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Magnetic properties of irradiated quasi 2D type II superconductors

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Persistent scaling behavior of magnetization in layered high T_c superconductors with short-range columnar defects is explained within the Ginzburg–Landau theory. In the weak field region, the scaling function differs from that of a clean sample and the critical temperature is renormalized due to defects. In the strong field region, defects are effectively suppressed and the scaling function, as well as the critical temperature are the same as in a clean superconductor. This picture is consistent with recent experimental results.

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1. Introduction

Layered high-temperature superconducting (HTSC) materials, such as $Bi_2Sr_2CaCu_2O_{8+\delta}$ and $Bi_2Sr_2Ca_2Cu_3O_{10}$, are known to exhibit experimentally 2D scaling magnetic properties [1,2] around the mean field transition line $H_{c2}(T)$. It is manifested by inspecting the magnetization M_0 as a function of temperature T and the (external) magnetic field H:

$$\frac{s\Phi_0}{A\sqrt{k_BTH}}M_0(T,H) = -2\gamma_0(x)\,, \qquad (1)$$

where s is an effective interlayer spacing, Φ_0 is the flux quantum, $x = AH'_{c2}\,[\,T - T_{c2}(H)]/\sqrt{k_BTH}$ is the scaling variable, $2\gamma_0(x)$ is so called scaling function, $H'_{c2} \equiv -dH_{c2}(T)/dT|_{T=T_{c0}}$, and T_{c0} is the zero field critical temperature. For a superconductor with Ginzburg–Landau (GL) parameter κ and Abrikosov geometric factor [3] β_A the constant $A = \sqrt{s\Phi_0/p}$, where $p = 16\pi\kappa^2\beta_A$.

The scaling function $\gamma_0(x)$ was firstly evaluated in the perturbative regime [4,5] for $x << 1$. Non-perturbative result for it was obtained later using the following arguments. The compounds mentioned above are strongly type II superconductors with large GL parameter $\kappa \approx 100$. Their effective interlayer separation $s = 1.5$ nm is larger than the effective superconducting coherence length $\xi(H, T)$ (if the magnetic field is not extremely close to the mean field transition field $H_{c2}(T)$), but is much smaller than the magnetic penetration depth. Hence the problem of fluctuations near $H_{c2}(T)$ becomes effectively two dimensional and can be represented theoretically in terms of the 2D GL mean-field theory projected onto the lowest Landau level (LLL) [6]. Such an approximation remains valid at least for $H > H_{c2}(T)/3$ when higher Landau levels are obviously irrelevant. Moreover, recent results [7] show that LLL projection is valid even for $H > H_{c2}(T)/13$. Tesanović [6] emphasized the crucial role played by the total amplitude of the order parameter in the critical region. Integration of the partition function over this amplitude, assuming that the Abrikosov factor β_A depends weakly on the vortex configuration, leads [8] to the scaling law (1). Moreover, putting this factor equal to constant from the very beginning, Tesanović et. al. obtained an explicit form of the scaling function [9]

$$\gamma_0(x) = \frac{1}{2}\,(\sqrt{x^2+2} - x)\ , \qquad (2)$$

which agrees, to a good accuracy, with the experimental magnetization data for $Bi_2Sr_2Ca_2Cu_3O_{10}$. Such form of the scaling function (2) implies the existence of a crossing point: at some temperature $T_0^* = T_{c0}(1 + k_B/(2A^2H'_{c2}))^{-1}$, the sample magnetization is independent on H, $M_0^* \equiv M_0(T_0^*, H) = -k_B T_0^*/(s\Phi_0)$. Later on, Tesanović and Andreev [10] took the fluctuations of β_A into account and generalized the approach developed in [9] to arbitrary type II superconductors.

Recently, the influence of columnar defects on the magnetic properties of superconductors has been studied experimentally [11,12] and theoretically [13–17]. Columnar defects emerge after heavy ion irradiation of the superconducting sample [11]. They serve as strong pinning centers, each one is able to pin a single vortex. The radius of a columnar defect can be larger or smaller than the coherence length (long-range or short-range defects, respectively). Strong columnar defects lead to the formation of multiquantum vortices in high temperature superconductors [13,14] and in conventional ones as well [16]. They also lead to additional magnetization jumps in mesoscopic samples [17]. Therefore it is interesting to understand how such defects influence both the scaling behavior and the existence of the crossing point.

