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Dynamical dark energy from extra dimensions

We consider multidimensional cosmological model with a higher-dimensional product manifold $M = \mathbf{R} \times \mathbf{R}^{d_0} \times \mathbf{H}^{d_1}/\Gamma$ where $\mathbf{R}^{d_0}$ is d_0-dimensional Ricci-flat external (our) space and $\mathbf{H}^{d_1}/\Gamma$ is d_1-dimensional compact hyperbolic internal space. M2-brane solution for this model has the stage of accelerating expansion of the external space. We apply this model to explain the late time acceleration of our Universe. Recent observational data (the Hubble parameter at the present time and the redshift when the deceleration parameter changes its sign) fix fully all free parameters of the model. As a result, we find that considered model has too big size of the internal space at the present time and variation of the effective four-dimensional fine structure constant strongly exceeds the observational limits.

ДИНАМІЧНА ТЕМНА ЕНЕРГІЯ З ДОДАТКОВИХ ВИМІРІВ, Баух В., Жук О. — Ми досліджуємо багатовимірну космологічну модель із різноманіттям у вигляді прямого добутку $M = \mathbf{R} \times \mathbf{R}^{d_0} \times \mathbf{H}^{d_1}/\Gamma$, де $\mathbf{R}^{d_0}$ є d_0-вимірним Річчі-плоским зовнішнім (нашим) простором, а $\mathbf{H}^{d_1}/\Gamma$ — d_1-вимірним компактним гіперболічним внутрішнім простором. М2-бранне рішення для цієї моделі має стадію прискореного розширення зовнішнього простору. Ми використовуємо цю модель для пояснення прискореного розширення нашого Всесвіту на пізніх етапах його еволюції. Сучасні спостережні дані (параметр Хаббла в даний момент часу та червоний зсув, що відповідає часу переходу від уповільненого до прискореного розширення Всесвіту) повністю фіксують вільні параметри моделі. У результаті ми показуємо, що досліджувана модель має занадто великий розмір внутрішнього простору в даний момент часу, а варіація ефективної 4-вимірної постійної тонкої структури сильно перевищує спостережувані обмеження.

ДИНАМИЧЕСКАЯ ТЕМНАЯ ЭНЕРГИЯ ИЗ ДОПОЛНИТЕЛЬНЫХ ИЗМЕРЕНИЙ, Баух В., Жук А. — Мы исследуем многомерную космологическую модель с многообразием в виде прямого произведения $M = \mathbf{R} \times \mathbf{R}^{d_0} \times \mathbf{H}^{d_1}/\Gamma$, где $\mathbf{R}^{d_0}$ является d_0-мерным Риччи-плоским внешним (нашим) пространством, а $\mathbf{H}^{d_1}/\Gamma$ — d_1-мерным компактным гиперболическим внутренним пространством. М2-бранное решение для этой модели имеет *стадию ускоренного расширения внешнего пространства. Мы используем эту модель для объяснения ускоренного расширения нашей Вселенной на поздних этапах ее эволюции. Современные наблюдательные данные (параметр Хаббла в настоящий момент времени и красное смещение, соответствующее времени перехода от замедленного к ускоренному расширению Вселенной) полностью фиксируют свободные параметры модели. В результате мы показываем, что исследуемая модель имеет слишком большой размер внутреннего пространства в настоящий момент времени, и вариация эффективной 4-мерной постоянной тонкой структуры сильно превышает наблюдаемые ограничения.*

INTRODUCTION

Recent astronomical observations abundantly evidence that our Universe underwent stages of accelerating expansion during its evolution. There are at least two of such stages: early inflation and late time acceleration. The latter began approximately at the redshift $z \sim 0.35$ (see e.g. [4]) and continues until now. Thus, the construction and investigation of models with stages of acceleration is one of the main challenge of the modern cosmology. Among such models, the models originated from fundamental theories (e.g. string/M-theory) are of most interest.

