Low Temperature Physics/Fizika Nizkikh Temperatur, 2018, v. 44, No. 12, pp. 1582–1587

Cooling of mechanical vibrations by heat flow

S.I. Kulinich

B. Verkin Institute for Low Temperature Physics and Engineering of the National Academy of Sciences of Ukraine
47 Nauky Ave., Kharkiv 61103, Ukraine
E-mail: kulinich@ilt.kharkov.ua

L.Y. Gorelik

Department of Physics, Chalmers University of Technology, Kemigården 1, SE-412 96 Göteborg, Sweden

Received July 27, 2018, published online October 26, 2018

We theoretically consider a nanomechanical link between two metallic leads subject to a temperature drop. It is shown that mechanical dynamics of such system can be strongly affected by a heat flow through it via the position dependent electron-electron interaction, even though the electronic transport between leads is blocked. In particular, it is demonstrated that, under certain conditions, the stationary distribution of the excitations in the mechanical subsystem has a Boltzmann form with an effective temperature, which is much lower than the temperature of the environment; this seems rather counterintuitive. We also find that a change in the direction of the temperature gradient can result in the generation of mechanical vibrations rather than the heating of the mechanical subsystem.

Keywords: nanoelectromechanical system, heat flow, Wigner distribution function.

Nanoelectromechanical systems (NEMS) promise to manipulate mechanical motion of the nanoobjects using electronic dynamics. There are many approaches to control nanomechanical (NMS) performance providing a number of new functionalities of nanodevice operations, in particular, pumping or cooling of the mechanical subsystem [1–5]. One of the main approaches exploits the dc electronic flow through the nanosystem induced either by the bias voltage or temperature drop between two electronic reservoirs connected by the NEMS [6,7]. Recently new type of nanomechanical heat engine, which working principle is based exclusively on the heat flow, was suggested in [8]. It was shown that the electron-electron coupling in NEMS may results in a nanomechanical instability controlled by a heat flow, even though the electronic transport between the leads is blocked. However, a semiclassical approach used in this paper didn't allow to investigate the operation of such NEMS in a cooling regime. In this paper, using reduced density matrix approach, we calculate a Wigner function characterizing a stationary state of the mechanical subsystem in the NEMS similar to one considered in [8]. We demonstrate, that, at certain conditions, it has a Boltzmann distribution form with an effective temperature which is lower than the temperatures of the adjacent electrodes.

There are many different experimental realizations of the general concept presented in the paper. To be concrete we chose one which, may be not the easiest, but the best, from our point of view, for the illustration of the basic physical phenomenon. This prototype NEMS device (Fig. 1) comprises two single-level quantum dots (QD) positioned sequentially between two electrodes — left (L) and right (R) — kept at different temperatures T_L and T_R, correspondingly. One quantum dot (right) clamped to the right electrode while another one (left) is attached to the left electrode by nanomechanical link and is free to move in horizontal direction. The electronic levels of the fabricated

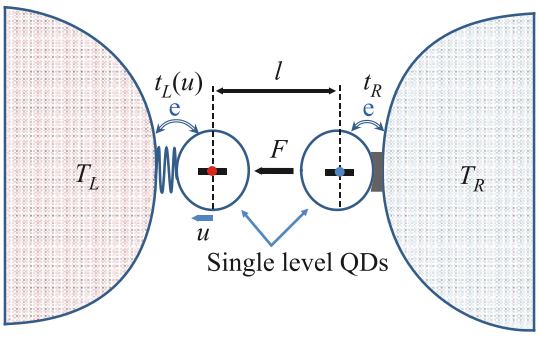

Fig. 1. The scheme of the device — two single-level quantum dots (left and right) is placed between the leads that are kept at different temperatures, $T_L \neq T_R$; $t_\kappa(u)$ is the tunneling amplitude of electron from the lead $\kappa = (L,R)$ to the corresponding dot, l is the distance between the dots and F is the strength caused by Coulomb interaction of the dots.

quantum dots are connected to the corresponding electrodes through high resistance tunnel barriers. At the same time the distance between the dots is too large to allow tunneling between them, but is short enough so that the Coulomb interaction between charged dots significantly affects their population. We made this assumption solely for the purpose of blocking electronic transportation between electrodes allowing at the same time the heat transfer between them. The mechanical deflection u of the left dot affects the electronic subsystem by changing the tunneling rate, $\Gamma_L(u)$, between the QD and the left electrode. Conversely, the electronic subsystem affects the deflection u via a Coulomb force acting on the charge residing on the QD if another one is also charged.