Specifically we refer to experiments performed by van der Beek et al. [12] who studied the thermodynamic properties of single crystals of $Bi_2Sr_2CaCu_2O_{8+\delta}$. The samples were irradiated with 5.8-GeV ions that produced columnar defects with radius $L = 3.5$ nm and 2D density $n_d = 5\cdot10^{10}$ cm^{-2}. Such density is small in the sense that the matching field $H_\Phi = n_d\Phi_0$, at which the number of vortices becomes equal to the number of defects, is much smaller than $H_{c2}(T)$. Magnetization was measured in the region of magnetic fields 0.2–5 T (which are also smaller than $H_{c2}(T)$) and temperatures 72–86 K. Within this interval of fields the defect radius is the smallest length scale in the problem and the defects can be treated as short-range ones. Measurements showed that columnar defects drastically change the reversible magnetization of the sample: there are now two scaling regimes pertaining to relatively weak ($H < H_\Phi$) and strong ($H > H_\Phi$) magnetic fields. These two regimes are described by the same form (2) of the scaling function as for clean sample but they correspond to two different zero field critical temperatures (used in Ref. 12 as fitting parameters) and two crossing points.

In this paper we propose an explanation of these results. Our arguments are based on the observation that the magnetic field serves as a control parameter for tuning an effective concentration $c = H_\Phi/H$ of defects (the number of defects divided by the number of vortices). In the weak field region, concentration is large ($c = 5$ for $\mu_0 H = 0.2$ T and n_d from [12]), each vortex is affected by a force emanating from many defects. On the average, this force leads to renormalization of the critical temperature T_c. Short-range defects are effectively weak and can be taken into account perturbatively. In first order they retain the same form of scaling function (2) as that of a clean sample, up to renormalization of the critical temperature mentioned above. Second order corrections indeed destroy the scaling behavior but in the vicinity of the crossing temperature scaling is approximately maintained. In the strong field region, concentration is small ($c = 0.2$ for $\mu_0 H = 5$ T and n_d from [12]), the renormalization is not needed and the standard concentration expansion [18] can be used. Here, strictly speaking, even the first order correction (with respect to small concentration) destroys the scaling behavior. However, a strong field effectively suppresses the defects, thus restoring the scaling behavior of a clean superconductor with the initial critical temperature T_{c0}. Identifying the two fitting temperatures of Ref. 12 with the renormalized critical temperature T_c and the initial one T_{c0} respectively, one finds for the dimensionless defect strength $\theta_1 = 0.49$, well inside its allowed range $0 \le \theta_1 \le 1$. This indicates a full consistence between the description constructed below and the experimental results of Ref. 12.

Our quantitative approach follows the one proposed and successfully used within the critical region in clean superconductors [8–10] and at low fields in disordered superconductors [15]. This approach is based on the LLL projection and on an assumption that the Abrikosov factor almost does not depend on the magnetic field. The latter assumption is evidently not valid in the vicinity of the matching field, but for fields much smaller or much larger than H_Φ it is valid. Indeed, columnar defects are strong pinning centers. Then, if the number of vortices is much less or much more than the number of defects, configurations close to the triangular Abrikosov lattice are always simultaneously compatible with any typical configuration of defects, and with the condition of complete (as possible) pinning. It is supported by noticing a remarkable difference between the number of vortices and the number of defects in both regions of fields. This enables us to take into account only such (Abrikosov-like) vortex configurations and fluctuations around them [19]. But for these configurations the Abrikosov factor almost coincides with its «triangular» value $\beta_A = 1.16$.

The next Sec. 2 contains the main body of the paper. Firstly we formulate the model (subsection 2.1) and then obtain the magnetization in the weak field region (subsection 2.2) and in the strong field region (subsection 2.3). Relation of our calculations to the experiment [12] is discussed in the Sec. 3. The last Sec. 4 summarizes the results obtained in this work.