In the present paper we consider a multidimensional cosmological model with a factorizable metric

$$g = -e^{2\gamma_0} d\tau \otimes d\tau + a_{0BD}^2 g^{(0)} + a_1^2 g^{(1)} = \Omega^2 (-dt \otimes dt + a_0^2 g^{(0)}) + a_1^2 g^{(1)}, \qquad (1)$$

which is defined on the manifold with product topology

$$M = \mathbf{R} \times \mathbf{R}^{d_0} \times \mathbf{H}^{d_1}/\Gamma, \qquad (2)$$

where $\mathbf{R}^{d_0}$ is d_0-dimensional Ricci-flat external (our) space with metric $g^{(0)}$: $R[g^{(0)}] = 0$ and scale factor a_0, and $\mathbf{H}^{d_1}/\Gamma$ is d_1-dimensional hyperbolic (compact) internal space with metric $g^{(1)}$: $R[g^{(1)}] = -d_1(d_{1-1})$ and scale factor a_1. (Negative constant curvature spaces are compact if they have a quotient structure: H^{d_i}/Γ_i, where H^{d_i} and Γ_i are hyperbolic spaces and their discrete isometry group, respectively.) Both a_0 and a_1 depend only on time. First line in (1) is a metric in the Brans—Dicke frame in the harmonic time gauge where $e^{\gamma_0} = a_{0BD}^{d_0} a_1^{d_1}$ [7]. Second line in (1) is a metric in the Einstein frame in the synchronous time gauge. The scale factors a_{0BD} of the external space in the Brans—Dicke frame is connected with the scale factor a_0 in the Einstein frame as follows: $a_0 = \Omega^{-1} a_{0BD}$, where conformal factor $\Omega = a_1^{-\frac{d_1}{d_0 - 1}}$.

Hereafter we consider 3-dimensional external space: $d_0 = 3$. Harmonic time τ is related to synchronous time t as $dt = f(\tau) d\tau$, where $f(\tau) = \Omega^{-1} a_{0BD}^3 a_1^{d_1} = a_{0BD}^3 a_1^{3d_1/2} = a_0^3$ [1].

With the standard York—Gibbons—Hawking boundary term S_{YGH}, an action for considered model reads

$$S = \frac{1}{2\kappa^2} \int_M d^D x \sqrt{|g|}\, R[g] + S_{YGH}, \qquad (3)$$

where $D = 1 + d_0 + d_1 = 4 + d_1$ is a total number of dimensions and κ_D is a D-dimensional gravitational constant.

Substituting the metric (1) into this action and minimizing obtained Lagrangian with respect to the scale factors, we get the the following solutions* of the equations of motion (in the harmonic time gauge):

$$a_0(\tau)=A_1^{\frac{d_1+2}{6}}\left(\sqrt{\frac{2\varepsilon}{|R_1|}}\right)^{\frac{d_1}{2(d_1-1)}}\times\frac{\exp\left(-\sqrt{\frac{d_1+2}{12(d_1-1)}}2\varepsilon\,\tau\right)}{\sinh^{d_1/[2(d_1-1)]}\left(-\sqrt{\frac{d_1-1}{d_1}}2\varepsilon\,\tau\right)}, \qquad (4)$$

and

$$a_1(\tau)=A_1\left(\sqrt{\frac{2\varepsilon}{|R_1|}}\right)^{\frac{1}{d_1-1}}\times\frac{\exp\left(-\sqrt{\frac{3}{(d_1-1)(d_1+2)}}2\varepsilon\,\tau\right)}{\sinh^{1/(d_1-1)}\left(-\sqrt{\frac{d_1-1}{d_1}}2\varepsilon\,\tau\right)}, \qquad (5)$$

where A_1 and ε are the constants of integration. The function $f(\tau)$ can be easily obtained from Eq. (4) via expression $f(\tau) = a_0^3(\tau)$.

We should note that solutions (4) and (5) for the metric (1) is a particular case of so called Sp-branes with $(p + 1)$-dimensional Ricci-flat external space. In the case $d_0 = 3$ we obtain $p = 2$. Therefore, if underlying model is $(D = 11)$-dimensional M-theory, we arrive at M2-branes where the number of internal dimensions is equal to 7. It is well known that such models with hyperbolic internal space undergo the stage of accelerating expansions (see e.g. [1] and references therein). However, the parameters of the model were not connected with observational data. So, in the present paper we want to use the modern cosmological data (the present day value for the Hubble parameter and the redshift when our external space transits from deceleration to acceleration) to fix all arbitrary parameters of the considered model and obtain corresponding dynamical behavior for the scale factors, the Hubble parameter, the deceleration parameter and the fine structure "constant".

