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Block-Parallel Chaotic Algorithms for Image Reconstruction

(Recommended by Prof. E. Dshalalow)

The paper is devoted to the elaboration and implementation of block-parallel asynchronous algorithms for computer tomography. The numerical reconstruction algorithms and numerical simulation results for a number of modeling objects and some particular systems of reconstruction are presented.

Разработаны и выполнены блочно-параллельные алгоритмы компьютерной томографии. Представлены численные алгоритмы восстановления и результаты численного моделирования для ряда тестовых задач и некоторых частных случаев систем реконструкции сбора данных.

K e y w o r d s: computer tomography, incomplete projection data, asynchronous algorithms, computer reconstruction, block-parallel algorithms.

Introduction. Some parallel implementations of iterative algebraic algorithms for image reconstruction for some particular reconstruction schemes which arise in some problems of engineering geophysics and mineral industry are considered in the paper. In such computing structure each elementary processor executes its independent calculations by means of the same simple algorithms connected with a set of corresponding equations.

It is assumed that each processor executes its calculations with its own pace and the communication channels are allowed to deliver messages out of order. In this case this results in the chaotic character of interactions in such computer parallel structure (CPS) which corresponds to some chaotic iterative algorithm. This algorithm realized on such CPS is based on the asynchronous methods [1—3].

In order to reduce the computation time and memory space of the computer other algebraic algorithms were proposed which allow their parallelization and may be realized on the fast massively parallel computing systems (MPCS) consisting of elementary processors and a central processor [4—6].

We represent in this paper some kinds of the block-parallel asynchronous algorithms for image reconstruction which are a certain generalization of parallel chaotic iteration methods considered by Bru, Elsner and Neumann [7].

Numerical simulation of the solving the problems of image reconstruction from incomplete projection data for some modeling objects, comparing the errors evaluations and rate of convergence of these algorithms are presented. It is shown that for some choice of parameters one can obtain a good quality of reconstruction with these algorithms, and that these algorithms have much higher rate of convergence in comparison with the corresponding synchronous algorithms.

Block-parallel iterative algorithms for image reconstruction. Certain parallel and block-iterative algorithms are used in the paper, some of which were considered in papers [8—10], for solving the system of linear equations

$$\mathbf{A}\cdot\mathbf{x}=\mathbf{p}, \tag{1}$$

where $\mathbf{A}=(a_{ij})\in\mathbf{R}^{m,n}$ is the matrix of coefficients; $\mathbf{x}=(x_1,x_2,...,x_n)^T\in\mathbf{R}^n$ is the image vector; $\mathbf{p}=(p_1,p_2,...,p_m)^T\in\mathbf{R}^m$ is the measurement vector of projection data; and system of linear inequalities

$$\mathbf{p}-\mathbf{e}\le\mathbf{A}\cdot\mathbf{x}\le\mathbf{p}+\mathbf{e}, \tag{2}$$

where $\mathbf{e}=\{\varepsilon_1,\varepsilon_2,...,\varepsilon_m\}$ is a non-negative vector.

Denote by

$$\mathbf{P}_i(\mathbf{x})=\mathbf{x}-\frac{((\mathbf{a}^i,\mathbf{x})-p_i-\varepsilon_i)^+-(p_i-\varepsilon_i-(\mathbf{a}^i,\mathbf{x}))^+}{\left\|\mathbf{a}^i\right\|^2}\mathbf{a}^i, \tag{3}$$

where

$$s^+=\begin{cases} s, & \text{if } s\ge 0;\\ 0, & \text{otherwise}\end{cases}$$

and

$$\mathbf{P}_i^{\omega}=(1-\omega)\mathbf{I}+\omega\mathbf{P}_i, \tag{4}$$

where $\mathbf{a}^i$ is i-th row of a matrix $\mathbf{A}$, ω is a relaxation parameter.

A l g o r i t h m 1 (PART).

1. $\mathbf{x}^{(0)}\in\mathbf{R}^n$ is an arbitrary vector.