2. Scaling behavior of the irradiated superconductor

2.1. The model

Consider an irradiated thin superconducting film (or one layer in a layered superconductor) with area S subject to perpendicular magnetic field (thus parallel to the defects). Columnar defects can be described as a local reduction of the critical temperature $\delta T_c(\mathbf{r}) = T_{c0} \sum t_j \exp(-(\mathbf{r}-\mathbf{r}_j)^2/2L^2)$. Here $\mathbf{r}$ is a two dimensional vector in the film plane, L is the defect radius, and the positions $\mathbf{r}_j$ of defects are uniformly and independently distributed over the film plane with density n_d. The value of n_d is assumed to be moderate so that for the pertinent region of temperature the matching field H_Φ is always much smaller than $H_{c2}(T)$. The dimensionless amplitudes of defects $0 \le t_j \le 1$ are also independent random quantities distributed with some probability density $p(t)$ whose first two moments θ_1 and θ_2 satisfy $0 \le \theta_{1,2} \le 1$. On the average, the defects lead to renormalization of the critical temperature

$$T_c = T_{c0} - \delta T_c \,, \tag{3}$$

where

$$\delta T_c \equiv \langle \delta T_c(\mathbf{r}) \rangle = 2\pi n_d L^2 \theta_1 T_{c0} \,.$$

The fluctuation of the shift of the critical temperature has zero mean value and variance

$$\langle (\delta T_c(\mathbf{r}) - \delta T_c)^2 \rangle = \pi n_d L^2 \theta_2 T_{c0}^2 \,.$$

The thermodynamic properties of a type II superconductor are described by its partition function

$$\mathcal{Z} \propto \int \mathcal{D}\{\Psi\} \exp\left(-\frac{G}{k_B T}\right), \tag{4}$$

where Ψ is the order parameter and G is the standard GL functional

$$G = s \int \left\{ \alpha(\mathbf{r})|\Psi|^2 + \frac{\beta}{2}|\Psi|^4 + \right.$$
$$\left. + \gamma\, |\partial_- \Psi|^2 + \frac{1}{8\pi}(\mathbf{B}-\mathbf{H})^2 \right\} d^2\mathbf{r} \,, \tag{5}$$
$$\partial_- = \frac{\hbar}{i}\nabla + \frac{2e}{c}\mathbf{A} \,, \quad \mathbf{B} = \nabla \times \mathbf{A} \,,$$

with the first GL coefficient $\alpha(\mathbf{r})$ depending on coordinate through a local change of the critical temperature.

Further simplifications will be done for the case of weak fields (in the case of strong fields slightly different simplifications are required − see subsection 2.3 below). Let us take into account large value of the GL parameter, project the system on the LLL corresponding to the external field, introduce scaling variable x as mentioned above, a scaled order parameter $\varphi \propto \Psi$, and dimensionless temperature fluctuations

$$\tau(\mathbf{r}) = \left(\frac{s\Phi_0}{p}\right)^{1/2} \frac{H_{c2}(T)}{\sqrt{k_B T H}} \, \frac{\delta T_c(\mathbf{r}) - \delta T_c}{T_c - T} \,.$$

(We emphasize that for the case of weak fields it is the renormalized critical temperature which enters the expression for the upper critical field as well as the definition of the scaling variable x.) This results in the following expression for the partition function

$$\mathcal{Z} \propto \int \mathcal{D}\{\varphi\} \exp\left\{ -N_v \left(x\overline{|\varphi|^2} + \frac{1}{4\beta_A}\overline{|\varphi|^4} + \overline{\tau(\mathbf{r})|\varphi|^2} \right) \right\}, \tag{6}$$

where N_v is the total number of vortices (i.e., the total number of flux quanta through the sample area S) and the bar denotes averaging over the sample area. The expression for the magnetization has the form

$$\frac{s\Phi_0}{A\sqrt{k_B T H}} M(T, H) = \frac{1}{N_v} \frac{\partial \ln \mathcal{Z}}{\partial x} \,. \tag{7}$$

These two formulas (6), (7) form the basis for the further calculation and analysis.

2.2. Magnetization: weak fields

The assumption that the Abrikosov factor is a constant enables us, following [15], to replace in Eq. (6) $\overline{|\varphi(\mathbf{r})|^4}$ by $\beta_A(\overline{|\varphi(\mathbf{r})|^2})^2$. This replacement, together with the simplest version of the Hubbard–Stratonovich transformation (introduction of an additional integration over some auxiliary field γ) turns the problem to be an exactly solvable one [15]. Indeed, expand the order parameter on the LLL subspace,

$$\varphi(\mathbf{r}) = \sum_{m=0}^{N_v} C_m L_m(\mathbf{r}) \;, \tag{8}$$

where $L_m(\mathbf{r})$ are normalized LLL eigenfunctions with orbital momentum m. Then after integration over the expansion coefficients C_m , the partition function (6) reads,

$$\mathcal{Z} \propto \int_\Gamma \exp\left\{-N_v\, \mathcal{L}(\gamma, x)\right\} d\gamma \;, \tag{9}$$

where

$$\mathcal{L}(\gamma, x) = -\gamma^2 + N_v^{-1}\, \mathrm{tr}\, \ln\, [(x+\gamma)\hat{I} + \hat{\tau}] \tag{10}$$

and $\hat{\tau}$ is a random matrix with elements:

$$\tau_{mn} = \int_S L_m^*(\mathbf{r})\tau(\mathbf{r})L_n(\mathbf{r})d^2\mathbf{r} \;. \tag{11}$$