DYNAMICAL BEHAVIOR OF THE MODEL

In this section, besides the external a_0 and internal a_1 scale factors described by Eqs. (4) and (5), we consider also the Hubble parameter for each of the factor spaces

$$H_0=\frac{1}{a_0}\frac{da_0}{dt}=\frac{1}{a_0 f(\tau)}\frac{da_0}{d\tau}=$$

$$=-\frac{\sqrt{2\varepsilon}}{f(\tau)}\left[\sqrt{\frac{d_1+2}{12(d_1-1)}}+\sqrt{\frac{d_1}{4(d_1-1)}}\coth\left(\sqrt{\frac{d_1-1}{d_1}}2\varepsilon\,\tau\right)\right], \qquad (6)$$

$$H_1=\frac{1}{a_1}\frac{da_1}{dt}=\frac{1}{a_1 f(\tau)}\frac{da_1}{d\tau}=$$

$$=-\frac{\sqrt{2\varepsilon}}{f(\tau)}\sqrt{\frac{3}{(d_1-1)(d_1+2)}}\left[1+\sqrt{\frac{d_1+2}{3d_1}}\coth\left(\sqrt{\frac{d_1-1}{d_1}}2\varepsilon\,\tau\right)\right], \qquad (7)$$

* The general method for this kind of models was elaborated in papers [2, 12, 13].

the external space deceleration parameter

$$q_0 = -\frac{d^2a_0}{dt^2}\frac{1}{H_0^2a_0} = -\frac{1}{f(\tau)}\frac{d}{d\tau}\left(\frac{1}{f(\tau)}\frac{da_0}{d\tau}\right)\frac{1}{H_0^2a_0} = -2\sinh^{-2}\left(\sqrt{\frac{d_1-1}{d_1}}2\varepsilon\ \tau\right) \times$$

$$\times\left[\sqrt{\frac{d_1+2}{3(d_1-1)}} + \sqrt{\frac{d_1}{d_1-1}}\ \coth\left(\sqrt{\frac{d_1-1}{d_1}}2\varepsilon\ \tau\right)\right]^{-2} + 2 \qquad (8)$$

and the variation of the fine structure constant (as a function of redshift z)

$$\Delta\alpha = \frac{\alpha(z)-\alpha(0)}{\alpha(0)} = \frac{a_1^{2d_1/(d_0-1)}(0)}{a_1^{2d_1/(d_0-1)}(z)} - 1. \qquad (9)$$

(It is well known that the internal space dynamics results in the variation of the fundamental constants such as the fine structure constant [3, 5, 8, 11]. For example, the effective four-dimensional fine-structure constant is inversely proportional to the volume of the internal space: $\alpha \propto V_1^{-1} \propto a_1^{-d_1}$). We assume also that the solution (4), (5) describes the M2-brane, that is $d_1 = 7$.

According to the recent observational data (see e.g. [4, 6]), the present acceleration stage began at redshift $z \approx 0.35$ and the Hubble parameter now is $H_0(t_p) \equiv H_p \approx 72$ km/s/Mpc = 2.33×10^{-18} s^{-1}. Hereafter, the letter p denotes the present day values. Additionally, at the present time the value of the external space scale factor can be estimated as $a_0(t_p) \approx cH_p^{-1} \approx 1.29\times10^{28}$ cm. We shall use these observational conditions to fix the free parameters of the model A_1 and ε (the constants of integration) and to define the present time t_p. (It is obvious that our model cannot pretend to describe the full history of the Universe. We try to apply this model to explain the late time acceleration of the Universe which starts at the redshift $z \approx 0.35$. Before this time, the Universe evolution is described by the standard Big Bang cosmology. Therefore, in our model $t = 0$ corresponds to $z = 0.35$ (i.e. q_0 ($z = 0.35$) = 0) and t_p is the time from this moment to the present day). Observational data also show that for different redshifts the fine structure constant variation does not exceed 10^{-5}: $|\Delta\alpha| < 10^{-5}$ [11].

