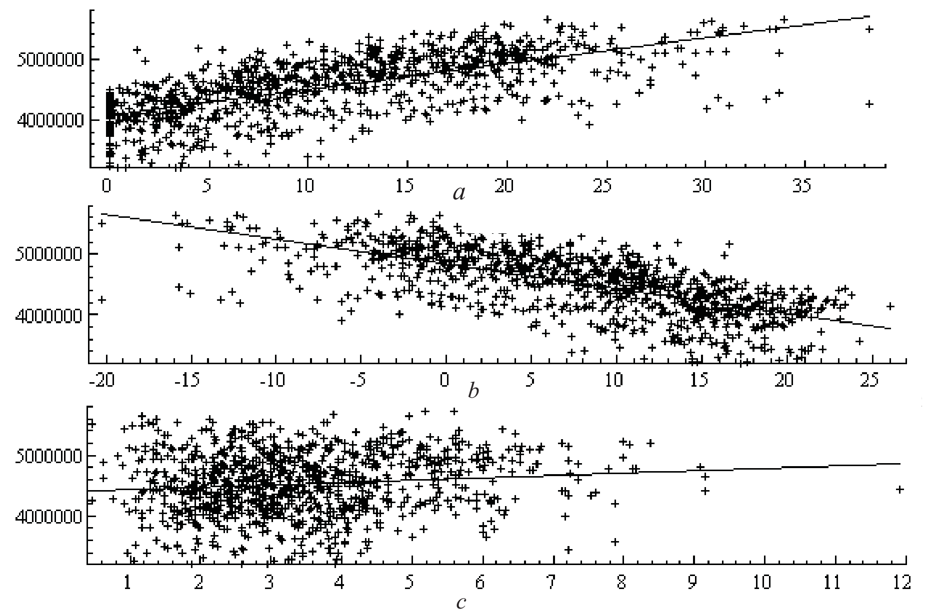

Fig. 4. Scatter plots of electricity load vs HDD, °C, (*a*), electricity load vs air temperature, °C, (*b*), electricity load vs wind speed, m/s, (*c*)

ature and HDD were included in further considerations and in the constructed econometric models. Similar conclusions can be drawn while analysing the scatter plots presented in Fig. 4.

It is worth mentioning that the direction of the correlation between the HDD index and electric energy consumption is positive, whereas the variable energy consumption and air temperature are correlated negatively.

An analysis of the properties of time series conducted in this part of the paper constitutes an important stage of econometric modeling, because identification of a regularity in the development of the examined variables brings effects in the form of a proper specification of equations of conditional mean and conditional variance of

Table 3. **Value of the Pearson correlation coefficients between analyzed variables**

Pearson correlation coefficient	Load
HDD	0.6737
Temperature	–0.6740
Wind	0.1235
CDD	–0.2727

the process of energy demand. In other words it enabled to construct a consistent econometric model according to the concept of Z. Zieliński.

Estimation and verification of electrical energy consumption models. Issues connected with the modeling of electric energy demand include three areas of searching for a proper specification of initial model of ARFIMAX(*P*, *D*, *Q*, *r*) — FIGARCH(*p*, *d*, *q*) class defined by the relations (4) — (7), namely:

specification of the deterministic part of the model connected with the conditional mean of the process $\mu_t = E(E_t \mid \Phi_{t-1})$;

specification of the stochastic part of the model including

equation of conditional variance of the process $h_t = \mathrm{var}(E_t \mid \Phi_{t-1})$;

the choice of the form of the density function with zero mean and unit variance for innovation process $z_t : z_t \sim iid\, D(0.1)$[1]

At the first stage of the research the authors tried to indicate a deterministic trend connected with the impact of the demographic, economic and social factors on the electric energy demand. The polynomial trend of the third degree was selected among the estimated various models of the development trend for daily electricity load, taking into account the value of the coefficient of determination and the significance of the assessments of the structural parameters of the models. Due to the occurrence of a strong linear correlation between the load and the HDD index, this weather variable was included as the explaining variable in the equation of conditional mean of the process of the energy demand.