The contour Γ in Eq. (9) is parallel to the imaginary axis and stretches from $\gamma^* - i\infty$ to $\gamma^* + i\infty$. To assure convergence of the integrals over the coefficients $\{C_m\}$ the real constant γ^* should satisfy the inequality $\gamma^* + x + \min \tau_n > 0$, where τ_n is the nth eigenvalue of the matrix τ_{mn} .

In the thermodynamic limit $S \to \infty$ with n_d and N_v/S fixed, the partition function (9) could be calculated in a saddle point approximation. This results in the following form for the magnetization

$$\frac{s\Phi_0}{A\sqrt{HT}}\, M(T, H) = -2\gamma(x) \;, \tag{12}$$

where $\gamma(x)$ is the solution of the saddle point equation

$$\partial\, \mathcal{L}(\gamma, x)/\partial\gamma = 0 \;. \tag{13}$$

In the case $\hat{\tau} = 0$ the two possible saddle points satisfy the equation

$$-2\gamma + \frac{1}{x+\gamma} = 0 \;,$$

but only one of them

$$\gamma_0(x) = \frac{1}{2}\left(\sqrt{x^2+2} - x\right) \tag{14}$$

can be reached by an allowed deformation of the contour Γ. Substitution of Eq. (14) into (12) yields the magnetization $M_0(T, H)$ of a clean sample (up to renormalization of the critical temperature (1), (2)) obtained in Ref. 9. The saddle point $\gamma_0(x)$ serves as the scaling function.

Returning to the disordered case we note that in the thermodynamic limit, the last term on the rhs of Eq. (10) has an explicit self-averaged structure N_v^{-1} tr (...) and therefore can be replaced by its average. This procedure modifies the saddle point equation

$$2\gamma = \frac{1}{x+\gamma} + \frac{\varepsilon(T)}{(x+\gamma)^3} \;, \tag{15}$$

and results in a magnetization

$$M(T, H) = M_0(T, H)\left(1 + \varepsilon(T)\,\frac{2\gamma_0(x)}{\sqrt{x^2+2}}\right), \tag{16}$$

where

$$\varepsilon(T) = \left\langle \frac{\mathrm{tr}\, \hat{\tau}^2}{N_v} \right\rangle = \frac{\theta_2}{p}\, n_d L^2\, \frac{(2\pi H'_{c2}\, T_{c0})^2 s L^2}{k_B T} \;. \tag{17}$$

Note that the parameter $\varepsilon(T)$ is proportional to the fourth power of the defect radius L thus justifying the perturbation approach for short-range defects.

In zeroth order approximation with respect to $\varepsilon(T)$ the magnetization has exactly the same form as for a clean sample, thus retaining both the scaling property and the existence of a crossing point. However, due to renormalization of the critical temperature, the crossing temperature $T^* = T_0^* - \delta T^*$ differs from its value T_0^* a clean sample without defects: $\delta T^* = \delta T_c(1 + (2A^2 H'_{c2})^{-1})^{-1}$. In the next order, scaling is virtually destroyed since the correction term (within the parenthesis in Eq. (16)) depends not only on the scaling variable x but also on temperature. But at the crossing temperature T^* the magnetization reads

$$M(T^*, H) = M_0(T^*)\left(1 + \varepsilon(T^*)\,\frac{2H^*}{H+H^*}\right), \tag{18}$$

where $H^* = H_{c2}(T^*) = k_B T^*/(2A^2)$. Therefore if the field is weak enough, $H << H^*$, then the crossing point is restored, T^* serves as a true crossing temperature and the magnetization at the crossing temperature differs from its unperturbed form $-2\gamma_0(x)$ merely by a multiplicative constant $1 + 2\varepsilon(T^*)$.