The Hamiltonian of the system has a form

$$\hat{H} = \hat{H}_l + \hat{H}_{\rm dot} + \hat{H}_{\rm tun} + \hat{H}_v, \tag{1}$$

where Hamiltonian $\hat{H}_l$ describes the noninteracting electrons in the leads,

$$\hat{H}_l = \sum_{k,\kappa} \varepsilon_{k,\kappa} a^\dagger_{k,\kappa} a_{k,\kappa}, \tag{2}$$

$a^\dagger_{k,\kappa}(a_{k,\kappa})$ is the creation (annihilation) operator of electron with the energies $\varepsilon_{k,\kappa}$ in the leads $\kappa = (L,R)$. The Hamiltonian $\hat{H}_{\rm dot}$ describes the electron states in the dot,

$$\hat{H}_{\rm dot} = E_L \hat{n}_L + E_R \hat{n}_R + U(u)\hat{n}_L \hat{n}_R, \tag{3}$$

where E_κ is the energy level on the corresponding dot, $\hat{n}_\kappa = d^\dagger_\kappa d_\kappa, d^\dagger_\kappa(d_\kappa)$ is the creation (annihilation) operator of the electron state on the dot $\kappa, U(u) \simeq U_0 - -Fu\ (F \sim e^2/l^2 > 0)$ is the energy of Coulomb interaction between the electrons on the dots, l is the distance between the dots. The Hamiltonian $\hat{H}_v$,

$$\hat{H}_v = \frac{p^2}{2m} + \frac{m\omega^2 u^2}{2}, \tag{4}$$

describes the mechanical dynamic of the left dot, p and u are the canonical conjugated momentum and coordinate, $[p,u] = -i\hbar$; m, ω are the mass and eigenfrequency of the dot, correspondingly.

The connection between the dots and the leads is described by the standard tunneling Hamiltonian,

$$\hat{H}_{\rm tun} = t_L(u)\sum_k d^\dagger_L a_{k,L} + t_R \sum_k d^\dagger_R a_{k,R} + \text{H.c.}, \tag{5}$$

where the tunneling amplitudes are $t_L(u) = t_L \exp(u/\lambda)$, $t_R(u) = t_R = \text{const}$ and $\lambda > 0$ is the tunneling length.

In this paper we will assume that the temperatures in the leads T_κ are much greater than the width of the electronic levels on the dots, $\hbar\Gamma_\kappa = 2\pi\nu_\kappa t_\kappa^2$ (here ν_κ are the density of states in the leads). This assumption allows one to neglect coherent mixing between electronic states of the dots and the leads and to factorize the total density matrix $\hat{\rho}(t)$

$$\hat{\rho}(t) \simeq \hat{\rho}_{\rm dot} \otimes \hat{\rho}_L \otimes \hat{\rho}_R, \tag{6}$$

where $\hat{\rho}_\kappa$ is an equilibrium (Gibbs) density matrix in the lead κ with the temperature T_κ and $\hat{\rho}_{\rm dot}$ is the density matrix of two dots which also describes the mechanical dynamics of the left dot.

Then following the standard procedure (see, for example, [2]) we get the following master equation for the reduced density matrix $\hat{\rho}_{\rm dot}$ (below index "dot" in $\hat{\rho}_{\rm dot}$ and $\hat{H}_{\rm dot}$ is ommited),

$$\frac{\partial\hat{\rho}(t)}{\partial t} + \frac{i}{\hbar}\left[\hat{H} + \hat{H}_v, \hat{\rho}(t)\right] = \hat{I}_L\{\hat{\rho}\} + \hat{I}_R\{\hat{\rho}\}, \tag{7}$$