Additionally, the classical equation of linear regression was extended with dummy variables which aim at describing the weekly periodicity, annual seasonality and the impact of holidays on the development of the electric energy demand. Finally, the following specification of the first equation of the I model of energy consumption was proposed:

$$E_t = \phi_0 + \phi_1 t + \phi_2 t^2 + \phi_3 t^3 + \lambda_1 \mathrm{HDD}_t + \sum_{i=1}^{6} \delta_i D_{it} + \sum_{j=2}^{12} \varphi_j M_{jt} +$$

$$+ \kappa_1 S_t + \kappa_2 S_{t-1} + \kappa_3 S_{t+1} + u_t, \qquad (8)$$

where t — time variable; δ_i — parameter measuring the periodic effect in a given phase of weekly cycle (day); φ_j — parameter measuring the seasonal effect in a given phase of annual cycle (month); D_{it} — dummy variable which equals 1 in day i and 0 otherwise; M_{jt} — dummy variable which equals 1 in month j and 0 otherwise; S_t — dummy variable which equals 1 on a holiday and 0 otherwise; S_{t-1} — dummy variable which equals 1 on the day preceding the

[1] Set Φ_{t-1} includes all information available till the t–1 moment; $D(0.1)$ symbol usually means in practice normal distribution, *t*-Student distribution or GED distribution.

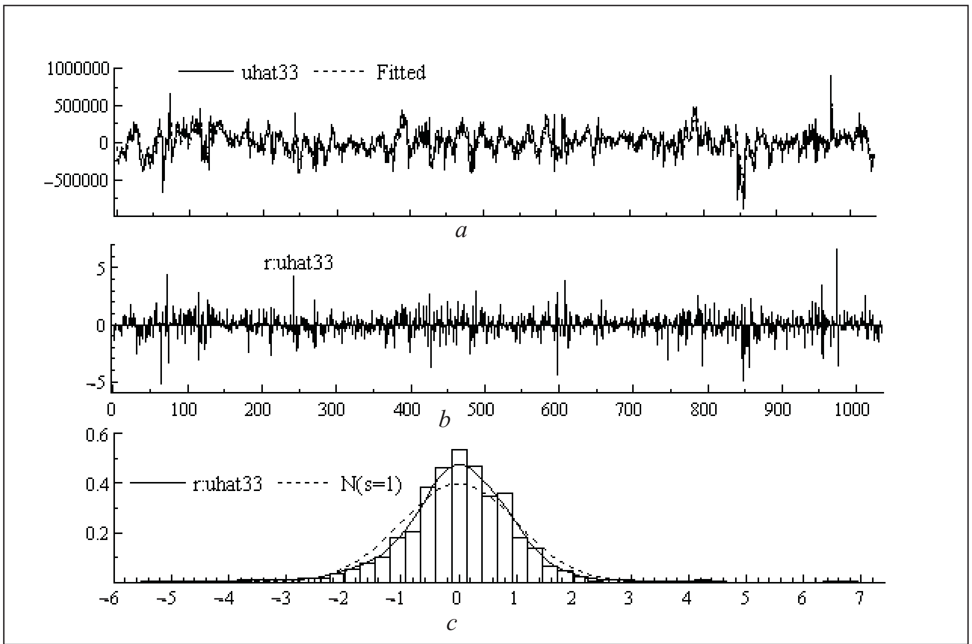

Fig. 5. The diagram of empirical and theoretical properties for the ARMA (1.1) model (*a*), scaled residuals of the ARMA (1.1) model (*b*) and the function of the density of scaled residuals of the ARMA (1.1) model (*c*)

holiday and 0 otherwise; S_{t+1} — dummy variable which equals 1 on the day following the holiday and 0 otherwise.