2.3. Magnetization: strong fields

When the magnetic field increases, the approach used above becomes inapplicable. Firstly, it fails in the vicinity of the matching field where the Abrikosov factor becomes very sensitive to the details of defect configuration. Secondly, higher order terms in the perturbation expansion for the saddle point equation (which were omitted in the Eq. (16)), grow with magnetic field. Fortunately, we have here a new small parameter because the dimensionless concentration c of defects in the strong field region is small. Therefore there is no sense in renormalizing the critical temperature and it is natural to use the concentration expansion [18]. Then in this region, the dimensionless temperature fluctuation $\tau(\mathbf{r})$ is now defined as

$$\tau(\mathbf{r}) = \left(\frac{s\Phi_0}{p}\right)^{1/2} \frac{H_{c2}(T)}{\sqrt{k_B TH}} \frac{\delta T_c(\mathbf{r})}{T_{c0} - T}. \qquad (19)$$

As mentioned above, the second term in the rhs of Eq. (10) is self-averaging and can be calculated using the limiting form of the density of states $\rho(\tau)$ of the matrix (11). For short-range defects in linear approximation with respect to c, this density of states reads

$$\rho(\tau) = (1 - c)\delta(\tau) + \frac{c}{\lambda}\, p\left(\frac{\tau}{\lambda}\right), \qquad (20)$$

where $\lambda = 2\pi L^2 T_{c0} A H'_{c2} \sqrt{H}\, (\Phi_0 \sqrt{k_B T})^{-1}$ and $p(t)$ is probability distribution of the dimensionless temperature t_j. Indeed, the matrix τ_{mn} is nothing but the Hamiltonian of a particle with charge $2e$ in a 2D system subject to a perpendicular magnetic field and containing short-range defects (projected on the LLL). The first and second terms in Eq. (20) correspond, respectively, to those states whose energy is stuck to the LLL (despite the presence of zero-range defects (see, e.g., [20])) and those states whose energies are lifted from the LLL by these defects. For sufficiently narrow distribution $p(t)$, the corresponding saddle-point equation leads to the magnetization

$$M = M_0\left[1 - \frac{c\lambda\theta_1}{(1 + 2\lambda\theta_1\gamma_0(x))\sqrt{x^2 + 2}}\right], \qquad (21)$$

were $M_0(T, H)$ is given by Eq. (1) with an initial critical temperature T_{c0}.

Rigorously speaking, scaling is destroyed since both the concentration c and the shifted eigenvalue $\lambda\theta_1$ depend explicitly on H and T. However, at strong field the correction term in Eq. (21) becomes negligibly small. This implies a restoration of the crossing point. Indeed, at temperature T_0^* the magnetization $M^* = M(T^*, H)$ assumes the form

$$M^* = M_0^*\left(1 - \frac{1}{1 + \eta}\, \frac{H_\Phi}{H + H^*}\right), \qquad (22)$$

with $\eta^{-1} = 2\pi L^2 H'_{c2} T_{c0}\theta_1/\Phi_0$. Therefore in the entire strong field region $H_\Phi << H << H^*$ the crossing temperature coincides with its initial value T_0^* and the magnetization in the crossing point practically coincides with its value M_0^* in a clean superconductor.

3. Discussion

According to the results obtained above, within the main approximation, the magnetization indeed manifests two separate scaling regimes in the regions of weak and strong magnetic fields. These regimes are described by the same scaling function that characterizes a clean sample, but with renormalized critical temperature in the weak region and initial critical temperature at high fields. These results are in complete qualitative correspondence with the experimental observations of van der Beek et al. Without pretending to account for a complete quantitative description of the above mentioned experiments, we nevertheless will show that some quantitative agreement can also be achieved.

Let us discuss the limits of applicability of our results and their relation to the experiment of Ref. 12.

1. In the weak field region, the important small parameters are $\varepsilon(T)$ (which enters the magnetization (18)) and $\varepsilon(T)/(x + \gamma_0(x))^2$ (which enters the saddle point equation). Using the parameters which were employed in the experimental work [12] ($k_B\mu_0 H'_{c2} = 1.15\ \mathrm{T{\cdot}K^{-1}}$, $T^* = 78.9$ K, and the rest of parameters which were already mentioned in Section 1) we find from Eq. (17) $\varepsilon(T^*) = 0.5\,\theta_2$ and $\varepsilon(T^*)/(x^* + \gamma_0(x^*))^2 \approx 0.25$ (the latter figure is obtained for $\mu_0 H = 0.2$ T). For quite plausible value $\theta_2 = 0.5$ one then finds $\varepsilon(T^*) = 0.5\,\theta_2 = 0.25$.