wherein the collision integral $\hat{I}_\kappa\{\hat{\rho}\}$ has the form

$$\begin{aligned}
\hat{I}_\kappa = {} & \frac{1}{2}\sqrt{\Gamma_\kappa(u)}\left[d^\dagger_\kappa \hat{\rho}(t) d_\kappa + d_\kappa \hat{\rho}(t) d^\dagger_\kappa\right]\sqrt{\Gamma_\kappa(u)} - \frac{1}{4}\left[\Gamma_\kappa(u), \hat{\rho}(t)\right]_+ + \\
& + \frac{i}{4}\left\{\int d\tau \frac{e^{i\beta_\kappa\mu\tau}}{\sinh\pi\tau}\sqrt{\Gamma_\kappa(u)} d_\kappa e^{-i\beta_\kappa(\hat{H}+\hat{H}_v)\tau}\hat{\rho}(t-\hbar\beta_\kappa\tau) d^\dagger_\kappa \sqrt{\Gamma_\kappa(u)} e^{i\beta_\kappa(\hat{H}+\hat{H}_v)\tau} + \right. \\
& + \int d\tau \frac{e^{-i\beta_\kappa\mu\tau}}{\sinh\pi\tau}\sqrt{\Gamma_\kappa(u)} d^\dagger_\kappa e^{-i\beta_\kappa(\hat{H}+\hat{H}_v)\tau}\hat{\rho}(t-\hbar\beta_\kappa\tau) d_\kappa \sqrt{\Gamma_\kappa(u)} e^{i\beta_\kappa(\hat{H}+\hat{H}_v)\tau} + \\
& + \int d\tau \frac{e^{i\beta_\kappa\lambda\mu\tau}}{\sinh\pi\tau}\sqrt{\Gamma_\kappa(u)} d_\kappa e^{-i\beta_\kappa(\hat{H}+\hat{H}_v)\tau}\sqrt{\Gamma_\kappa(u)} d^\dagger_\kappa \hat{\rho}(t-\hbar\beta_\kappa\tau) e^{i\beta_\kappa(\hat{H}+\hat{H}_v)\tau} + \\
& \left. + \int d\tau \frac{e^{-i\beta_\kappa\mu\tau}}{\sinh\pi\tau}\sqrt{\Gamma_\kappa(u)} d^\dagger_\kappa e^{-i\beta_\kappa(\hat{H}+\hat{H}_v)\tau}\sqrt{\Gamma_\kappa(u)} d_\kappa \hat{\rho}(t-\hbar\beta_\kappa\tau) e^{i\beta_\kappa(\hat{H}+\hat{H}_v)\tau} - \text{H.c.}\right\}.
\end{aligned} \tag{8}$$

Here μ is a chemical potential which is supposed to be the same for both leads, β_κ are inverse temperatures of the leads, $\Gamma_L(u) = \Gamma_L \exp(2u/\lambda)$, $\Gamma_R(u) = \Gamma_R = \text{const}$ and $[A,B]_+ = AB + BA$ denotes the anticommutator of the operators A, B.

In the following we will use the dimensionless variables,

$$\frac{u}{u_0}\to u,\ \frac{u_0 p}{\hbar}\to p,\ t\omega\to t,\ \frac{\Gamma_\kappa}{\omega}\to\Gamma_\kappa,\ \frac{\lambda}{u_0}\to\lambda,$$

$$\hat H_v\to\hat H_v=\frac{p^2+u^2}{2},\ \frac{Fu_0}{\hbar\omega}\to F, \tag{9}$$

where $u_0=\sqrt{\hbar/m\omega}$ is an amplitude of zero-point oscillations. We restrict our consideration to the case of relatively large temperatures, $\beta_\kappa\hbar\omega\ll 1$, which allows us to replace $\rho(t-\hbar\beta_\sigma\tau)\to\rho(t)$ and neglect the vibronic Hamiltonian $\hat H_v$ in Eq. (8). In addition we assume that $F\ll 1$ and neglect the coordinate dependence of the Coulomb interaction.

The reduced density matrix describes the oscillator at different population of the dots and may be presented in the form

$$\hat\rho(t)=(1-\hat n_L)(1-\hat n_R)\rho_0+\hat n_L(1-\hat n_R)\rho_L+$$
$$+\hat n_R(1-\hat n_L)\rho_R+\hat n_L\hat n_L\rho_2. \tag{10}$$

Here the mechanical density matrix $\rho_0=\rho_0(u,u')$ describes the state of the oscillator when both dots are "empty", $\rho_\kappa=\rho_\kappa(u,u')$ describes the state of the oscillator when only one dot is occupied, and $\rho_2=\rho_2(u,u')$ is the density matrix of the mechanical subsystem when both levels on the dots are occupied. Substituting Eq. (10) into Eqs. (7), (8) we get the following system of equations describing the evolution of the mechanical subsystem:

$$\frac{\partial\rho_0}{\partial t}+i\left[\hat H_v,\rho_0\right]=(1-f_L^{(1)})\sqrt{\Gamma_L(u)}\rho_L\sqrt{\Gamma_L(u)}-$$
$$-\frac{f_L^{(1)}}{2}\left[\Gamma_L(u),\rho_0\right]_+ +\Gamma_R(1-f_R^{(1)})\rho_R-\Gamma_R f_R^{(1)}\rho_0, \tag{11}$$