Below, all quantities are measured in the Hubble units. For example, the scale factors are measured in cH_p^{-1} and synchronous time t is measured in H_p^{-1}. Therefore, $a_0(t_p) = 1$ and $H_p = 1$.

To fix all free parameters of the model, we use the following logic chain. First, from the equation $q_0(\tau) = 0$ we obtain the harmonic time τ_{in} of the beginning of the stage of acceleration. We find that this equation has two roots which describe the beginning and end of the acceleration. Second, we define the constant of integration A_1 from the equation $z = 0.35 = 1/a_0(\tau_{in}) - 1$ where we use the condition that acceleration starts at $z = 0.35$ and that $a_0(\tau_p) = 1$. Third, we find the present harmonic time τ_p from the condition $a_0(\tau_p) = 1$. It is worth of noting that τ_{in}, τ_p and A_1 are the functions of ε. To fix this parameter, we can use the condition $H_0(\tau_p) = 1$. Finally, to find the value of the present synchronous time, we use the equation $t_p = \int_{\tau_{in}}^{\tau_p} f(\tau)d\tau$ where $f(\tau) = a_0^3(\tau)$. In the case $d_1 = 7$, direct calculations give for the constants of integration $A_1 = 1.23468$ and $\varepsilon = 1.53097$. It results in $t_p = 0.296 \sim 4$ Gyr, $q_0(t_p) = -0.960572$ and for the internal space $a_1(t_p) = 1.24319$, $H_1(t_p) = 0.0500333$.

Dynamical behavior of the considered model is depicted in figures 1—3.

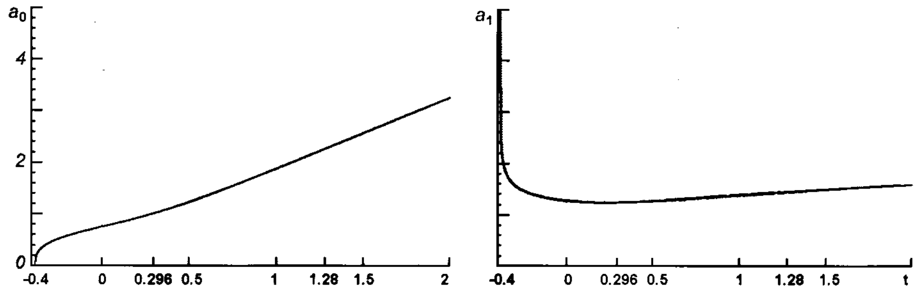

Fig. 1. The scale factors of the external space (left panel) and internal space (right panel) versus synchronous time

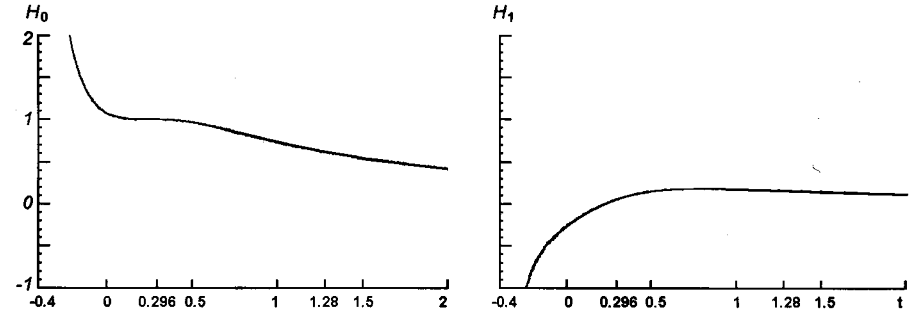

Fig 2. The Hubble parameters of the external space (left panel) and internal space (right panel) versus synchronous time