2. The $k+1$-th iteration is calculated in accordance with such a scheme:

$$y^{k,i}=\mathbf{P}_i^{\omega_k}\mathbf{x}^{(k)}\quad (i=1,2,...,m), \tag{5}$$

$$\mathbf{x}^{(k+1)} = \mathbf{C}\sum_{i=1}^{m}\mathbf{B}_i^k \mathbf{y}^{k,i}, \tag{6}$$

where $\mathbf{P}_i^{\omega_k}$ are operators defined by (3) and (4), ω_k are relaxation parameters, $\mathbf{C}$ is a constrained operator and $\mathbf{B}_i^k$ are matrices of dimension $n\times n$ with real nonnegative elements and

$$\sum_{i=1}^{m}\mathbf{B}_i^k = \mathbf{E}, \quad \sum_{i=1}^{m}\left\|\mathbf{B}_i^k\right\| \le 1,$$

for all $k\in N$, where $\mathbf{E}$ is the unit matrix of dimension $n\times n$.

The parallel implementation of this algorithm may be organized as follows:

begin
  $\mathbf{x}^{(0)}$=initial
  for $k = 0, 1, \ldots$ until convergence
  do
    for i-th processor, $i = 1$ to m
    do
      $y^i = \mathbf{P}_i^{\omega_k}\mathbf{x}^{(k)}$
    enddo
      $\mathbf{x}^{(k+1)} = \mathbf{C}\sum_{i=1}^{m}\mathbf{B}_i^k \mathbf{y}^i$
  enddo
end

Let $\mathbf{B}_i^k = (\gamma_{jj}^i)_{j=1}^n$ be a diagonal matrix with elements $0<\gamma_{jj}^i<1$. If $\gamma_{jj}^i = \gamma_i$ for each $j\in J$, $i\in I$, $\mathbf{C}=\mathbf{I}$, then there results the Cimmino algorithm [11].

The sufficient conditions of convergence of algorithm 1 are given by the following theorem.

Theorem 1. If system (2) is consistent and $0<\omega_k<2$, then the sequence $\{\mathbf{x}^{(k)}\}_{k=1}^{\infty}$ defined by algorithm 1 converges to some solution of the system (2).

For many practical applications $\mathbf{x}\ge 0$, the elements of a matrix $\mathbf{A}=(a_{ij})$ are nonnegative real numbers and $p_i>0$ for all $i\in I$. In this case the following parallel multiplicative algorithm is proposed for solving a system of linear inequalities (2).

A l g o r i t h m 2 (MARTP).

1. $\mathbf{x}^{(0)}\in\mathbf{R}^n$ and $\mathbf{x}^{(0)}>0$.
2. The $k+1$-th iteration is calculated in accordance with such a scheme:

$$x_j^{(k+1)} := x_j^{(k)}\prod_{i=1}^{m} y_j^{k,i}, \tag{7}$$

where

$$y_j^{k,i} := \left(\frac{p_i}{(\mathbf{a}^i, \mathbf{x}^{(k)})} \right)^{\gamma_{ij}^k a_{ij}} \qquad (8)$$

$(i = 1, 2, ..., m;\ j = 1, 2, ..., n)$, γ_{ij}^k are positive real numbers such that

$$0 < \sum_{i=1}^{m} a_{ij} \gamma_{ij}^k \leq 1$$

for every j, k.

The parallel realization of this algorithm may be given in such a form:

begin
$x_j^{(0)}$ = initial
for $k = 0,1, ...$ until convergence
do
for i-th processor, $i = 1$ to m
do
for j-th point, $j = 1$ to n
do

$$y_j^{k,i} := \left(\frac{p_i}{(\mathbf{a}^i, \mathbf{x}^{(k)})} \right)^{\gamma_{ij}^k a_{ij}} \quad (i = 1, 2, ..., m)$$

enddo
enddo

$$x_j^{(k+1)} := x_j^{(k)} \prod_{i=1}^{m} y_j^{k,i}$$

enddo
end

The similar algorithms for image reconstruction were considered in works [11—12].

The sufficient conditions of convergence of the algorithm 2 are given by the following theorem.

Theorem 2. Let system (2) be consistent and have if only one positive solution. If $a_{ij} \geq 0$, $\mathbf{a}^i \neq 0$, $p_i > 0$,

$$\gamma_{ij}^k \geq \varepsilon_i > 0, \quad 0 < \sum_{i=1}^{m} a_{ij} \gamma_{ij}^k \leq 1$$

for all i, j, k, $\mathbf{x}^{(0)} \in (R^+)^n$, the sequence $\{\mathbf{x}^{(k)}\}_{k=1}^{\infty}$ defined by the algorithm 2 converges to some solution of the system (2).

The proofs of theorems 1 and 2 are presented in [9].