$$\frac{\partial\rho_L}{\partial t}+i\left[\hat H_v,\rho_L\right]=f_L^{(1)}\sqrt{\Gamma_L(u)}\rho_0\sqrt{\Gamma_L(u)}-$$
$$-\frac{1-f_L^{(1)}}{2}\left[\Gamma_L(u),\rho_L\right]_+ +\Gamma_R(1-f_R^{(2)})\rho_2-\Gamma_R f_R^{(2)}\rho_L, \tag{12}$$

$$\frac{\partial\rho_R}{\partial t}+i\left[\hat H_v,\rho_R\right]=(1-f_L^{(2)})\sqrt{\Gamma_L(u)}\rho_2\sqrt{\Gamma_L(u)}-$$
$$-\frac{f_L^{(2)}}{2}\left[\Gamma_L(u),\rho_R\right]_+ -\Gamma_R(1-f_R^{(1)})\rho_R+\Gamma_R f_R^{(1)}\rho_0, \tag{13}$$

$$\frac{\partial\rho_2}{\partial t}+i\left[\hat H_v-Fu,\rho_2\right]=f_L^{(2)}\sqrt{\Gamma_L(u)}\rho_R\sqrt{\Gamma_L(u)}-$$
$$-\frac{1-f_L^{(2)}}{2}\left[\Gamma_L(u),\rho_2\right]_+ -\Gamma_R(1-f_R^{(2)})\rho_2+\Gamma_R f_R^{(2)}\rho_L. \tag{14}$$

In Eqs. (11)–(14) we used the notations $f_\kappa^{(1)}=f_\kappa(E_\kappa)$, $f_\kappa^{(2)}=f_\kappa(E_\kappa+U_0)$, where

$$f_\kappa(E)=\left[1+\exp\beta_\kappa\left(E-\mu\right)\right]^{-1} \tag{15}$$

is the Fermi–Dirac distribution function.

To analyze the system (11)–(14) we will use the Wigner representation

$$W_A(u,p)=\frac{1}{2\pi}\int d\xi e^{-ip\xi}\rho_A(u+\xi/2,\,u-\xi/2). \tag{16}$$

Defining operators $\hat T_1$, $\hat T_2$ as

$$\hat T_1 f(u,p)=-\frac{\partial f(u,p)}{\partial t}+\left(u\frac{\partial}{\partial p}-p\frac{\partial}{\partial u}\right)f(u,p), \tag{17}$$

$$\hat T_2 f(u,p)=\frac{f(u,p+i/\lambda)+f(u,p-i/\lambda)}{2}, \tag{18}$$

one finds that the equations of motion, Eqs. (11)–(14), in the Wigner representation take the form

$$\hat T_1 W_0+(1-f_L^{(1)})\Gamma_L(u)W_L-f_L^{(1)}\Gamma_L(u)\hat T_2 W_0+$$
$$+\Gamma_R(1-f_R^{(1)})W_R-\Gamma_R f_R^{(1)}W_0=0, \tag{19}$$

$$\hat T_1 W_L+f_L^{(1)}\Gamma_L(u)W_0-(1-f_L^{(1)})\Gamma_L(u)\hat T_2 W_L+$$
$$+\Gamma_R(1-f_R^{(2)})W_2-\Gamma_R f_R^{(2)}W_L=0, \tag{20}$$

$$\hat T_1 W_R+(1-f_L^{(2)})\Gamma_L(u)W_2-f_L^{(2)}\Gamma_L(u)\hat T_2 W_R-$$
$$-\Gamma_R(1-f_R^{(1)})W_R+\Gamma_R f_R^{(1)}W_0=0, \tag{21}$$

$$\hat T_1 W_2+F\frac{\partial W_2}{\partial p}+f_L^{(2)}\Gamma_L(u)W_R-(1-f_L^{(2)})\Gamma_L(u)\hat T_2 W_2-$$
$$-\Gamma_R(1-f_R^{(2)})W_2+\Gamma_R f_R^{(2)}W_L=0. \tag{22}$$