Due to the fact that the residuals u_t in the equation (8) show strong autocorrelation and the occurrence of the ARCH effect, the ARMA(P, Q) — GARCH(p, q) models was used in their modeling. The order of the ARMA(P, Q) model was chosen on the basis of the information criteria: Akaike's (AIC), Bayes extension Akaike's (BIC), Schwartz's (SC), Hannan — Quinn's (HQ), which minimize the volatility of the remaining element of the model and the significance of the model parameters [19]. For the time series analysed in this paper it was assumed that $P = Q = 1$: [1]

$$u_t = \theta_1 u_{t-1} + \varepsilon_t + \psi_1 \varepsilon_{t-1},$$

where ε_t — random parameter in the ARMA (1.1) model.

[1] The authors also assessed the ARFIMA (P, D, Q) model for the residuals of the model (8), however the D parameter responsible for the occurrence of the effect of long memory process to be statistically insignificant.

Table 4. **Results of estimation of parameters of electric energy consumption models** (8)

Parameter	Equation (1)		Equation (2)		Equation (3)	
	Coefficient	*p*-value	Coefficient	*p*-value	Coefficient	*p*-value
ϕ_0	4933810	0.0000 ***	4933810	0.0000 ***	4841330	0.0000 ***
ϕ_1	–1147.36	0.0000 ***	–1147.36	0.0000 ***	–1179.82	0.0000 ***
ϕ_2	1.99800	0.0000 ***	1.99800	0.0000 ***	2.04483	0.0000 ***
ϕ_3	–0.000803	0.0080 ***	–0.000803	0.0080 ***	–0.000822	0.0052***
λ_1	19004.3	0.0000 ***	19004.3	0.0000 ***	6935.10	0.0016***
λ_2					9098.65	0.0022***
λ_3					7989.53	0.0003***
δ_1	–214921	0.0000 ***	–214921	0.0000 ***	–217467	0.0000 ***
δ_2	–760854	0.0000 ***	–760854	0.0000 ***	–765886	0.0000 ***
δ_3	–205569	0.0000 ***	–205569	0.0000 ***	–215784	0.0000 ***
δ_4	47048.1	0.0227**	47048.1	0.0227**	–57524.7	0.0040***
δ_5	–9088.51	0.6604	–9088.51	0.6604	–13289.0	0.5060
δ_6	1844.44	0.9290	1844.44	0.9290	438.427	0.9825
φ_2	–165255	0.0000 ***	–165255	0.0000 ***	–153460	0.0000 ***
φ_3	–168648	0.0000 ***	–168648	0.0000 ***	–143445	0.0000 ***
φ_4	–270178	0.0000 ***	–270178	0.0000 ***	–221569	0.0000 ***
φ_5	–555488	0.0000 ***	–555488	0.0000 ***	–477985	0.0000 ***
φ_6	–674895	0.0000 ***	–674895	0.0000 ***	–584053	0.0000 ***
φ_7	–561600	0.0000 ***	–561600	0.0000 ***	–465870	0.0000 ***
φ_8	–551100	0.0000 ***	–551100	0.0000 ***	–449232	0.0000 ***
φ_9	–410895	0,0000 ***	–410895	0,0000 ***	–327723	0,0000 ***
φ_{10}	–225479	0.0000 ***	–225479	0,0000 ***	–168370	0,0000 ***
φ_{11}	–66476.4	0.0146 **	–66476.4	0,0146 **	–33547,1	0,2071
φ_{12}	36979.9	0.1572	36979.9	0,1572	36569,6	0,1473
κ_1	–789853	0.0000 ***	–789853	0,0000 ***	–778551	0,0000 ***
κ_2	–182665	0.0000 ***	–182665	0,0000 ***	–177341	0,0000 ***
κ_3	–359959	0.0000 ***	–359959	0,0000 ***	–372871	0,0000 ***
λ_{r2}			15825.0	0,0000 ***		
λ_{r3}			–4021.01	0,0000 ***		
λ_{r4}			9686.06	0,0000 ***		
λ_{r5}			20713.3	0,0000 ***		
λ_{r6}			2798.30	0,0000 ***		
λ_{r7}			–14294.1	0,0000 ***		
ω_1			–81339.7	0,0000 ***		
ϖ			249.234	0,0000 ***		

Table 4 (continued)