These algorithms may be realized on parallel computing structures consisting of m elementary processors and one central processor. On each $(k + 1)$-th step of iteration every i-th elementary processor computes the coordinates of vector $\mathbf{y}^{k,i}$ in accordance with formula (5) or (8) and then the central processor computes the $(k + 1)$-th iteration of the image vector $\mathbf{x}$ in accordance with formula (6) or (7).

The main defect of parallel algorithms considered above is their practical realization on parallel computational structures because it needs a lot of local processors in such MPCS. In order to reduce the number of required local processors we consider a block-iterative additive and multiplicative algorithms.

For this purpose decompose the matrix $\mathbf{A}$ and the projection vector $\mathbf{p}$ into M subsets in accordance with decomposition $\{1, 2, ..., m\} = H_1 \cup H_2 \cup ... \cup H_M$, where

$$H_t = \{m_{t-1} + 1, m_{t-1} + 2, ..., m_t\}, \; 0 = m_0 < m_1 < ... < m_M = m. \qquad (9)$$

A l g o r i t h m 3.

1. $\mathbf{x}^{(0)} \in \mathbf{R}^n$ is an arbitrary vector.

2. The k +1-th iteration is calculated in accordance with such a scheme:

$$\mathbf{x}^{(k+1)} = \mathbf{C} \sum_{i \in H_{t(k)}}^{m} \mathbf{B}_i^k \mathbf{P}_i^{\omega_k} \mathbf{y}^i,$$

where $t(k) = k \pmod M + 1$, $\mathbf{P}_i^{\omega_k}$ are operators defined by (3) and (4), ω_k are relaxation parameters, $\mathbf{C}$ is a constrained operator and $\mathbf{B}_i^k$ are matrices of dimension $n \times n$ with real nonnegative elements and

$$\sum_{i \in H_{t(k)}}^{m} \mathbf{B}_i^k = \mathbf{E}, \quad \sum_{i \in H_{t(k)}}^{m} \left\| \mathbf{B}_i^k \right\| \le 1, \qquad (10)$$

ffor all $k \in N$, where $\mathbf{E}$ is the unit matrix of the dimension $n \times n$. The parallel implementation of this algorithm can be described as follows:

$$y^{k,i} = \mathbf{P}_i^{\omega_k} \mathbf{x}^{(k)} \quad i \in H_{t(k)}, \quad \mathbf{x}^{(k+1)} = \mathbf{C} \sum_{i \in H_{t(k)}}^{m} \mathbf{B}_i^k \mathbf{y}^{k,i},$$

or may be given by such a form:

begin
 $\mathbf{x}^{(0)}$ = initial
 for $k = 0, 1, ...$ until convergence
 do
  $t(k) = k(\text{mod } M) + 1$
  do
  for i-th processor, $i \in H_t$

do

$$y^{k,i} = \mathbf{P}_i^{\omega_k} \mathbf{x}^{(k)}$$

enddo

$$\mathbf{x}^{(k+1)} := \mathbf{C} \sum_{i \in H_t}^{m} \mathbf{B}_i^k \mathbf{y}^i$$

enddo

end

The conditions of convergence of algorithm 3 may be given by the following theorem.

Theorem 3. If system (2) is consistent and $0 < \omega_k < 2$, then for every point $\mathbf{x}^{(0)} \in \mathbf{R}^n$ the sequence $\{\mathbf{x}^{(k)}\}_{k=1}^{\infty}$ defined by algorithm 3 converges to some solution of the system (2).

The block-iterative algorithms represent examples of sequential-parallel algorithms. They may be considered as an intermediate version between sequential algorithms and full parallel ones. In each step of iterative process the block-iterative algorithm uses simultaneously information about all equations concerning the given block.

Block-iterative algorithms may be also considered in the case of multiplicative algorithms. In this case the following algorithm is obtained.

A l g o r i t h m 4 (BMART).

1. $\mathbf{x}^{(0)} \in \mathbf{R}^n$ and $\mathbf{x}^{(0)} > 0$.

2. The $k + 1$-th iteration is calculated in accordance with such a scheme:

$$x_j^{(k+1)} := x_j^{(k)} \prod_{i \in H_{t(k)}} \left(\frac{p_i}{(\mathbf{a}^i, \mathbf{x}^k)} \right)^{\gamma_{ij}^k a_{ij}},$$

where γ_{ij}^k are positive real numbers such that

$$0 < \sum_{i \in H_{t(k)}} a_{ij} \gamma_{ij}^k \le 1$$

for every j, k; $H_{t(k)}$ are defined in accordance with (9), and $t(k)$ is almost cycle control sequence. If $\gamma_{ij}^k = \gamma_i$ for all k, j and $0 \le a_{ij} \le 1$, $\sum_{i \in H_{t(k)}} \gamma_i = 1$, then as a result the block-iterative multiplicative algorithm proposed in [6] is obtained. The conditions of convergence are the same as for the algorithm 2.