We are interested in the steady state regime of the vibrational subsystem in the limit when the parameters F and $1/\lambda$ are small. To find the solution of Eqs. (19)–(22) to leading order in these parameters it is convenient to introduce linear combinations of the Wigner distribution functions as follows:

$$W_\Sigma=W_0+W_L+W_R+W_2, \tag{23}$$

$$R_1=W_L+W_2,\ R_2=W_R+W_2,\ R_3=W_2. \tag{24}$$

In addition, it is convenient to change from (u,p) to polar coordinates (A,φ) so that $u-\bar u=A\sin\varphi$ and $p=A\cos\varphi$, where $\bar u\sim F$ is the equilibrium position of the left dot. In polar coordinate the steady state equation for the Wigner distribution function $W_\Sigma(A,\varphi)$ that describes the vibrational degree of freedom is given by the equation

$$-\frac{\partial W_\Sigma}{\partial\varphi}+\hat L\left(\bar u W_\Sigma-FR_3\right)+\tilde\Gamma_L\left(1-\hat T_2\right)Y=0, \tag{25}$$

$$Y=Y(A,\varphi)=f_L^{(1)}(W_\Sigma-R_1-R_2+R_3)+$$
$$+(1-f_L^{(1)})\left(R_1-R_3\right)+f_L^{(2)}(R_2-R_3)+(1-f_L^{(2)})R_3, \tag{26}$$

where $\tilde{\Gamma}_L = \tilde{\Gamma}_L(A,\varphi) = \Gamma_L \exp(2A\sin\varphi/\lambda)$ and differential operator $\hat{L}$ is defined according to the expression

$$\hat{L} = \cos\varphi\frac{\partial}{\partial A} - \frac{\sin\varphi}{A}\frac{\partial}{\partial\varphi}. \tag{27}$$

Equation (25) for the oscillator Wigner function $W_\Sigma(A,\varphi)$ is coupled to the steady state equation for the vector-function $|\mathbf{R}\rangle = (R_1, R_2, R_3)^T$. The equation for $|\mathbf{R}\rangle$ takes the form

$$\frac{\partial|\mathbf{R}\rangle}{\partial\varphi} + \hat{M}(A,\varphi)|\mathbf{R}\rangle = |\mathbf{F}\rangle, \tag{28}$$

$$\hat{M}(A,\varphi) = \begin{pmatrix} \tilde{\Gamma}_L & (f_L^{(1)} - f_L^{(2)})\tilde{\Gamma}_L & 0 \\ (f_R^{(1)} - f_R^{(2)})\Gamma_R & \Gamma_R & 0 \\ -f_R^{(2)}\Gamma_R & -f_L^{(2)}\tilde{\Gamma}_L & \tilde{\Gamma}_L + \Gamma_R \end{pmatrix}, \tag{29}$$

$$|\mathbf{F}\rangle = W_\Sigma \begin{pmatrix} f_L^{(1)}\tilde{\Gamma}_L \\ f_R^{(1)}\Gamma_R \\ 0 \end{pmatrix} + \hat{L}\begin{pmatrix} \bar{u}R_1 - FR_3 \\ \bar{u}R_2 - FR_3 \\ \bar{u}R_3 - FR_3 \end{pmatrix}. \tag{30}$$

Equations (25)–(30) have to be solved subject to the periodic boundary conditions $W_\Sigma(A,\varphi+2\pi) = W_\Sigma(A,\varphi)$, $R_i(A,\varphi+2\pi) = R_i(A,\varphi)$. We will find the analytical solutions of these equations in the limit of small-amplitude vibrations, $A \ll \lambda$.

For small vibrations an analytical solution can be found by perturbation theory in terms of the small parameters $\varepsilon = \{F, 1/\lambda\}$. In this case Eqs. (25), (28)–(30) take the form

$$-\frac{\partial W_\Sigma}{\partial\varphi} + \hat{L}\left(\bar{u}W_\Sigma - FR_3\right) + \frac{\Gamma_L}{2\lambda^2}\hat{L}^2 Y = 0, \tag{31}$$

$$\frac{\partial|\mathbf{W}\rangle}{\partial\varphi} + \hat{M}_0|\mathbf{W}\rangle = |\mathbf{f}\rangle, \quad \hat{M}_0 = \hat{M}(A=0,\varphi), \tag{32}$$

$$|\mathbf{f}\rangle = W_\Sigma \begin{pmatrix} f_L^{(1)}\Gamma_L \\ f_R^{(1)}\Gamma_R \\ 0 \end{pmatrix} + \hat{L}\begin{pmatrix} \bar{u}R_1 - FR_3 \\ \bar{u}R_2 - FR_3 \\ \bar{u}R_3 - FR_3 \end{pmatrix} + $$

$$+ \frac{2\Gamma_L A}{\lambda}\sin\varphi \begin{pmatrix} f_L^{(1)}W_\Sigma - R_1 - (f_L^{(1)} - f_L^{(2)})R_2 \\ 0 \\ f_L^{(2)}R_2 - R_3 \end{pmatrix}. \tag{33}$$

We solve these equations by perturbation expansions

$$R_i(A,\varphi) = R_i^{(0)}(A,\varphi) + R_i^{(1)}(A,\varphi) + \ldots \tag{34}$$

(and similarly for the function $W_\Sigma(A,\varphi)$), where $R_i^{(n)}$ is of nth order in ε.