Parameter	Equation (1)		Equation (2)		Equation (3)	
	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value
θ_1	0.730415	0.0000 ***			0.739969	0.0000 ***
ψ_1	−0.158129	0.0000 ***			−1069.63	0.0000 ***
α_0	2.62155	0.0000 ***	2626830000	0.0000 ***	6030390000	0.0000 ***
α_1	0.135355	0.0000 ***	0.134292	0.0000 ***	0.238152	0.0000 ***
β_1	0.721565	0.0000 ***	0.726271	0.0000 ***	0.397007	0.0000 ***
(t –Student) df	4.94570	0.0000 ***	4.43292	0.0000 ***	5.65441	0.0000 ***
Adjusted R^2	0.877352		0.877352		0.885469	
HMSE	8.13597		10.1826		9.18767	
AIC	26.279748		26.2787753		26.3823341	
$\alpha + \beta$	0.85692		0.860563		0.635159	
Effect ARCH	0.52449 [0.5920]		0.14178 [0.8678]		0.26156 [0.7699]	
L–B (32)	33.742 [0.3833]		33.272 [0.4051]		29.064 [0.6159]	

Source: own calculation in PcGive; p-value in brackets.

Since the conducted diagnostic tests for the random component in the ARMA (1.1) model indicate a lack of its autocorrelation and occurrence of the ARCH effect, in the next phase the GARCH(1.1) model was proposed with the t-Student distribution to the description of the phenomenon of volatility clustering and thick tails of empirical distribution (Fig. 5):

$$\varepsilon_t = z_t \sqrt{h_t},\ z_t \sim IID(0.1),$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_t^2 + \beta_1 h_{t-1}, \tag{9}$$

where $\alpha_0 \geq 0, \alpha_1 > 0, \beta_1 > 0.$

Assessments of model I parameters and results of diagnostic tests related to properties of the random component is placed in Table 4. While assessing the adjustment of the model to the empirical data, the following criteria were taken into account: the significance of the model parameters verified by the t-Student test, determination coefficient (R^2), heteroscedasticity — adjusted mean square error (HMSE), Akaike's criterion (AIC).

Model II constitutes a modification of model I by taking into account the periodic structure of the random component of the ARMA(1.1) model and introducing in the equation a conditional variance (9) of additional regressor in the form of weather variable HDD: [1]

$$\varepsilon_t = \sum_{i=2}^{7} \lambda_{ri} D_{ri} + \omega_r S_r + z_t \sqrt{h_t}, \;\; z_t \sim IID(0.1)\,, \tag{10}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_t^2 + \beta_1 h_{t-1} + \varpi \mathrm{HDD}_t, \tag{11}$$

where D_{ri} — the dummy variable which describes the periodicity of the random component with weekly cycle (e. g. $D_{r2} = 1$ if r is Tuesday and $D_{r2} = 0$ otherwise); S_r — dummy variable which describes the effect of holidays in the development of the random component ($S_r = 1$ if r is a holiday, $S_r = 0$ otherwise), $\alpha_0 \geq 0, \alpha_1 > 0, \beta_1 > 0, \varpi > 0.$

Model III is the modification of model I consisting in the introduction of the second order delays of the HDD index, while assuming the impact of air temperature from previous days on the development of electric energy consumption:

$$E_t = \phi_0 + \phi_1 t + \phi_2 t^2 + \phi_3 t^3 + \sum_{k=0}^{2} \lambda_{k+1} \mathrm{HDD}_{t-k} + \sum_{i=1}^{6} \delta_i D_{it} + \sum_{j=2}^{12} \varphi_j M_{jt} +$$

$$+ \kappa_1 S_t + \kappa_2 S_{t-1} + \kappa_3 S_{t+1} + u_{2t},$$

$$u_{2t} = \theta_1 u_{2t-1} + \varepsilon_{2t} + \psi_1 \varepsilon_{2t-1},$$

$$\varepsilon_{2t} = z_{2t} \sqrt{h_t},\; z_{2t} \sim IID(0.1)\,,$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{2t}^2 + \beta_1 h_{t-1},$$

where $\alpha_0 \geq 0, \alpha_1 > 0, \beta_1 > 0.$

While analyzing the results placed in Table 4, especially while taking into account the considered comparative criteria and tests verifying the properties of scaled residuals, one can indicate that model I described in the best way the development of electric energy consumption in the South Poland region.