2. The condition of convergence of the integral over the expansion coefficients $\{C_m\}$ can be written as $H > 0.25\, H_\Phi \theta_1^2/\theta_2$ and even in the worst case $\theta_1^2 = \theta_2$ it reads $\mu_0 H \approx 0.25$ T.

3. Then, one has $\mu_0 H^* \approx 6.4$ T and applicability of the LLL projection requires $\mu_0 H > 0.5$ T. The weak field region of Ref. 12 corresponds to $\mu_0 H = 0.2{-}0.02$ T. Thus, in the weak field region, only the condition for applicability of the LLL projection is slightly violated, but the deviation is not dramatic.

4. In the strong field region we find $\eta \approx 2.9$ and therefore the correction term in parenthesis of Eq. (22) is less than three percents so that in this region our assumptions are fully satisfied.

Hence, up to non significant mismatch for the very weak fields our theoretical assumptions and simplifications are completely consistent with the experimental parameters of Ref. 12. Using the same set of parameters we display in Fig. 1 the quantity $M/\sqrt{TH}$ as a function of the scaling variable for weak field (inset) and strong field (main part). We used here the maximal value $\theta_1 = 1$. In the strong field region, the deviation form clean sample scaling behavior is negligibly small for all three values of the strong magnetic field, in complete agreement with our results. In the weak field region, the scaling functions for three different fields can hardly be distinguished. This means that scaling is undoubtedly valid in a vicinity of the crossing temperature. At the same time the scaling function differs from its form in a clean sample (1) by a multiplicative constant (see the parenthesis in Eq. (16)). Note that scaling in the weak field region (which was experimentally established) is less pronounced than that in the strong field region.

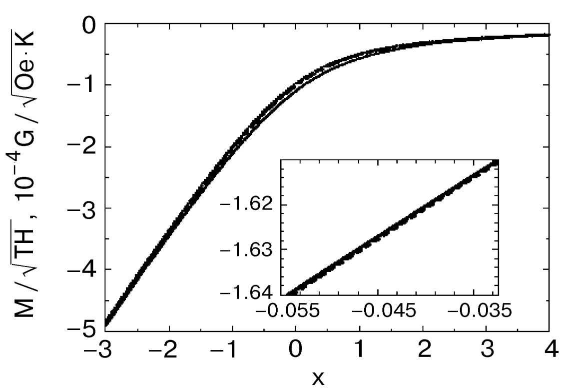

Fig. 1. The quantity $M/\sqrt{HT}$ as a function of scaling variable x. Dashed, dotted and dot-dashed lines correspond to $\mu_0 H = 3$, 4 and 5 T (main figure) and correspond to $\mu_0 H = 0.02$, 0.1 and 0.2 T (insetion). The solid line corresponds to the clean-sample scaling function (strong field region only).

Apparently, the reason is that the experimental data are fitted to account for the clean sample scaling function. Nevertheless if we identify the fitted temperature 82.6 K (found in Ref. 12 in the weak field region) with the renormalized critical temperature $T_c = T_{c0} - \delta T_c$, and the fitted critical temperature 84.2 K in the strong field region [12] with T_{c0} , then, even within such a rough approximation, we obtain $\theta_1 \approx 0.5$. Recalling that θ_1 should be positive and less than unity, the above result strongly supports the applicability of our theory to the pertinent experiment [12].

4. Summary

In summary, we calculated the magnetization of an irradiated superconductor below the meanfield transition line $H_{c2}(T)$, using the approach developed in Refs. 8–10, 15. It was shown that, from a rigorous point of view, disordered short-range defects are expected to destroy the scaling behavior and prevent the existence of crossing point in both regions of weak and strong magnetic fields (with respect to matching field H_Φ). And yet, within the framework of the parameters which were employed in the experimental work [12] the deviation from scaling behavior appears to be negligibly small and crossing points exist in both field regions, in complete agreement with the experimental findings. The two fitting critical temperatures introduced in Ref. 12 for the strong and weak field regions correspond, in our formalism, to the initial and renormalized critical temperatures.

This paper is devoted to the memory of Lev Shubnikov, whose outstanding contributions to many branches of low temperature physics have shaped its development ever since.

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