It is evident from Eqs. (31)–(33) that the functions $W_\Sigma^{(0)}(A,\varphi)$, $R_i^{(0)}(A,\varphi)$ do not depend on φ. Hence, $W_\Sigma^{(0)}(A,\varphi) = W_\Sigma^{(0)}(A)$ and

$$R_1^{(0)} = n_L W_\Sigma^{(0)}, \; R_2^{(0)} = n_R W_\Sigma^{(0)}, \; R_3^{(0)} = n_2 W_\Sigma^{(0)}, \tag{35}$$

where $n_i = n_i(\Gamma_\kappa, \beta_\kappa)$. The exact expression for the functions n_i is given in Appendix.

To first order in perturbation theory Eq. (31) determines the equilibrium position of the dot, $\bar{u} = Fn_2$. To second order in perturbation theory Eq. (31) after averaging over φ takes the form

$$F\frac{\partial}{\partial A}\left(A\left\langle\cos\varphi R_3^{(1)}\right\rangle\right) + \frac{D_1}{2\lambda^2}\frac{\partial}{\partial A}\left(A\frac{\partial W_\Sigma^{(0)}}{\partial A}\right) = 0. \tag{36}$$

Here the coefficient D_1 is the function of the tunneling rates and temperatures of the electrodes (the exact form of this function is given in Appendix). The brackets, $\langle f(A,\varphi)\rangle$, in Eq. (36) denote the zeroth Fourier component of the 2π-periodic function $f(A,\varphi)$. Deriving Eq. (36) we used the property

$$\langle\hat{L}f(A,\varphi)\rangle = \frac{1}{A}\frac{\partial}{\partial A}\left(A\langle\cos\varphi\, f(A,\varphi)\rangle\right). \tag{37}$$

Therefore, to get a closed equation for $W_\Sigma^{(0)}(A)$ one needs to know the function $R_3^{(1)}(A,\varphi)$. This function can, to first order in perturbation theory, be determined from Eqs. (32) and (33). As a result one gets the stationary Fokker–Planck equation for the oscillator Wigner distribution function $W_\Sigma^{(0)}(A)$,

$$\mathcal{G}\frac{\partial}{\partial A}\left(A^2 W_\Sigma^{(0)}\right) + \mathcal{D}\frac{\partial}{\partial A}\left(A\frac{\partial W_\Sigma^{(0)}}{\partial A}\right) = 0, \tag{38}$$

where the drift $\mathcal{G}$ and diffusive $\mathcal{D}$ coefficients take the form

$$\mathcal{G} = \frac{F}{\lambda}G, \quad \mathcal{D} = \frac{D_1\lambda^{-2} + D_2F^2}{2}, \tag{39}$$

the exact form for the coefficients G, D_2 is given in Appendix. The solution of Eq. (38) at small values of amplitude has the form of a Boltzmann distribution function, $W_\Sigma(A) \sim \exp(-\beta A^2/2)$, where $A^2/2$ is the vibrational energy and coefficient β plays a role of an effective temperature, and is given by equation

$$\beta = \mathcal{G}/\mathcal{D} = \frac{2\lambda F}{D_1 + D_2(\lambda F)^2}G. \tag{40}$$

It is a matter of direct verification to prove that the coefficients D_i are positive, while $G \sim \sinh(\beta_R - \beta_L)U_0/2$. Therefore the sign of the effective temperature depends on the direction of the temperature drop. Negative temperature corresponds to the situation of the nanomechanical instability considered early in [8]. If the temperature of the

left electrode is greater than the temperature of the right electrode one can compare the effective temperature of the mechanical subsystem with the temperatures of the leads. It is appropriate at this point to recall that the drift coefficient $\mathcal{G}$ was calculated above with accuracy to $\beta_\kappa\hbar\omega$. In the cases $\lambda \to \infty, F \to 0$ or $\beta_L = \beta_R$, drift coefficient $\mathcal{G} \to 0$, and the corrections of the order of $\beta_\kappa\hbar\omega$ are of first importance. One can show that in this situation the effective temperature is of the same order as temperatures in the leads.