Block-parallel asynchronous algorithms for computer tomography. In this section the generalized model ofa asynchronous iterations, considered in [13], is applied for implementation of block-parallel algorithms on nonsynchronous computer structure. We shall use the basic notions of the theory of asynchronous iterations which were introduced by Chazan and Miranker in [3] and Baudet in [1] (see, also [14]).

Applying the generalized model of asynchronous iterations for implementation of algorithm BPART on nonsynchronous computer structure results in obtaining the following algorithm, where the numbers of operators are chosen in the chaotic way:

A l g o r i t h m 5.

1. $\mathbf{x}^{(0)} \in \mathbf{R}^n$ is an arbitrary vector.

2. The $k+1$-th iteration is calculated in accordance with such a scheme:

$$\mathbf{x}^{k+1,i} = \mathbf{C} \sum_{i \in H_{t(k)}} \mathbf{B}_i^k \mathbf{P}_i^{\omega_k} \mathbf{x}^{(\sigma^i(k))},$$

where $\mathbf{P}_i^{\omega_k}$ are operators defined by (3) and (4), ω_k are relaxation parameters, $\mathbf{C}$ is a constrained operator, $t(k) = I_k$, $I = \{I_k\}_{k=0}^{\infty}$ is a sequence of chaotic sets such that $I_k \subset \{1, 2, ..., m\}$ and $\mathbf{B}_i^k$ are matrices of dimension $n \times n$ with real nonnegative elements which satisfy the conditions of (10), $J_i = \{\sigma^i(k)\}_{k=1}^{\infty}$ are sequences of delays.

The convergence of this algorithm is given by the following theorem.

Theorem 4. Let system (1) be consistent, $I = \{I_k\}_{k=0}^{\infty}$ be a regular sequence of chaotic sets $I_k \subset \{1,2,...,m\}$ with the number of regularity T, $J_i = \{\sigma^i(k)\}_{k=1}^{\infty}$ be sequences with limited delays and $\sigma_j^i(k) = \sigma_i(k)$, and let the number of delays be equal to T. Then for every point $\mathbf{x}^{(0)} \in \mathbf{R}^n$ the sequence $\{\mathbf{x}^{(k)}\}_{k=1}^{\infty}$ defined by algorithm 5 converges to some point $\mathbf{x}^* \in H$, which is a fixed point of orthogonal projection operators $\mathbf{P}_i$ ($i = 1, 2, ..., m$).

The full proof of this theorem one can find in [9]. In this paper the constrained operator $\mathbf{C}$ is given in the form $\mathbf{C} = C_1 C_2$, where

$$(C_1[\mathbf{x}])_i = \begin{cases} a, & \text{if } x_i < a; \\ x_i, & \text{if } a \le x_i \le b; \\ b, & \text{if } x_i > b; \end{cases}$$

$$(C_2[\mathbf{x}])_j = \begin{cases} 0, & \text{if } p_i = 0 \text{ and } a_{ij} \ne 0; \\ x_j, & \text{otherwise.} \end{cases}$$

Computer simulation and experimental results. In this section we present some numerical results of applying the special cases of block-parallel algorithm BPART-3 and chaotic block-parallel algorithm CHBP-3 considered in the previous sections for the reconstruction of high contrast objects from incomplete projection data in the case when they are not available at each angle of view and they are a few-number limited. The influence of various parameters of these algorithms such as a pixel initialization, relaxation parameters, the number of iterations and noise in the projection data on reconstruction quality and convergence of these algorithm are also studied there.