Introduction of the GARCH structure with the conditional t-Student distribution (characterized by tails thicker than in the normal distribution) resulted each time in the elimination of volatility clustering effect which was present in the residuals of the ARMA model. By including dummy variables in the equation (10) and the weather variable in the equation of conditional volatility (11) it was possible to model the periodicity of the volatility of the random component,

[1] The daily volatility of the energy consumption and of the HDD index (compare to Fig. 3) is characterized by periodicity, which can be explained by the weekday effect, holiday effect orannual seasonality. Seasonality in volatility for time series of energy consumption can also develop under the influence of the seasonality in the volatility of air temperature hence the idea of including in the stochastic part of the model II the dummy variables and the weather factor – HDD index.

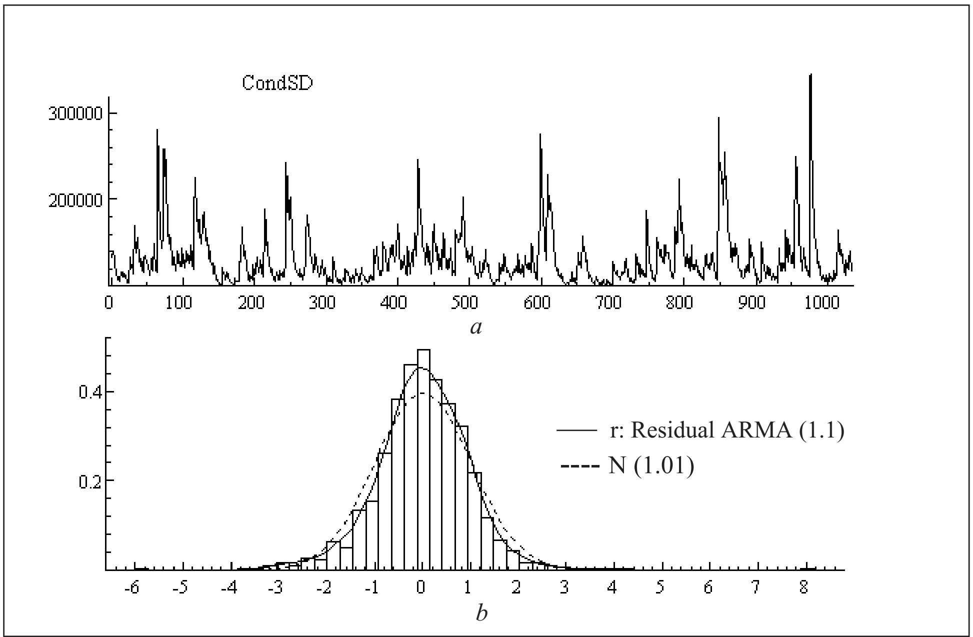

Fig. 6. Conditional volatility (*a*) and the function of the density of empirical distribution of standardized residuals (*b*) of the GARCH (1.1) model with *t*-Student distribution

however it did not cause any significant improvement of the properties of model II in relation to the specification I or III from the perspective of AIC, HMSE criterion. Also the introduction of the autoregressive structure for the HDD weather variable in model III did not bring any effects in the form of significant reduction of the values of Akaike criterion or HMSE criterion.

The extension of ARMA-GARCH models with the cause and effect relation which occurs between air temperature and energy consumption made it possible to assess the sensitivity of the electric energy demand in respect of the weather factor, such as the HDD index. Assessments of parameters at dummy variables which model the periodicity of a weekly cycle in the electric energy demand, except for Friday and Saturday, are statistically significant and indicate that the days with the greatest energy consumption in a given region of Poland are: Thursday, Saturday and Sunday. All parameters at dummy variables which relate to holidays and days preceding and following holidays are statistically significant and negative, which proves that the electric energy consumption is lower in the holiday period than on weekdays. In case of dummy variables related to monthly seasonal effects, all parameter assessments are significant and negative, except for coefficients related to December. Such results allow one to conclude that the electric energy demand in the analyzed region of Poland is the greatest in January and December. It should be also emphasized that the trend, dummy variables modeling calendar effects and seasonal fluctuations as well as the weather index explain the variability of the electric energy consumption by 87.7 %.