From Eq. (40) one can see that inverse effective temperature as a function of the parameters λ and F takes a maximum value, $\beta = \beta_{\max}$, when $\lambda F = \sqrt{D_1/D_2}$ and

$$\beta_{\max} = \frac{G(\beta_\kappa,\Gamma_\kappa)}{\sqrt{D_1(\beta_\kappa,\Gamma_\kappa)D_2(\beta_\kappa,\Gamma_\kappa)}}. \qquad (41)$$

In this paper we will analyze particular case when the heat flow between electrodes generated by electronic subsystem takes a maximum value at fixed Γ_κ. This is the case when the following condition are satisfied: $\Gamma_L = \Gamma_R = \Gamma/2$, $E_L \simeq E_R \simeq U_0/2$ and $T_R \ll U_0 \ll T_L$. In that case $f_L^{(1)} \cong f_L^{(2)} \cong 1/2$, $f_R^{(1)} \cong 1$, $f_R^{(2)} \cong 0$ and the maximum inverse effective temperature is defined by the equation

$$T_{\text{eff}}^{\min} = \sqrt{\frac{3(\Gamma^2+1)}{4} + \left(\frac{\Gamma^2+1}{\Gamma^2+4}\right)^2 \left(\Gamma^2+12\right)}. \qquad (42)$$

From this expressions it follows that in the case of the weak coupling between the dots and the leads ($\Gamma \ll 1$), the effective temperature of the dot is $T_{\text{eff}}^{\min} \simeq \sqrt{3/2}\hbar\omega$ (restoring dimension). Thus it is valid to say that we have a near ground state cooling of the mechanical subsystem. In opposite case $\Gamma \gg 1$ the effective temperature $T_{\text{eff}}^{\min} \simeq \sqrt{7/4}\hbar\Gamma$.

In conclusion, we theoretically studied the NEMS system which is comprised of two quantum dots sequentially positioned between two bulk electrodes kept at a different temperature. It was shown that electron-electron interaction between dots inducing the heat transfer between the electrodes significantly affects the state of the mechanical subsystem even though the electronic transfer between electrodes is blocked. We found that at certain direction of the temperature gradient between the electrodes the stationary distribution of the excitation in the mechanical subsystem has a Boltzmann form with an effective temperature dependent on the parameters of the system. As this take place, the minimal possible effective temperature is $T_{\text{eff}} \sim 1.2\hbar\omega$ and might be much less than the temperature of the "cold" electrode. Therefore it was demonstrated that the heat flow through NEMS might significantly suppress the thermal fluctuation in the mechanical subsystem. At this point we have to note that in all our consideration we suggest that the mechanical subsystem interacts only with electronic one. In realistic experimental situation there is also phonon's environment which affects the state of the mechanical subsystem. However the strength of the interaction between vibronic mode and phonons in the leads depend on the quality of the nanomechanical link and might be much weaker than the coupling between vibronic mode and electronic subsystem.

SIK acknowledge financial support from NAS Ukraine (Grant No 4/18-N and Scientific Programme No. 1.4.10.26.4/F26-4).

Appendix

The expressions for the functions $n_i = n_i(\Gamma_\kappa,\beta_\kappa)$, Eq. (35), have the form

$$n_L = \frac{f_L^{(1)} - f_R^{(1)}(f_L^{(1)} - f_L^{(2)})}{\Delta}, \qquad (43)$$

$$n_R = \frac{f_R^{(1)} - f_L^{(1)}\left(f_R^{(1)} - f_R^{(2)}\right)}{\Delta}, \qquad (44)$$

$$n_2 = \frac{f_R^{(2)}\Gamma_R n_L + f_L^{(2)}\Gamma_L n_R}{\Gamma}, \qquad (45)$$

where $\Gamma = \Gamma_L + \Gamma_R$, and $\Delta = 1 - (f_L^{(1)} - f_L^{(2)})(f_R^{(1)} - f_R^{(2)})$.