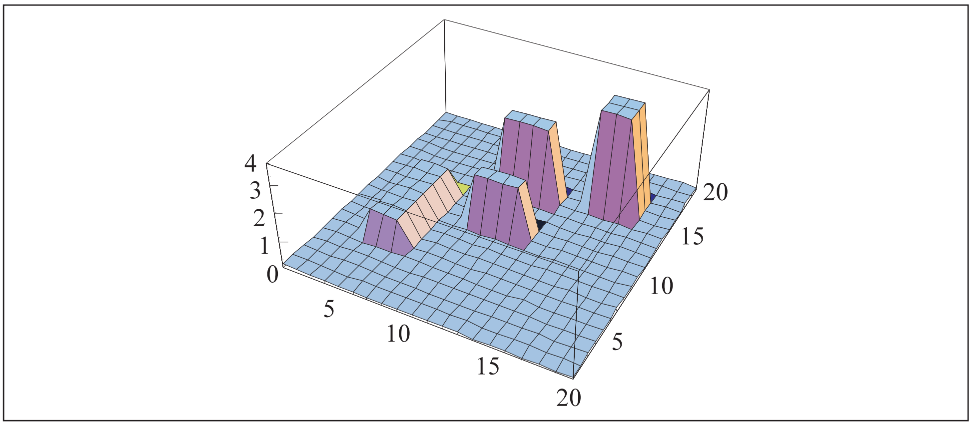

Fig. 1. The original function $f(x, y)$

In dependence on obtaining the system of projections there are many image reconstruction schemes. In some practical problems, in engineering for example, it is impossible to get projections from all directions because of the existing of some important reasons (such as situation, size or impossibility of an access to a research object). This situation arises, for example, in the coal bed working.

In this paper the goodness of the applied algorithm of reconstruction was tested for different kinds of geometric figures and reconstruction schemes.

The discrete functions with high contrast were chosen to illustrate the implementation of these algorithms working with incomplete data. The results presented in this paper are given for the following function:

$$f(x,y)=\begin{cases}1, & (x,y)\in D_1\subset E\subset \mathbf{R}^2;\\ 2, & (x,y)\in D_2\subset E\subset \mathbf{R}^2;\\ 3, & (x,y)\in D_3\subset E\subset \mathbf{R}^2;\\ 4, & (x,y)\in D_4\subset E\subset \mathbf{R}^2;\\ 0, & \text{otherwise,}\end{cases}$$

where E is a square $E=\{(x,y){:}-1\le x,y\le 1\}$, and D_i are subsets of E of the following form:

$$D_1 = [-0.7,-0.4]\times[-0.5,0.2],\quad D_2= [-0.2,0.2]\times[-0.1,0.1],$$

$$D_3 = [-0.2,0.2]\times[0.3,0.5],\quad D_4 = [0.4,0.7]\times[0.4,0.7].$$

The plot of this function is given in Fig. 1. The results for image reconstructions are represented for only two different schemes, which are described in [14]

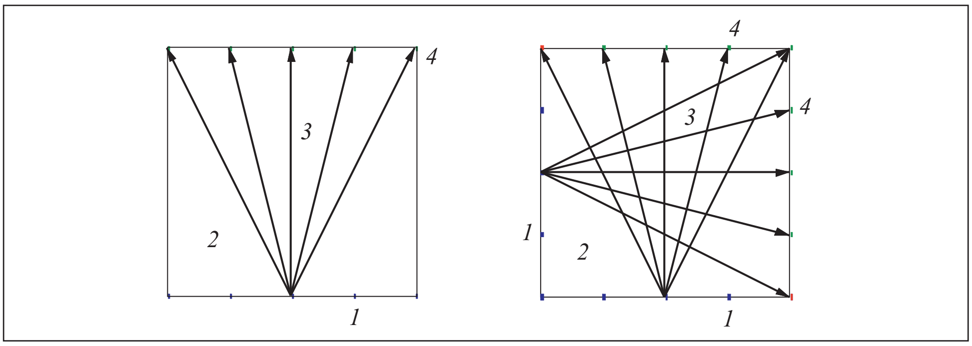

Fig. 2. The schemes of obtaining projection data: system (1 × 1) and system (1 × 1, 1 × 1): *1* — sources of rays; *2* — research object; *3* — rays; *4* — detectors

and were called system (1 × 1) and (1 × 1, 1 × 1). Both of these systems are shown in Fig. 2.

In the first scheme of obtaining the projection data, which we shall call the system (1 × 1), we have an access to the research object from only two opposite sides. This situation often arises in engineering geophysics. In this case the sources of rays are located only on one side and the detectors are located on the opposite side of the research part of a coal bed. This scheme of information obtaining is shown in Fig. 2.