Moreover, the assessment of the ARMA-GARCH models on the basis of the residuals of the model (8) made it possible to assess the conditional volatility of the process of electric energy demand. With the use of conditional volatility one can measure the volatility of the electric energy demand, i.e. risk related to unpredictable change in the energy consumption under the influence of e.g. changing weather conditions. Periods which can be seen in Fig. 6 and correspond to a high value of conditional volatility should be understood as the periods in which the energy consumption was significantly changing under the influence of unpredictable factors (i.e. all the variables omitted in the equation (8)).

Conclusions. Demonopolization in power industry has forced the power engineering companies to prepare and implement internal procedures of managing the risk involved in the energy sales. Such activities do not serve the purpose of generating income but rather the purpose of limiting potential losses resulting from other demand for the electric energy than it was planned by the company. Companies of this industry branch are more exposed to short-term fluctuations of the weather conditions, since each deviation of the given weather factor from conditions considered normal may have a direct impact on the energy consumption by final consumers and may thus result in the deterioration of their financial results. That is why the companies are increasingly using weather derivative instruments to protect themselves against weather risk consequences, since such actions allow them to make their financial results independent of the change in weather conditions. The decisions concerning the security level are taken on the basis of, among others, breakdown of the costs involved in the hedge transaction, using weather derivative instruments as well as current and forecast situation in the energy market, especially information on the amount of energy demand in the future.

While reviewing the research on modeling the electric energy demand, one can notice a lot of interest, both on the part of scientists and practitioners in analysing the sensitivity of the energy demand to weather factors. Analyses of the impact of selected weather factors on the electric energy consumption conducted by the authors, were related only to a selected region of the South Poland. Unfortunately, in Polish conditions, gaining access to this type of data involves high purchase costs, whereas in many countries databases concerning weather variables are made available free of charge on the weather stations' websites or state units responsible for the collection of this type of data.

It should be emphasized that from the analysed weather variables only the air temperature and the HDD index determined on its basis have proved to have a substantial impact on the level of energy consumption. In subsequent research studies the authors will verify the impact of the remaining weather variables, i.e. clouds or precipitation on the electric energy consumption and will perform modeling for the HDD index defined in relation to another threshold temperature. Specific nature of the time series used in this paper concerning the electric energy consumption and the weather variables, requires application of a special class of models — ARMAX—GARCH. Dummy variables were applied in order to eliminate the deterministic seasonality. However in further research studies, alternative methods of elimination of periodicity should be applied (analysis of periodicity indices, harmonic analysis, differentiation method with an adequate length of delay, rolling volatility). In order to confirm the research results presented in this paper, the analysis should be extended to cover the area of the whole country and the data frequency should be increased (e.g. by introducing hourly data) due to the specific nature of the energy sales. One can also take into consideration the distinction of hourly profiles in the energy sales and create models for each hour of the day separately; such an approach renders unnecessary the consideration of models with the double periodic component: daily and weekly.

While extending the analysis of the impact of weather factors on the functioning of a power industry branch company, one should apply the Value at Risk methodology to measure the weather risk. Such an approach will make companies dealing with the energy production and sales aware of the potential losses they may suffer as a result of unexpected change of weather factors.

Розглянуто новий підхід до аналізу впливу температури, швидкості вітру та індексу ступеню денного нагріву на змінність добової потреби в електроенергії в районі Сілезії. Властивості використаних часових рядів, пов’язані з споживанням електроенергії та погодними змінними, такими як температура чи індекс ступеню денного нагріву, обумовлюють застосування спеціального класу моделей — ARFIMAX—GARCH (узагальнена авторегресійна умовна гетероскедастична модель).

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