The expressions for the functions D_i, G, Eqs. (36), (39), take the form

$$D_1 = \Gamma_L\left[(1-f_L^{(1)})n_L + (f_L^{(1)} - f_L^{(2)})n_2\right], \qquad (46)$$

$$G = \frac{\Gamma_L\Gamma_R}{\Gamma(1+\Gamma^2)} \times$$

$$\times \frac{f_R^{(1)} f_L^{(2)}\left(1-f_L^{(1)} - f_R^{(2)}\right) - f_L^{(1)} f_R^{(2)}\left(1-f_R^{(1)} - f_L^{(2)}\right)}{\Delta}, \qquad (47)$$

$$D_2 = n_2\left[\frac{\Gamma}{1+\Gamma^2}\frac{1-n_2}{\Delta} + \xi_1(1-n_L) + \xi_2(1-n_R)\right], \qquad (48)$$

$$\xi_1 = \frac{\Gamma}{\Delta}\left[f_R^{(2)} + f_L^{(2)}(f_R^{(1)} - f_R^{(2)})\right] \times$$

$$\times\left[\frac{1}{\Gamma^2 + (1-\Delta\Gamma_L\Gamma_R)^2} - \frac{1}{1+\Gamma^2}\right] - \Gamma_R f_R^{(2)} \frac{1-\Delta\Gamma_L\Gamma_R}{\Gamma^2 + (1-\Delta\Gamma_L\Gamma_R)^2}, \qquad (49)$$

$$\xi_2 = \frac{\Gamma}{\Delta}\left[f_L^{(2)} + f_R^{(2)}(f_L^{(1)} - f_L^{(2)})\right] \times$$

$$\times\left[\frac{1}{\Gamma^2 + (1-\Delta\Gamma_L\Gamma_R)^2} - \frac{1}{1+\Gamma^2}\right] - \Gamma_L f_L^{(2)} \frac{1-\Delta\Gamma_L\Gamma_R}{\Gamma^2 + (1-\Delta\Gamma_L\Gamma_R)^2}. \qquad (50)$$

1. L. Gorelik, A. Isaksson, M.V. Voinova, B. Kasemo, R.I. Shekhter, and M. Jonson, *Phys. Rev. Lett*. **80**, 4526 (1998).
2. S.I. Kulinich, L.Y. Gorelik, A.N. Kalinenko, I.V. Krive, R.I. Shekhter, Y.W. Park, and M. Jonson, *Phys. Rev. Lett*. **112**, 117206 (2014).
3. D. Fedoretz, L.Y. Gorelik, R.I. Shekhter, and M. Jonson, *Europhys. Lett*. **58**, 99 (2002).
4. F. Santandrea, L.Y. Gorelik, R.I. Shekhter, and M. Jonson, *Phys. Rev. Lett*. **106**, 186803 (2011).
5. T. Novotny, A. Donarini, and A.-P. Jauho, *Phys. Rev. Lett*. **90**, 256801 (2003).
6. O.A. Ilinskaya, S.I. Kulinich, I.V. Krive, R.I. Shekhter, H.C. Park, and M. Jonson, *New J. Phys*. **20**, 063036 (2018).
7. P. Stadler, W. Belzig, and G. Rastelli, *Phys. Rev. Lett*. **117**, 197202 (2016).
8. A. Vikström, A.M. Eriksson, S.I. Kulinich, and L.Y. Gorelik, *Phys. Rev. Lett*. **117**, 247701 (2016).

Охолодження механічних коливань тепловим потоком

С.І. Кулинич, Л.Ю. Горелик

Розглянуто наномеханічний зв’язок між двома металевими електродами, які підтримуються при різній температурі. Показано, що механічна динаміка в такій системі істотно визначається тепловим потоком. Існування ненульового теплового потоку обумовлено електрон-електронною взаємодією, при цьому потік заряду між електродами відсутній. Встановлено, що за певних умов стаціонарний розподіл збуджень в механічній підсистемі має вигляд больцманівської функції розподілу з ефективною температурою, яка значно нижча за температуру електродів. Також показано, що зміна напрямку градієнта температури призводить до механічних коливань, а не до нагрівання механічної підсистеми.

Ключові слова: наноелектромеханічна система, тепловий потік, вігнерівська функція розподілу.

Охлаждение механических колебаний тепловым потоком

С.И. Кулинич, Л.Ю. Горелик

Рассмотрена наномеханическая связь между двумя металлическими электродами, которые поддерживаются при различной температуре. Показано, что механическая динамика в такой системе существенным образом определяется тепловым потоком. Существование ненулевого теплового потока обусловлено электрон-электронным взаимодействием, при этом поток заряда между электродами отсутствует. Установлено, что при определенных условиях стационарное распределение возбуждений в механической подсистеме имеет вид больцмановской функции распределения с эффективной температурой, которая значительно ниже температуры электродов. Также показано, что изменение направления градиента температуры приводит к механическим колебаниям, а не к нагреву механической подсистемы.

Ключевые слова: наноэлектромеханическая система, тепловой поток, вигнеровская функция распределения.