The convergence characteristics of image reconstruction are given in a view of plots for the following measures of errors:

the absolute error: $\delta(x,y)=\left|f(x,y)-\widetilde{f}(x,y)\right|$;

the maximal absolute error: $\Delta=\max\limits_i\left|f_i-\widetilde{f}\right|$;

the maximal relative error:

$$\delta_1=\frac{\max\limits_i\left|f_i-\widetilde{f}_i\right|}{\max\limits_i\left|f_i\right|}100\,\%;$$

the mean absolute error:

$$\delta_2=\frac{1}{n}\sum_i\left|f_i-\widetilde{f}_i\right|,$$

where f_i is the value of the given modeling function in the center of the *i*-th pixel and $\widetilde{f}_i$ is the value of the reconstructed function in the *i*-th pixel. As the result of computer simulation it was assumed, that n is the number of pixels, i.e. the number of variables; m is the number of rays, i.e. the number of equations; M is the number of blocks; *iter* is the number of full iterations.

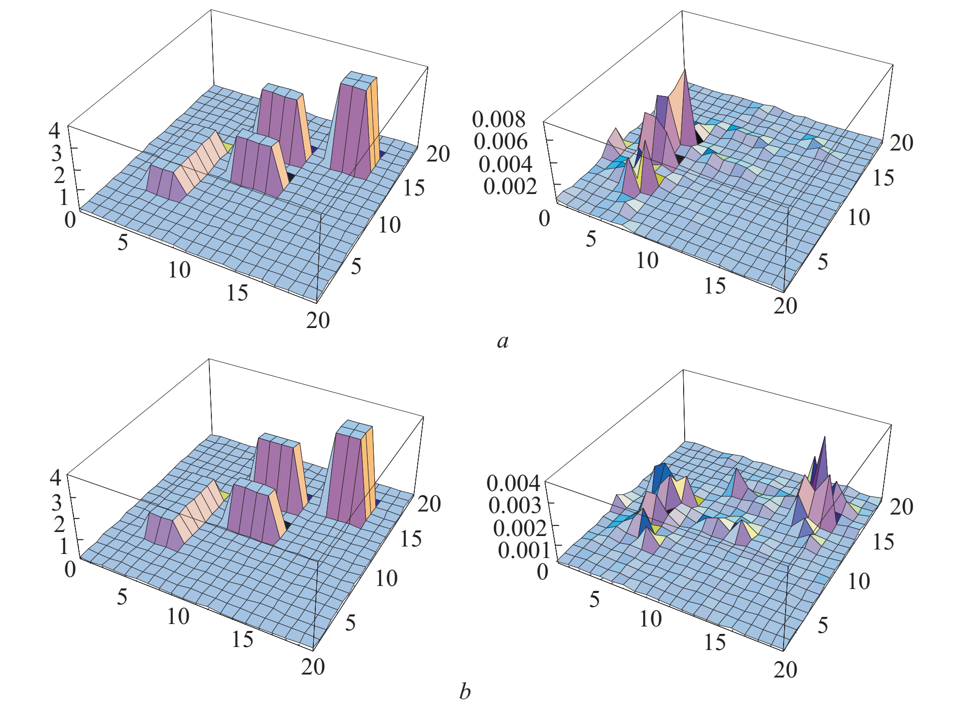

Fig. 3. The image reconstruction and the absolute error for $f(x, y)$ obtained with algorithm BPART-3 (*a*) and algorithm CHBP-3 (*b*) for $n = 20 \times 20$, $m = 644$, $M = 36$, *iter* = 75 in the system (1 × 1, 1 × 1)

In all experiments it was also assumed that M is equal to the number of detectors; the sequence of chaotic sets I_k has the form $\{\xi_k\}$, where ξ_k is an integer random variable in the interval $[1, m]$ with uniform distribution; the reconstruction domain $E = \{(x, y){:}-1 \le x, y \le 1\}$ was divided into $n = 20 \times 20$ pixels; the number of projections m in the system (1 × 1) is equal to 788, and in the system (1×1, 1×1) the number $m = 644$.

The results of image reconstructions for $f(x, y)$ with block-parallel algorithm BPART-3, and chaotic block-parallel algorithm CHBP-3 in the system (1×1, 1×1) for the same parameters are given in Fig. 3.

The plots, which are presented in Fig. 4, illustrate the dependence of the maximum relative error and the mean absolute error on the number of iterations of image reconstruction of $f(x, y)$ with algorithms BPART-3 and CHBP-3 in the system (1×1, 1×1). Table 1 shows the dependence of the maximum absolute error Δ on the number of iterations for algorithms BPART-3 and CHBP-3 in the system (1×1, 1×1).