Fig. 1 shows the dynamics of the external space scale factor $a_0(t)$ (left panel) and the internal space scale factor $a_1(t)$ (right panel). Here, $t = 0.296$ is the present time, and $t = 0$ and $t = 1.28$ correspond to the beginning and end of the stage of acceleration, respectively. It follows that the internal space is the same order of magnitude as the external one at the present time. However, for the standard Kaluza-Klein models there is the experimental restriction on the size of extra dimensions: $l_{extra} \leq 10^{-17}$ cm. That is $a_0/a_1 \geq 10^{45}$. Obviously, our model does not satisfy this condition. One of the possible way to avoid this problem consists in proposal that the Standard Model matter is localized on a brane. In this case the extra dimensions can be much bigger that 10^{-17} cm (even an infinite). However, such model requires the generalization of our metric (1) to the non-factorizable case and this investigation is out of the scope of the present paper.

We plot in Fig. 2 the evolution of the Hubble parameters $H_0(t)$ (left panel) and $H_1(t)$ (right panel). We can see that their values are comparable with each other. Thus, the internal space is rather dynamical and this fact is the main reason of too large variations of the fine structure constant (see Fig. 3).

We present in Fig. 3 the evolution of the deceleration parameter $q_0(t)$ (panel *a*) and the variation of the fine structure constant $\Delta\alpha(t)$ (panel *b*). Left picture clearly shows that the acceleration stage has the final period for the considered model. It starts at $t = 0$ and finishes at $t = 1.28$. The right picture demonstrates that $\Delta\alpha$ does not satisfy the observable restrictions $|\Delta\alpha| < 10^{-5}$. There is the

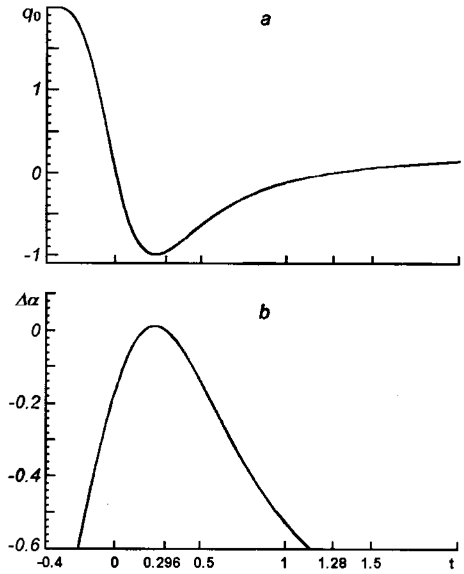

Fig. 3. The deceleration parameter of the external space (*a*) and variation of the fine structure constant (*b*) versus synchronous time

only very narrow region in the vicinity of $z = 0.13$ (or equivalently $t = 0.17$ in synchronous time) where $\Delta\alpha$ changes its sign. However, it is the exceptional region but restriction $|\Delta\alpha| < 10^{-5}$ works for very large diapason of redshifts z [11].

CONCLUSIONS

In the present paper we investigate multidimensional cosmological model with Ricci-flat external space and compact hyperbolic internal space. Such pure gravitational model has the exact solution with the stage of accelerating expansion for our external space (so called Sp-brane solution). This solution depends on a number of free parameters. It is remarkable that observable cosmological parameters (such as the Hubble parameter, the parameter of deceleration) gives a possibility to fix all these free parameters and completely determinate this model. We perform this analysis in the case of M2-brane with $d_1 = 7$ extra dimensions. For obtained parameters, we describe the dynamical behavior of the considered model. It is shown that our external space really has the finite stage of the accelerating expansion starting at $z = 0.35$ and lasting until now. However, this model has two significant drawbacks. On the one hand, the internal space is too big with respect to the standard Kaluza-Klein restrictions $a_{internal} \leq 10^{-17}$ cm and, on the other hand, this space is not sufficiently constant to satisfy the observable limits on the fine structure constant variations. Thus, these drawbacks rule out this model from the scope of viable theories. (In papers [9, 10], it is investigated the effects of matter overdensities on the time and space variations of α. It is shown that a far slower evolution of α will be found in virialised regions (such as the Earth or the Solar

System) than in the cosmological background. Thus, this effect gives a possible way to alleviate our conclusions concerning incompatibleness of the considered model with the observations on variations of alpha. However, the problem of too big size of the extra dimensions still remains).

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