The results of reconstruction of the function $f(x, y)$ with algorithm BPART-3 and CHBP-3 in the system (1×1, 1×1) is shown in Fig. 5.

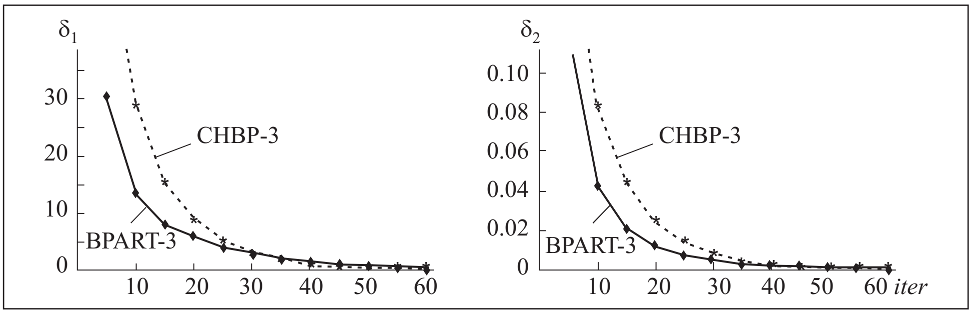

Fig. 4. Dependence of the mean absolute error δ_2 and the maximum relative error δ_1 on the number of iterations for image reconstruction of $f(x, y)$ with algorithm BPART-3 and CHBP-3 in the system (1 × 1, 1 × 1)

The plots presented in Fig. 6 illustrate the dependence of the maximum relative error and the mean absolute error on the number of iterations of image reconstruction of $f(x, y)$ with algorithms BPART-3 and CHBP-3 in the system (1 × 1).

Table 2 shows the dependence of the maximum absolute error Δ on the number of iterations for algorithms BPART-3 and CHBP-3 in the system (1 × 1).

All experimental results in the case of reconstruction of objects from limited projection data show that the errors of reconstruction with algorithms BPART-3 and CHBP-3 are constantly reduced with increasing the number of iterations.

Table 1

Iter	BPART-3	CHBP-3
10	0.4640	0.2112
20	0.1973	0.0478
40	0.0293	0.0054
50	0.0113	0.0018
100	0.0001	0.000001

Table 2

Iter	BPART-3	CHBP-3
100	0.1902	0.2668
200	0.0883	0.1345
500	0.0146	0.0168
1000	0.0007	0.0006
2000	2.109×10^{-6}	7.872×10^{-7}

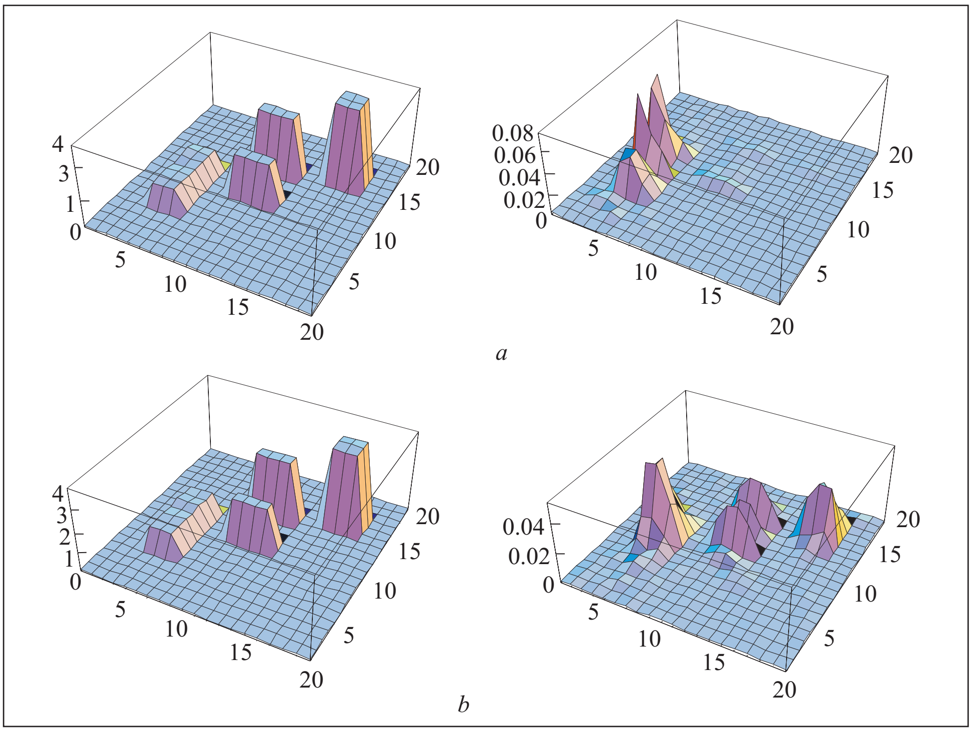

Fig. 5. The image reconstruction and the absolute error for $f(x, y)$ obtained with BPART-3 (a) and algorithm CHBP-3 (b) for $n = 20 \times 20$, $m = 788$, $M = 28$ in the system (1×1): a — $iter = 600$; b – $iter = 200$

The obtained results also show that chaotic algorithm CHBP-3 gives better results as compared with block-parallel algorithm BPART-3.

Table 3 shows the number of iterations required for obtaining the image reconstruction with a given maximum relative error δ_2 for the considered algorithms BPART-3, CHBP-3 and for the considered systems.

Table 3

(1×1, 1×1)		(1×1)		δ_2, %
BPART-3	CHBP-3	BPART-3	CHBP-3	
13	24	74	95	<10
23	30	178	148	<5
47	46	953	271	<1
60	53	271	340	<0,5

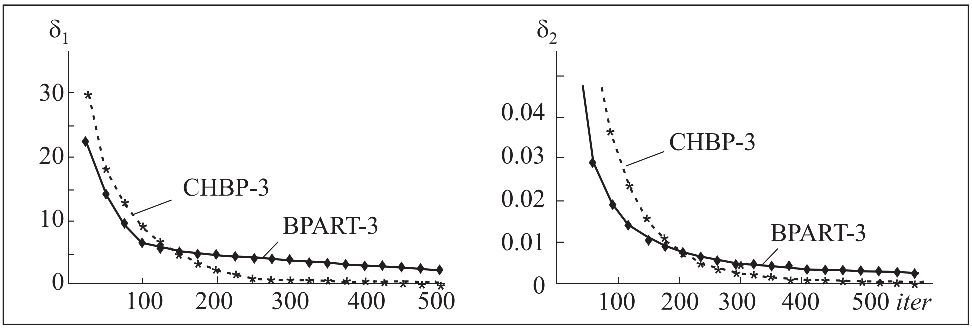

Fig. 6. Dependence of the mean absolute error δ_2 and the maximum relative error δ_1 on the number of iterations for image reconstruction of $f(x, y)$ with algorithm BPART-3 and CHBP-3 in the system (1 × 1)

All algorithms were implemented on IBM/PC (processor AMD Duron XP, 1600 MHz) by means of C++ and MATHEMATICA 5.1. One iteration by means of Mathematica 5.1 was implemented approximately 1s for algorithm BPART-3 and CHBP-3, and in C++ one iteration for both algorithms is implemented in a real time.

Conclusion. New chaotic iterative algorithms for image reconstruction are presented in the paper. These algorithms can be realized on a parallel computing structure consisting of elementary processors and some central processor, all of which are connected with shared memory. The quality and convergence of these algorithms were studied by computing simulation on sequential computer. The experimental results show that convergent characteristics of block-parallel chaotic algorithm CHBP-3 are better as compared with block-parallel algorithm BPART-3. Taking into account that the time of implementation of block-parallel computer algorithm on parallel computer is approximately M times less (where M is the number of processors) in comparison with a sequential computer, it follows from results of computer simulation that the time characteristics of block-parallel algorithms are better as compared with sequential ART-3. It also follows from our experiments that the configuration (1×1, 1×1) is considerably better as compared with the scheme (1×1). And for each considered scheme of reconstruction there exist the parameters which allow to obtain a rather good quality of reconstruction after some number of iterations, but this number is considerably larger than that for reconstruction with complete projection data. The number of iterations for achieving the stable reconstruction is approximately two times more for the second scheme in comparison with the first one. And this number is approximately 10 times more for the scheme (1×1, 1×1) in comparison with the case of complete data.

Розроблено та виконано блочно-паралельні алгоритми комп'ютерної томографії. Наведено чисельні алгоритми відновлення та результати чисельного моделювання для тестових задач і деяких окремих випадків систем реконструкції збирання даних.

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